22.38 PS #3 Solutions

2.40) $P(1H)\,P(F/H) = .3\,(.25) = .075$

$P(2H)\,P(F|H) \times 2 = 2(.25)(.05) = .025$

$P(F_{snow}) = .1$

$P(\text{Flood}) = .075 + .025 + .1 = \boxed{.2}$ $\Rightarrow$ 20% chance of flood per year.

2.43) $P(\text{detect diseased tree}) = P(A) + P(B\bar{A}) = .8 + P(B/\bar{A})P(\bar{A})$

a) B & A are independent $\therefore$

$P(\text{detect}) = .8 + (.9)(.2) = \boxed{.98}$

b) $P(\text{detected by 1 device}) = P(A\bar{B}) + P(B\bar{A})$

$= (.8)(.1) + (.9)(.2) = \boxed{.26}$

c) $P(\text{locating diseased tree}) = P(A_D)P(A_L)P(\bar{B}) + P(B_D)P(B_L)P(\bar{A}) + P(AB)$

where $P(X_D)$ = detector X detected it, $P(X_L)$ = detector X located it

$P(\text{locating}) = .8(.7)(.1) + (.2)(.9)(.1) + .8(.9) = \underline{.848}$

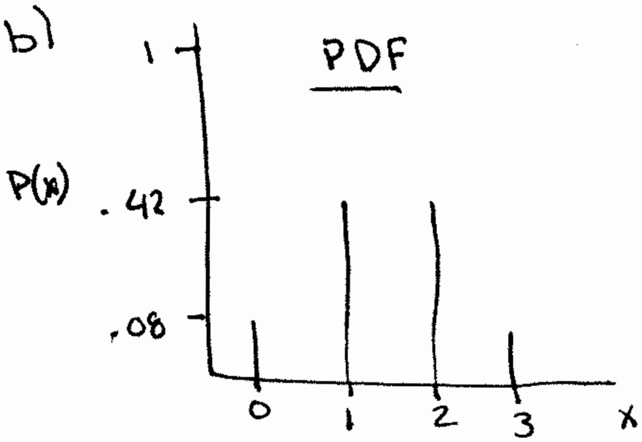
3.1) a) $X=0$: $P(\overline{ABC}) = (1-.05)(1-.8)(1-.2) = \underline{0.08}$

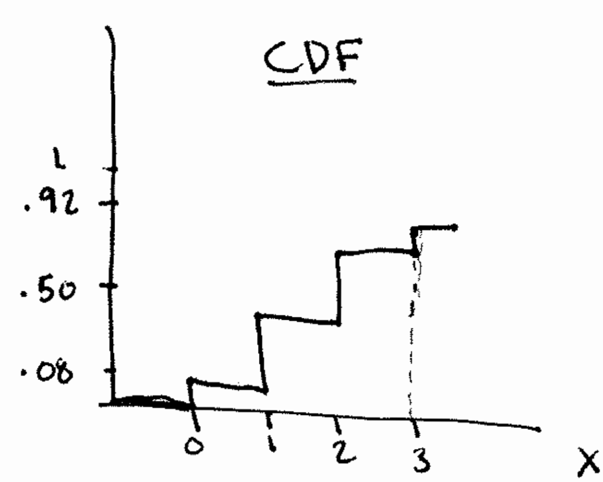
$X=1$: $P(A\overline{BC}) + P(\bar{A}\cdot B\cdot\bar{C}) + P(\overline{AB}\cdot C) = \underline{0.42}$

$X=2$: $P(AB\bar{C}) + P(A\bar{B}C) + P(\bar{A}BC) = \underline{0.42}$

$X=3$: $P(ABC) = (.5)(.8)(.2) = \underline{0.08}$

b)





3.1 continue) c) $P(X \le 2) = .42 + .42 + .08 = .92$

d) $P(0 \le X \le 2) = .42 + .42 = .84$

3.12) a) i) mode : $\tilde{X} = 300$ vehicles

ii) mean : $\langle x \rangle = \int_{-\infty}^{\infty} x f(x) dx = \int_{0}^{300} x f_1(x) dx + \int_{300}^{400} x f_2(x) dx$

where $f_1(x) = \frac{P}{300} X$; $f_2(x) = \frac{-P}{100} X + 4p$

where $\frac{1}{2} p(300) + \frac{1}{2}(p)(100) = 1$ due to normalization $\Rightarrow p = \frac{1}{200}$

$\langle x \rangle = \int_0^{300} (1.67 \times 10^{-5}) x^2 + \int_{300}^{400} -(5 \times 10^{-5}) x^2 + .02x \; dx$

$= 150 - 617 + 700 = 233$ cars

iii) median ; $F_x(X_{50}) = \int_0^{X_{50}} f_1(x) dx$ (note, this is from the fact that by inspection we know the median is in region 1)

$F_x(X_{50}) = .5 = \frac{1.67 \times 10^{-5}}{2} (X_{50}^2)$

$\Rightarrow X_{50} = 245$ cars

iv) $F_x(X_{90}) = .9 = \int_0^{300} f_1(x) dx + \int_{300}^{x} f_2(x) dx$

$\Rightarrow -2.5 \times 10^{-5} x^2 + 2 \cdot 10^{-2} x - 3.9 = 0$

$X_{90} = 337$ cars

associated probabilites of exceedance $= \int_x^{400} f(x) dx$

i) $\tilde{X} = 300$; $P(X > \tilde{x}) = 0.25$

ii) $\langle x \rangle = 233$; $P(X > \mu) = 0.546$

iii) $X_{50} = 245$; by definition $P(X > X_{50}) = .5$

iv) $X_{90} = 337$; by definition $P(X > X_{90}) = .1$

b) $P(>300) = \frac{1}{2}(100)\left(\frac{1}{200}\right) = .25$

$P(>350) = \frac{1}{2}(50)\left(\frac{-350}{100 \cdot 200} + \frac{1}{50}\right) = .0625$

$P(\text{exceed}) = .2(.25) + .8(.0625) = .1 \Rightarrow 10\%$

3.14) a) $\langle X \rangle = \sum_{i=1}^{4} X_i P_X(X_i) = 100{,}000\,[5(.5) + 6(0.3) + 7(.1) + 7(.1)]$

$= \underline{\$570{,}000}$

b) $\sigma^2 = \mathrm{Var}(X) = 100{,}000\,[(5-5.7)^2(.5) + (6-5.7)^2(.3) + 2(7-5.7)^2(.1)]$

$= \cancel{\$78{,}0}\ \ 6.1 \times 10^9$

$\sigma = \sqrt{\sigma^2} = \sqrt{6.1 \times 10^9} = \underline{\$78{,}000}$

3.19) Known info: $\mu = 30$; $F(40) = .9 \Rightarrow X_{90} = 40$

a) $S = \frac{X-\mu}{\sigma} \Rightarrow .9 = \Phi\left(\frac{40-\mu}{\sigma}\right) - \Phi(-\infty)$

$\Rightarrow 1.285 = \frac{40-\mu}{\sigma} \Rightarrow \sigma = \frac{10}{1.285} = \underline{7.78 \text{ days}}$

$P(X \le 50) = \Phi\left(\frac{50-30}{7.78}\right) = \Phi(2.57) = \underline{0.9949}$

b) $P(X \le 0) = \Phi\left(\frac{0-30}{7.78}\right) = \Phi(-3.856) = 1 - \Phi(3.856) = 1 - .999942 = \underline{5.8 \times 10^{-5}}$

this probability is sufficently small that a normal distribution can be considered a reasonable approximation.

c) $S = \frac{\ln x - \lambda}{\xi}$; $\xi^2 = \ln(1 + \sigma^2/\mu^2)$; $\lambda = \ln\mu - \frac{1}{2}\xi^2$

$\xi^2 = \ln\left(1 + \frac{7.78^2}{30^2}\right) = 6.5 \times 10^{-2} \Rightarrow \underline{\xi = .255}$; $\underline{\lambda = 3.369}$

$S = \frac{\ln(50) - 3.369}{.255} = 2.13 \Rightarrow \Phi(2.13) = \underline{.983414 = P(X \le 50)}$

3.22) ~~Lognormal distribution:~~ $\mu = 30$; $COV = \frac{\sigma}{\mu} = \frac{\sqrt{36}}{30} = 0.2$

$\xi^2 = \ln(1 + \sigma^2/\mu^2) = \ln(1 + 36/30^2) = 0.0392 \Rightarrow \underline{\xi = .198042}$

$\lambda = \ln\mu - \frac{1}{2}\xi^2 = \underline{3.38159}$

a) $P(40 = \text{adequate}) = P(H \ge 39) = 1 - \Phi\left(\frac{\ln 39 - \lambda}{\xi}\right) = 1 - \Phi(1.424) = 1 - .92 \approx \underline{0.08}$

b) $P(44 \ge H > 39) = \Phi\left(\frac{\ln 44 - \lambda}{\xi}\right) - \Phi\left(\frac{\ln 39 - \lambda}{\xi}\right) = .058$

$P(H \le 44 \mid H > 39) = \frac{P(44 \ge H > 39)}{P(H > 39)} = \frac{.058}{\Phi\left(\frac{\ln 39 - \lambda}{\xi}\right)} = \frac{.058}{.92} = \underline{0.72}$

3.26) Note: there are several ways to do this problem;

1) plot the table and fit a curve to obtain a continuous distribution then do the calculations using numerical methods to integrate.

2) plot the table and notice that it closely resembles a normal distribution, and use the properties of a normal distribution (calculated μ and σ) to do the probability calculations. (can ~~also~~ use binomial dist. once you have P_{fail})

3) ~~find the fail~~ define a "failure" criterion and then find the probability of failure from the experimental data and ~~apply~~ treat them as Bernoulli trials using the binomial distribution to calculate the probabilities.

4) define a "failure" criterion and then find the failure rate from the experimental data and apply the Poission distribution to find the probabilites.

In these solutions I will use ~~both~~ method ~~3 & 4~~ 3 to solve the problem:

a) $\dfrac{\#\text{ occurances } x > 80}{20 \text{ years}} \Rightarrow p = \dfrac{3}{20} = \underline{0.15}$ $\Rightarrow$ [would also be λ if choose method 4]

b) in the next 10 years,

$P(X=3) = \binom{10}{3}(0.15)^3(1-0.15)^7 = \underline{0.1298}$ $\left[\dfrac{n!}{X!\,(n-X)!}\, p^{x}(1-p)^{n-x}\right] \Rightarrow$ is Binomial

c) $P(X \geq 1 \text{ for 3 years life}) = 1-(X=0) = 1-\binom{3}{0}(1-.15)^3 = \underline{0.3859}$

d) if "failure" is $v = 85$ kph, then $p = \frac{1}{20} = .05$

$P(\geq 1 \text{ for 3 years}) = 1-(X=0) = 1-\binom{3}{0}(1-.05)^3 = \underline{0.1426}$