

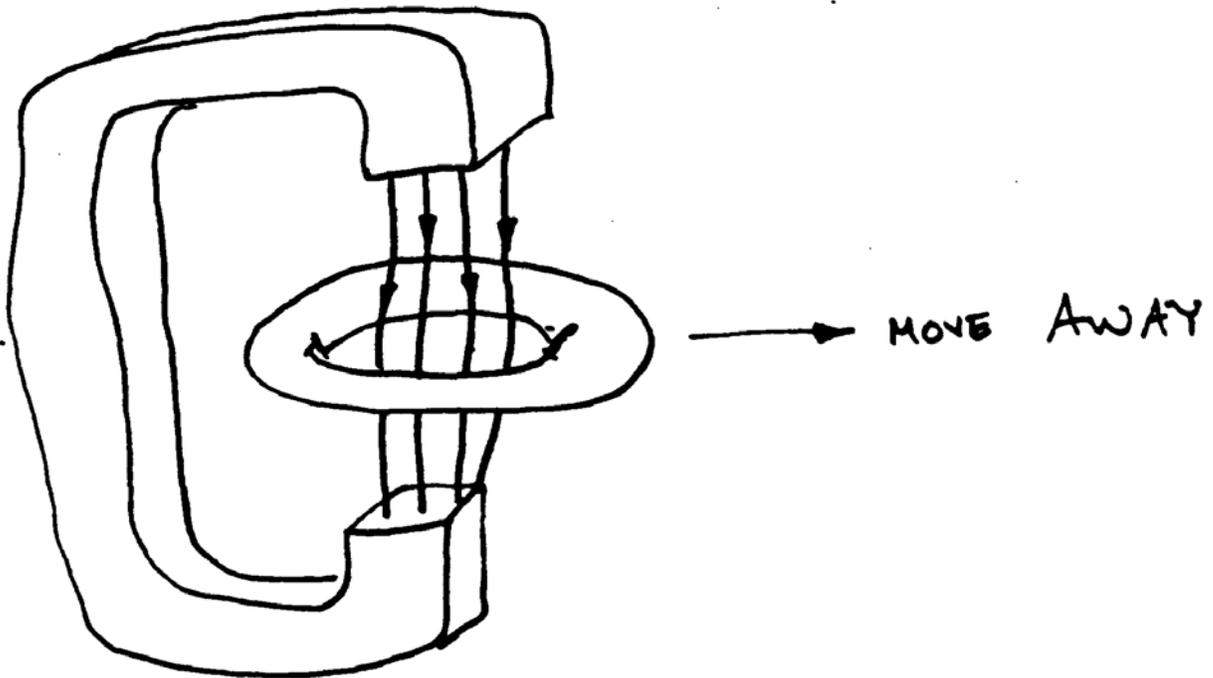
Fall Term 2002
Introduction to Plasma Physics I

22.611J, 6.651J, 8.613J

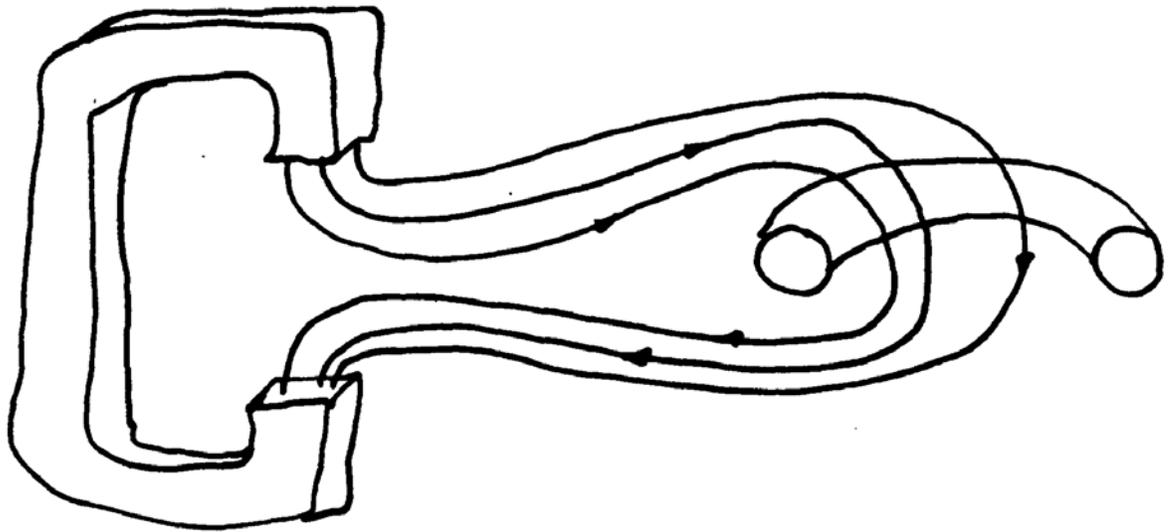
Problem Set #2

1. Field Line Topology Paradox

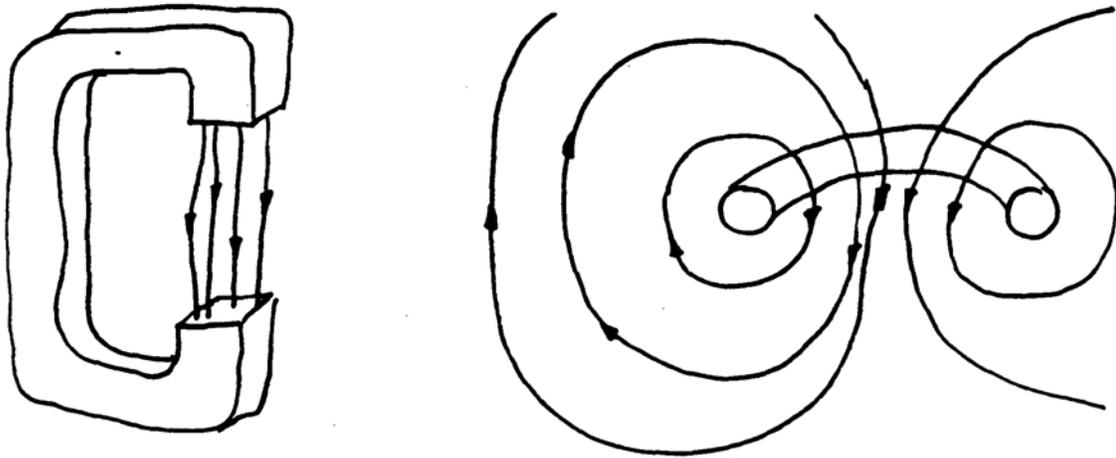
Consider the magnetic field of a permanent “C” magnet, whose field threads an Aluminum torus as shown in the drawing below. The torus is then cooled to $1\text{ }^\circ\text{K}$ and becomes a superconductor. The problem is to determine what happens to the magnetic field as the torus is pulled away a large distance to the right.



There are two arguments one can make that lead to very different pictures for the magnetic lines. First, following from the property that magnetic flux cannot pass through a perfect conductor, one might conclude that the lines get stretched to the right indefinitely and arrive at the picture below.



Second, one could observe that as the torus is pulled away a large distance the field threading it will be determined predominantly by the current that has been induced in the torus. Since the flux through the torus must be held constant this current must approach a constant at large distances, and will generate a dipole field. Then the field between the poles of the magnet will be perturbed only slightly and we arrive at the following picture.



Which picture is correct? If you endorse case 1 you must explain how the large currents in the torus required to modify the permanent magnet field, from such a distance are produced. If you endorse case 2 you must explain how the lines that initially threaded inside torus succeeded in getting outside it without breaking, forming ends, and violating $\nabla \cdot \mathbf{B} = 0$.

2. Polarization Drift

Since the $\mathbf{E} \times \mathbf{B}$ drift is perpendicular to the electric field, \mathbf{E}_\perp , a particle subject to this drift cannot change its energy. However, if the electric field changes slowly in time, the energy associated with the drift motion, $\frac{1}{2}m\mathbf{v}_E^2$, must change. Show that an additional drift must be present if the work done by the electric field on the particle is to account for the energy change, and that for small $\partial \ln E / \partial t$ the drift is given by,

$$\mathbf{v}_{pol} = \frac{c}{\Omega B} \frac{d\mathbf{E}_\perp}{dt}$$

3. Fermi Acceleration of Cosmic Rays

A cosmic ray proton is trapped between two moving mirrors with mirror ratio, $R = 5$, and initially has energy 1 KeV and, $v_\perp = v_\parallel$, at the midplane. The mirrors are separated by a distance L and move toward the midplane with a velocity $v_M = 10 \text{ km/sec}$.

- (a) Using the loss cone formula and the invariance of the magnetic moment, μ , find the energy to which the proton will be accelerated before it escapes.
- (b) How long will it take to reach that energy?

Notes:

- i. Treat the mirrors as flat pistons and assume the time spent bouncing is negligible compared to the time spent traversing the stationary field between the mirrors.
- ii. You may assume that the distance, L , changes negligibly during the acceleration.
- iii. Compute the number of bounces and time (to within a factor of two) to traverse L that many times.

4. Numbers for Drifts

Evaluate the magnitude of the, $E \times B$, grad-B, and curvature drifts for the ions in a typical tokamak plasma. Write the dimensionless ratio of these quantities relative to the ion thermal velocity. Estimate quantities from the outer midplane of the tokamak taking parameters from the MIT Alcator C-mod experiment:

$$\begin{aligned} R &= .66 \text{ m} \\ a &= .21 \text{ m} \\ B &\sim B_T = 4.5 \text{ T} \\ n &= 4 \times 10^{14} \text{ cm}^{-3} \\ T &= 2 \text{ KeV} \\ E_r &= 1 \text{ KV/cm} \end{aligned}$$

You may use the low beta assumption in computing magnetic field gradients. Note that the electric field here is radial. *Apologia:* The units are mixed as is typical in practise, so you will have to unravel the units. . . some helpful conversions are: $1 \text{ StatVolt (cgs)} = 300 \text{ Volts (si)}$, $1 \text{ Tesla (si)} = 10,000 \text{ Gauss (cgs)}$. Recall that in cgs units the electric and magnetic fields have the same dimensions, so that, $\frac{1 \text{ StatVolt/cm}}{1 \text{ Gauss}} = 1$.