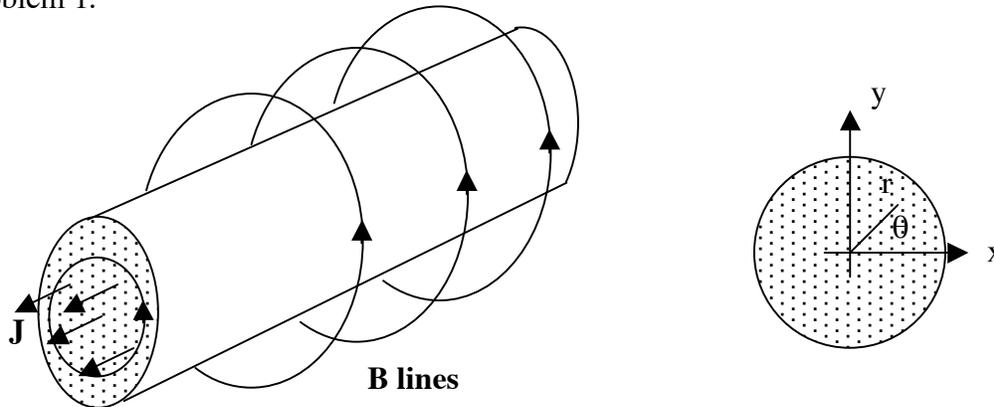


Problem Set 5

Reminder: The Mid-term quiz will be on Tuesday, October 31, 2006. It will be 1.5 hours in length and closed book, but you may bring one 8.5"x11" paper of notes and formulas.

Problem 1.



The figure depicts a long z-pinch with current density $\vec{J} = \hat{z}J(r)$, magnetic field $\vec{B} = \hat{\theta}B(r)$, and pressure $p = p(r)$. There is no flow and the resistivity is assumed to be zero. Also $n_e = n_i, p_e = p_i$ and $Z_i = 1$.

- What is the relation between p, B and J required to assure MHD equilibrium?
- Calculate the current density \vec{J}_D associated with the electron and ion drifts.
- The relation between J and \vec{J}_D in parts a) and b) is not at all obvious and in fact they are not the same. The missing link is the so-called diamagnetic current which is given by

$$\vec{J}_M = -\nabla \times \vec{M}$$

where the magnetization is $\vec{M} = \frac{nw_{\perp}}{B} \hat{b} = \frac{p}{B} \hat{b}$. Compute $\vec{J}_M + \vec{J}_D$ and compare with \vec{J} as determined in part a).

Note: In cylindrical coordinates: $\nabla \times \vec{A} = \hat{r} \left(\frac{\partial A_z}{r \partial \theta} - \frac{\partial A_{\theta}}{\partial z} \right) + \hat{\theta} \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{z} \left(\frac{\partial A_{\theta}}{\partial r} + \frac{A_{\theta}}{r} - \frac{\partial A_r}{r \partial \theta} \right)$

Problem 2.

A 10 turn coil is wound around a plasma column (an axi-symmetric screw pinch with $\partial / \partial \theta, \partial / \partial z = 0$) and is connected to an ideal integrator. The output voltage from the integrator is given by

$$v_o = \frac{1}{\tau} \int_{-\infty}^t v_{coil} dt$$

where τ is the time constant of the integrator and v_{coil} is the voltage across the coil terminals. The plasma was initiated sometime in the past and the output voltage is steady at π Volts. The total plasma current is 1 MA and the external field is 1 T.

a) Calculate the total plasma energy per meter stored in the plasma column, assuming the plasma is a) diamagnetic and b) paramagnetic. Take $\tau = 0.1$ s and further assume that the perturbation in the axial field due to the plasma is small compared to the external field.

b) If the density is constant across the plasma cross-section and equal to 10^{20} m⁻³, and the cross-sectional area of the column is 0.25 m², what is the area-averaged plasma temperature in the two cases in part a)?