Massachusetts Institute of Technology

Subject 6.651J/8.613J/22.611J R. Parker
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Problem Set 1

1. The three-dimensional Maxwellian velocity distribution function is

$$f_m(\vec{v}) = \frac{n}{(v_t\sqrt{\pi})^3}\exp\left(\frac{-v^2}{v_t^2}\right)$$

where n is the particle density and $v_t^2 = \frac{2kT}{m}$. Determine the following:

i) the average of each velocity component, e.g., $\langle v_x \rangle$;

ii) the average speed $\langle v \rangle$;

iii) the rms speed $\sqrt{\langle v^2 \rangle}$;

iv) the average kinetic energy in any one direction, e.g., $\left\langle \frac{1}{2} m v_x^2 \right\rangle$; and

v) the flux crossing a planar surface in one direction.

2. In class we worked out the potential associated with a sheet of charge immersed in a plasma with electron temperature T_e and ion temperature T_i and obtained the result

$$\varphi = \frac{\sigma\lambda_D}{2\varepsilon_0}\exp\left(-\frac{|x|}{\lambda_D}\right)$$

where σ is the charge density, x is the coordinate perpendicular to the charge sheet and λ_D is the Debye length. A key assumption in our analysis was $\frac{e|\varphi|}{kT_{e,i}} << 1$ so that the exponentials appearing in Poisson's equation could be well represented by the first two terms in a Taylor series. The purpose of this problem is to remove this restriction for a plasma in which $T_e = T_i = T$.

i) Use the quadrature method discussed in class to solve Poisson's equation (without approximations) for $\frac{d\varphi}{dx}$ and then integrate to get $x(\varphi)$ in terms of V, the potential of the charge sheet.

ii) What is V in terms of σ?

iii) Let x_s be the distance from the sheet where the potential has fallen to Ve^{-3}. Sketch x_s vs. $\frac{eV}{kT}$.

How does it compare with the result of the calculation for the case $\frac{e|\varphi|}{kT_{e,i}} << 1$?

3. A spherical ball of plasma consists of *cold* $(T_e = T_i = 0)$ electrons and singly charged ions with uniform density, $n_e = n_i = n_0$. The sphere of electrons is perturbed a distance $\vec{\delta}$ from its equilibrium position, i.e., where the electron sphere would coincide with that of the ions. Find the frequency at which the electron sphere oscillates. (Note: one particular form of a nearly spherical plasma occurs naturally and is known is known as "ball lightning".)

4. Consider the parameters of the plasmas listed in the table below, taken from notes for this subject prepared by Prof. A. Bers. Complete the table by calculating the Debye length λ_D, the plasma parameter $\Lambda = n\lambda_D^3$ and the plasma frequency f_p. Which plasmas are strongly coupled? Which need a quantum mechanical treatment?

Plasmas	$Log_{10}ne$	$Log_{10}Te$	λ_D(m)	Λ	f_p(Hz)	Strongly coupled?	QM?
Weakly ionized							
Ionosphere, D layer 70 km	9	2.5					
Gas discharge, weak current	17	4					
Gas discharge, strong current	21	5					
MHD energy convertor	22	3					
Strongly ionized							
Interstellar gas	0	3.5					
Solar wind	6.5	5					
Ionosphere, F_2 layer 250 km	11.5	3					
Solar corona (R_O ~1.5)	13	6.5					
Tokamaks	20	8					
Alkali plasmas – surface ionization	18	3					
Laser plasmas	25	5					
Nuclear explosions	26	6					
Magnetosphere of pulsars	18	16					
Dense plasmas							
Electrons in metals	29	2.5					
Interior of stars	33	7.5					
Interior of white dwarfs	38	7					

Note: T_e in °K, n_e in m^{-3}.