SNOWFALL IN BOSTON

Snowfall Accumulation During Storm: 13:00, 3/31/99 – 14:00, 4/1/99

The storm lasted for 25 hours.

Assume that the snowfall yield is Poisson distributed, with frequency parameter, λ.

Then the vote of the class can be intepreted as saying that the expectation of the storm's snow accumulation, $\langle x \rangle = \lambda t$, ranges over the interval [1, 30], with the distribution shown in Fig. 1.

Forecast of class for snow accumulation, x:

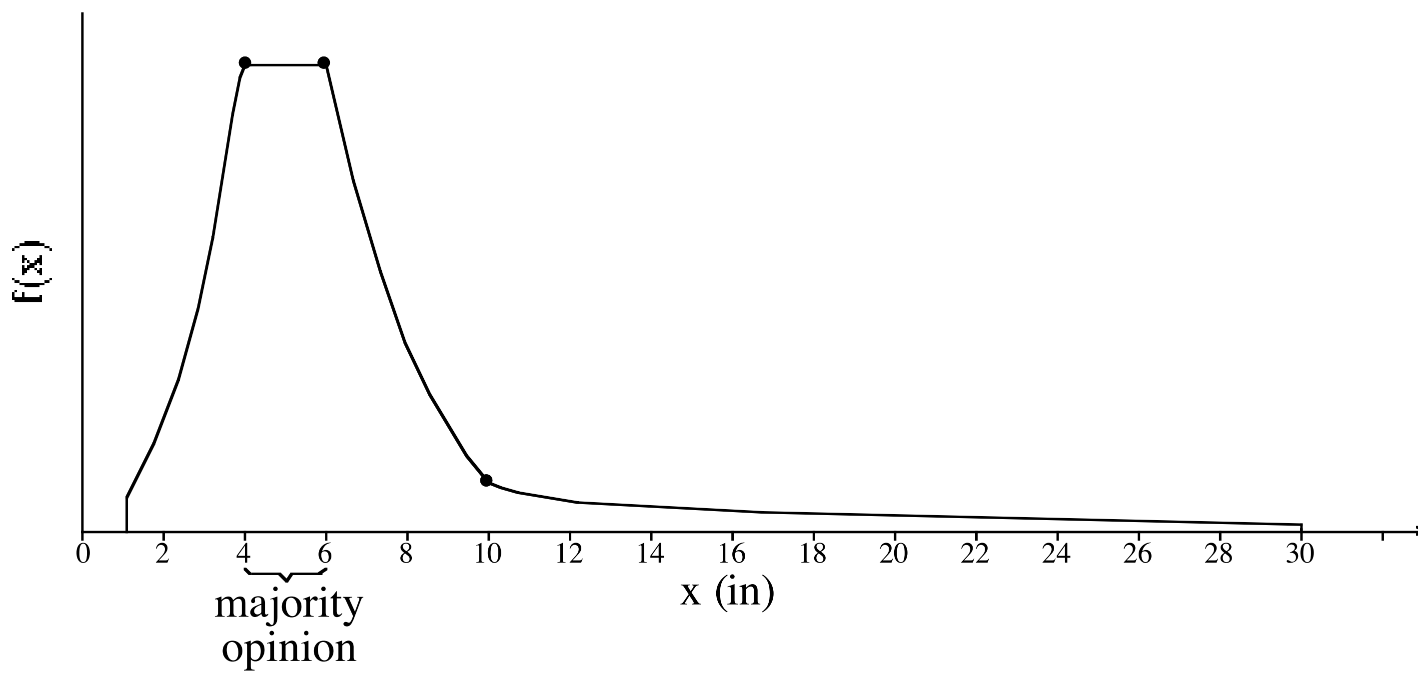


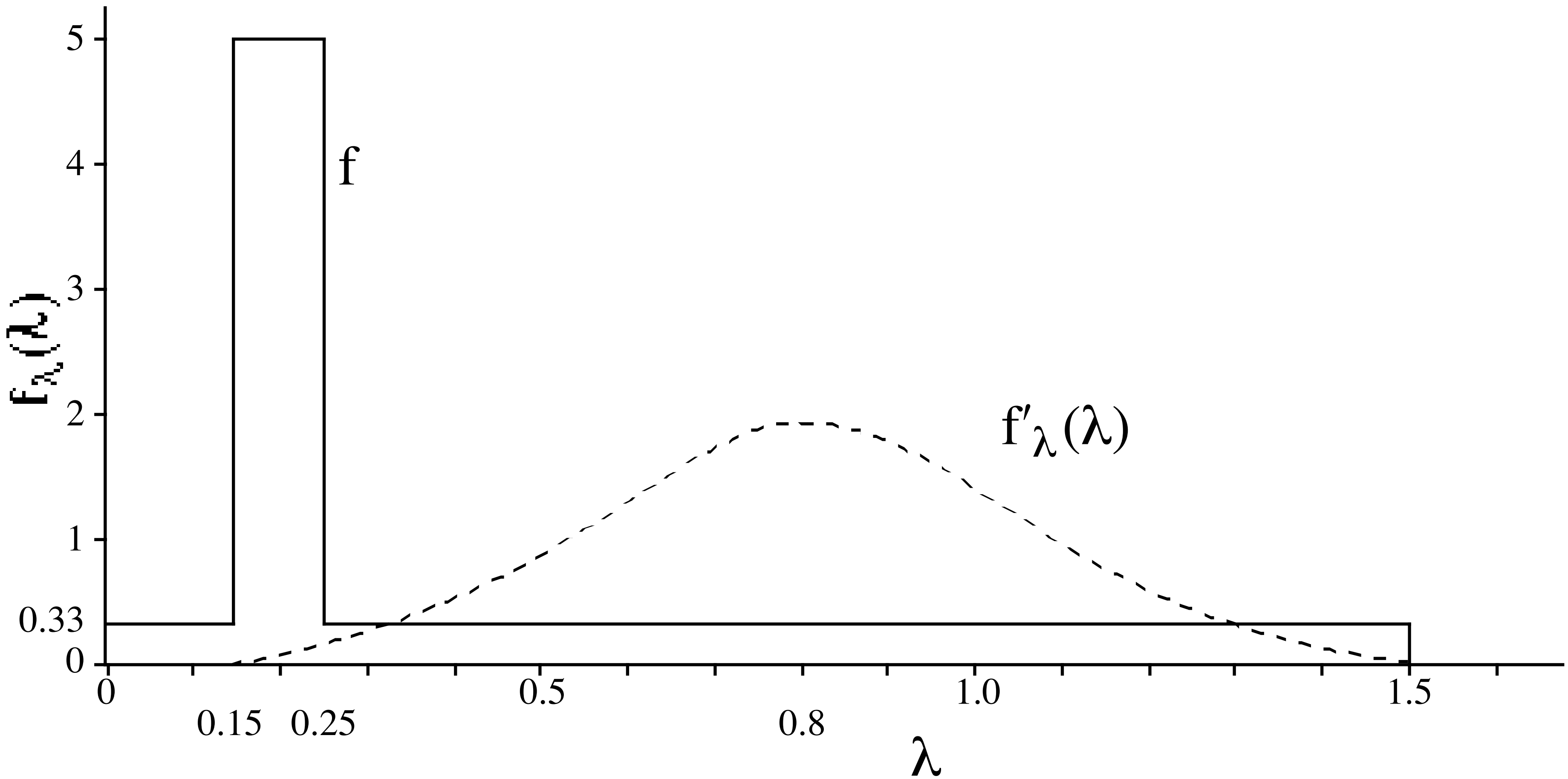
Fig. 1. Distribution of Expected Snow Accumulation

$$P_i(x) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}\ , \quad x = \text{integer}\ .$$

From the vote of the class, most likely accumulations are in the range [4, 6] in.

Example: $\langle x \rangle = 5 \Rightarrow \lambda = 5$ in/25 hr = 0.20 in/hr.

Fig. 2 assumed prior distribution of λ.



For $\lambda = 0.2$ (in/hr)

x	P(x)
0	0.25
1	0.034
2	0.084
3	0.140
4	0.175
5	0.175
6	0.146
7	0.104
8	0.065
9	0.036
10	0.018
15	1.6E-4
20	2.6E-7
25	1.3E10
30	2.1E-12

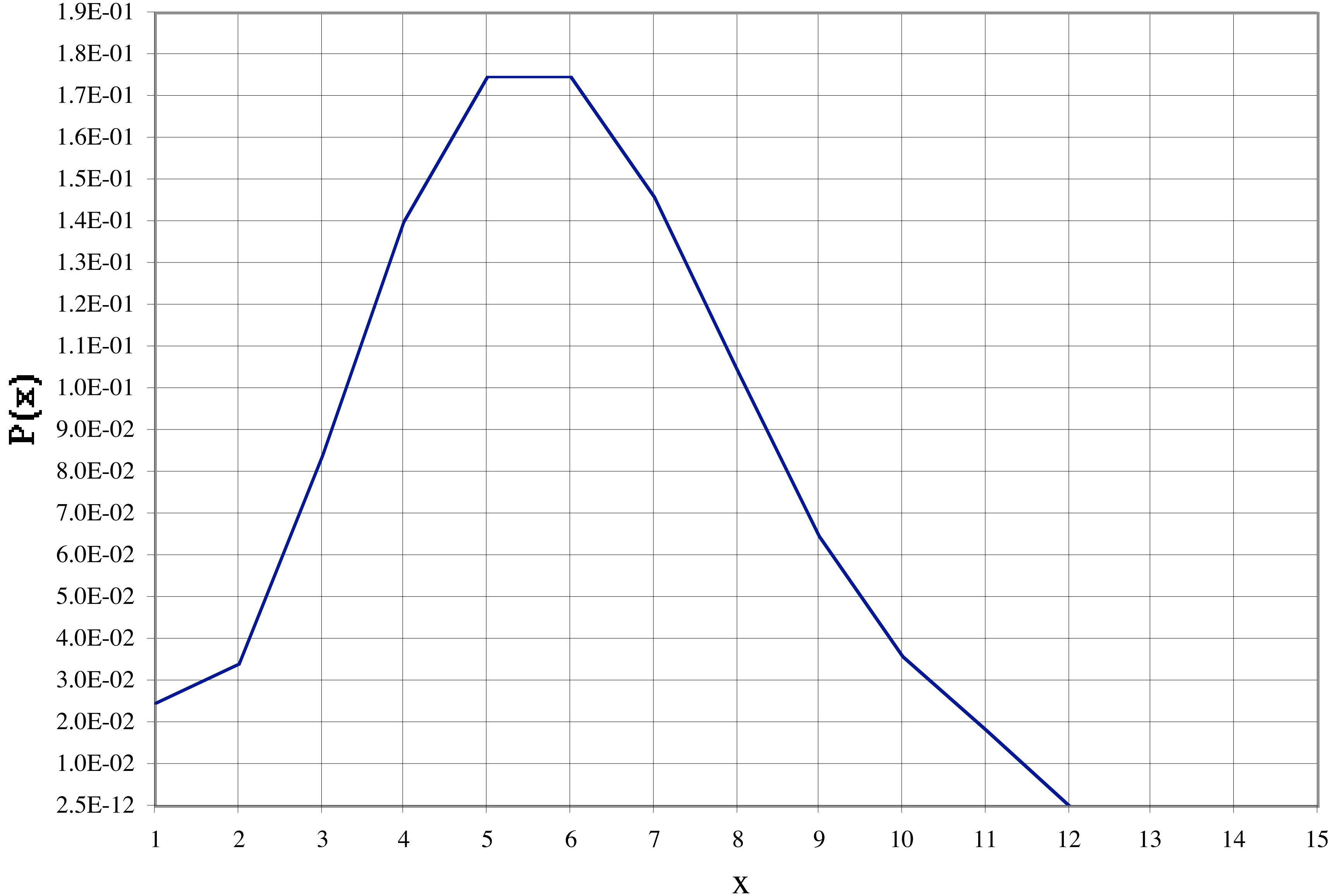


Fig. 3. Poisson distribution for x, when $\lambda = 0.2$ in/hr.

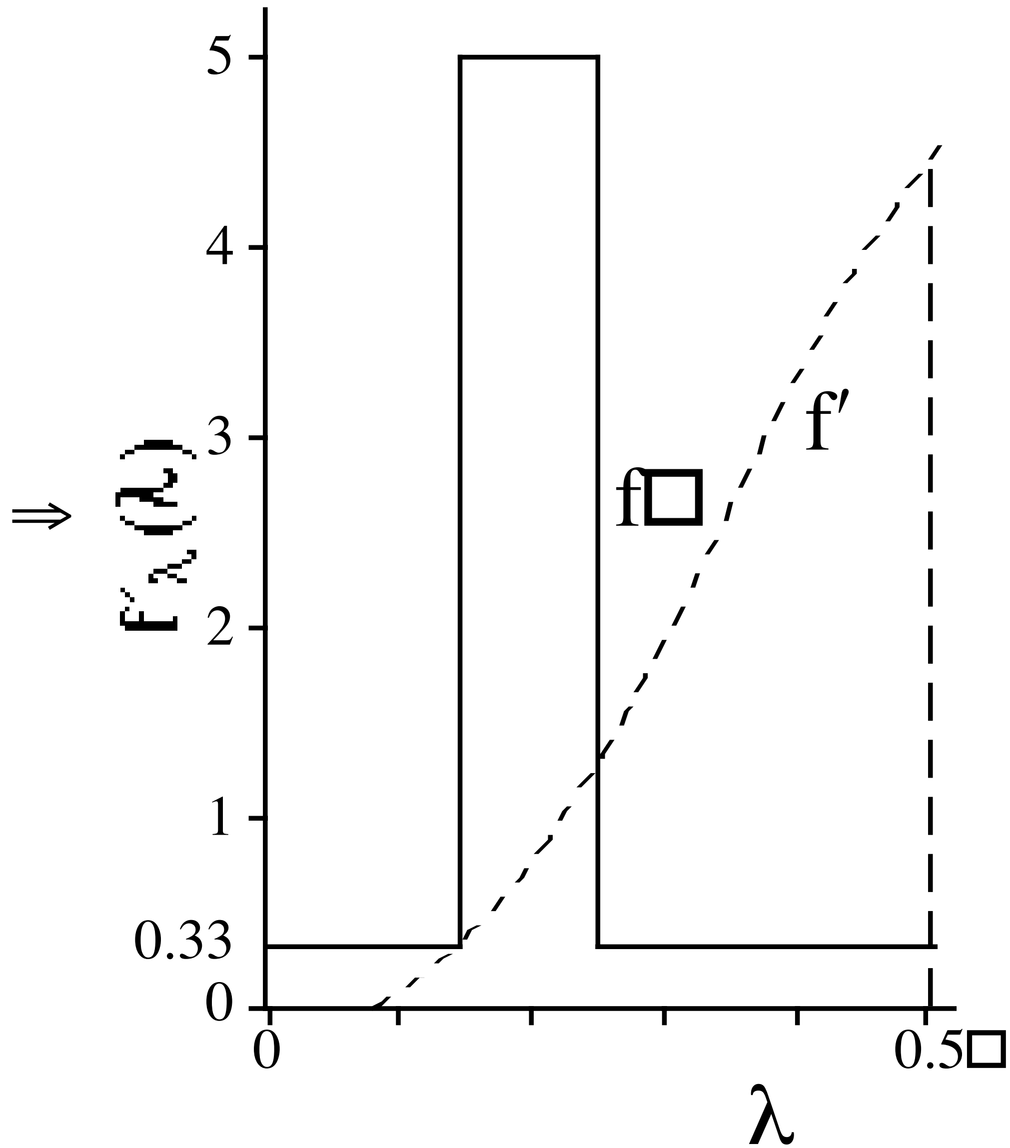
$$P(\lambda|E) = \frac{P(E|\lambda)P(\lambda)}{P(E)}$$

$$P(\lambda|E) = \frac{\left[e^{-\lambda t}(\lambda t)^{x}\right]\left[f_{\lambda}(\lambda)d\lambda\right]}{\int_{\lambda_{min}}^{\lambda_{max}} d\lambda f_{\lambda}(\lambda)\left[\frac{e^{-\lambda t}(\lambda t)^{x}}{x!}\right]} = f'(\lambda|E)$$

Here the evidence is x = 20 in, t = 25 hr.

$$f_{\lambda}'(\lambda) = f_{\lambda}(\lambda)\frac{\frac{\left[e^{-(25\lambda)}(25\lambda)^{20}\right]}{20!}}{\int_{0}^{1.5} d\lambda f_{\lambda}(\lambda)\left[\frac{\left[e^{-(25\lambda)}(25\lambda)^{20}\right]}{20!}\right]}.$$

What if the range of the prior distribution had been too short?



TYPES OF UNCERTAINTY

Aleatory, due to the variation in outcomes of random trials. Is from "Alea jacta est," or "the die are cast." Attributed to Julius Ceasar upon crossing the Rubicon, thereby invading the territory of Rome when he siezed power as dictator.

Epistimic, due to uncertainty in our state of knowledge about phenomena and data. Is from Greek term for knowledge, "epistemikos." Is the root of epistimic and epistemological.

Consider events

$\Theta = \theta_i$, $E = e_j$.

$$P\left[\Theta = \theta_i, E = e_j\right] = P\left(\Theta = \theta_i \middle| E = e_j\right) P\left(E = e_j\right) = P\left(E = e_j \middle| \Theta = \theta_i\right) P\left(\Theta = \theta_i\right)$$

$$\Rightarrow P\left(\Theta = \theta_i \middle| E = e_j\right) = \frac{P\left(E = e_j \middle| \Theta = \theta_i\right) P\left(\Theta = \theta_i\right)}{P\left(E = e_j\right)}$$

OR

$$L'\left(\Theta = \theta_i \middle| E = e_j\right) = \frac{P\left(E = e_j \middle| \Theta = \theta_i\right) L\left(\Theta = \theta_i\right)}{P\left(E = e_j\right)}$$

Bayes' Theorem

Now $E = e_j$ is possible for various values of θ_i, as

$$P\left[\left(E = e_j\right)\right] = \sum_{i=1}^{n} P\left(E = e_j \middle| \Theta = \theta_i\right) P\left(\Theta = \theta_i\right)$$

$$\boxed{L'\left(\Theta = \theta_i \middle| E = e_j\right) = \frac{P\left(E = e_j \middle| \Theta = \theta_i\right) L\left(\Theta = \theta_i\right)}{\sum_{i=1} P\left(E = e_j \middle| \Theta = \theta_i\right) P\left(\Theta = \theta_i\right)}}$$

Let the evidence, E, following a random trial be equal to e_i. We wish to know the change in our knowledge of P(Θ) due to new evidence, $P(\Theta_i = \theta_i | E = e)$.

Consider, prior to the trial, that Θ may take several values, as $\Theta = [\theta_1, \theta_2, ..., \theta_n]$. Also consider that the outcome of a random trial, E, is governed by a "model of the world," where Θ is a model parameter that takes a specific value, Θ_i.

BAYESIAN UPDATING

Which value of Θ actually obtains is unknown, but we (an observer) will have a "model of belief" regarding the likelihood of the alternative possible values, $L(\Theta)$, where

$$L(\Theta = \theta_i) = \Pr(\Theta = \theta_i).$$

Let the alternative possible outcomes from the random trial be designated $E = [e_1, e_2, ..., e_k]$.

Let the specific outcome of the next random trial be e_i, i.e., $E = e_i$ is the "evidence" of the next trial.

Then, we wish to know the effect of this result upon our "model of belief," concerning the likelihood of the possible values of Θ, $L'(\Theta)$.

USE OF EXPERTS & BAYES' THEOREM

Experts can be used either to provide

1. Model of belief (e.g., our subjectively obtained distribution of alternative causes of Egypt Air plane crash).
2. Model of the world ⇒ assignment of most likely alternatives or outcome (i.e., provides evidence as his/her expert opinion as if the evidence were resulting from a random trial).

Former Case: New evidence can be used to update a subjective model of belief just as with any other model of belief.

Latter Case: $P(\Theta = \theta_i | E = e_j)$ is posterior probability distribution based upon evidence obtained from an expert.

$$= \frac{P(E = e_j | \Theta = \theta_i) P(\Theta = \theta_i)}{P(E = e_j)}, \text{ where}$$

$P(E = e_i | \Theta = \theta_j)$ is observer's judgment of the probability that expert will give evidence, e_j, when observer believes that $\Theta = \theta_j$.