1. Flux Flow and Flux Flow Resistivity

The phenomena of flux motion and flux flow resistivity are really the heart of the dissipative mechanism in superconductors. In order to understand the phenomena it must be shown that the flux vortices do, indeed move and if they move, the action is dissipative. This can be shown by considering the case of an unpinned ideal type II superconductor carrying a transport current in a uniform background field as shown in, Fig. 2.14. The transport current interacts with the flux quanta to exert a Lorentz force on each flux vortex of the form

$$\boldsymbol{f_L = J_t \times \phi_o}.$$

In the above expression f_L is the force per unit length on the flux vortex, J_t is the transport current density and ϕ_o is the quantum of flux in the vortex. The moving flux vortices will induce an electric field E, given by

$$\boldsymbol{E = v_L \times B = v_L \times n\phi_o} \qquad \textbf{(1)}$$

where v_L, is the velocity of flux lines and n is the number of flux lines per unit area such that $B{=}{\sim}n\phi_o$. Since there is no pinning force to balance the Lorentz force, we might assume that the flux lines accelerate and thus E would increase with time. However, the observed voltage indicates that E is constant in time implying a drag force must exist to balance the Lorentz force. This force balance was proposed by Kim in the form

$$\boldsymbol{\eta v_L = f_L = J_t \times \phi_o} \qquad \textbf{(2)}$$

where η is a viscous damping constant. The damping force is dissipative and thus requires a power input $P= J{\bullet}E$ to maintain the transport current. Equations (1) and (2) can be combined to define the "flux flow resistivity":

$$\rho_f = \frac{\boldsymbol{E}}{\boldsymbol{J}} = \frac{\boldsymbol{B\phi_o}}{\eta} \qquad \textbf{(3)}$$

Experimental measurement of ρ_f determines the viscosity coefficient η.

So far, no mention has .been made of exactly what is the dissipative mechanism due to a moving vortex. Experimental measurements of ρ_f by Kim, Hempstead and Strnad show that it is a function of temperature, and as T goes to 0, it is well correlated by the expression

$$\rho_f = \rho_n \frac{\boldsymbol{H}}{\boldsymbol{H_{c2}}} \qquad \textbf{(4)}$$

where ρ_n is the resistivity of the normal material [-]. The ratio H/H_{c2} is just the fraction of the normal core area. This is a very interesting result because it implies that the dissipation is due to the transport current flowing through the normal cores of the moving flux vortices.

Actually there have been several theories about the dissipative mechanism but one of the more successful and simpler models was derived by Bardeen and Stephen [-]. The model is very simple in that it assumes that all the dissipation takes place by normal resistivity in the central core of the flux vortices where the normal radius is assumed to be equal to ξ • They solve for the electric field distribution due to a flux vortex moving across the specimen and then derive the current distribution and energy dissipation due to this induced electric field. The resulting expression for the flux flow resistivity agrees with the empirically determined expression (Eq. 4) given by Kim.

Other more rigorous analyses of this dissipative mechanism were given by Schmid [-], Carol i and Maki [-], and Hu and Thompson [-] by using time-dependent Ginzburg-Landau theory. A different approach was taken by Clem [-] who derived the dissipation due to the irreversible entropy generation at the trailing and leading edges of the moving flux vortex where the normal-superconducting transitions occur. There is no consensus as to which analysis is correct, if the different mechanisms are additive, or if they are all different ways of calculating the same thing. No matter, the important point is that dissipation does indeed occur whenever the flux vortices are set in motion and the simple empirical form for the flux flow resistivity is useful to describe this effect.

The flux flow resistivity is the underlying dissipative mechanism that is responsible for the ac loss but it is crucial to note that dissipation only occurs when the vortices are set in motion. In the next section it is shown how this flux motion is caused by a changing external magnetic field or transport current. However the flux flow resistivity is not explicitly included in the ac loss calculation.

Now that the dissipative mechanism is quantified a lot of the behavior of type II superconductors can be explained. For instance, an ideal type II superconductor has a very low value of critical current, because, in the absence of pinning sites to restrain the flux motion, energy dissipation increases as the square of the current, thereby excessively heating the sample above T_c and destroying the superconductivity.

By contrast, in a non-ideal type II superconductor, the number and strength of the pinning sites prevent the flux motion until very high values of current cause the Lorentz force to exceed the pinning force. Experimental verification of the flux flow was done by Van Ooijen and Van Gurp [-], who showed that the flux is actually pinned in "bundles" of many flux quanta. Flux motion is also involved in two other interesting phenomena, namely flux creep and flux jumping. These will be discussed in conjunction with the critical state model.

2 The Critical State

The concepts of flux pinning, flux motion and viscous damping have been used to develop the critical state model. Again 1et us consider a non-ideal type II superconductor with pinning. In reality, not every vortex is pinned, just some of them. However, due to the repulsion between fluxons, the vortex lattice is fairly rigid, thereby effectively pinning all the vortices. Thus one can use an average pinning force per unit length of core, f_p . As long as the Lorentz force due to the transport current is less than the pinning force, i.e. $J_t\phi_o < f_p$, the system is stable.

The critical state then is just the state where the Lorentz force is exactly equal to the pinning force and is governed by the equation:

$$J_c \phi_o = f_p \tag{5}$$

where J_c is called the critical current density. If the current exceeds the critical current, flux flow and dissipation occur.

C.P. Bean [-] suggested that J_c = constant, whereas Kim [-] suggested that J_c vary as 1/B , these two being the most widely used forms of the critical state model. Each form gives accurate results for different conditions of magnetic field.

When J is raised above J_c the Lorentz force exceeds the pinning force and the flux vortices are set in motion. Kim, et al. [-] and later Irie and Yamafuji [-], successfully treated this condition by proposing a force balance between the viscous force, the Lorentz force and the pinning force in the form,

$$\eta v_L = f_L - f_p = \phi(J - J_c)$$

The induced electric field is

$$E = \frac{B\phi_o}{\eta}(J - J_c) \tag{6}$$

Since the dissipative mechanism is the same as that just discussed in the previous section, Eq. ( ) can be rewritten in the form

$$E = \rho_f (J - J_c) = \left(\rho_n \frac{H}{H_{c2}} \right)(J - J_c) \tag{7}$$

This equation is very useful in computing the energy loss in a superconducting filament carrying a transport current.

Experimental evidence exists to back up Eq, (7) [-]. Figure 2.15a shows a typical plot of voltage along a specimen versus current. The different curves show the effect of increasing the impurities for pinning sites and thus increasing J_c . In Fig. 2.15b the effect of H in Eq. (7) is shown. The flux flow resistivity is plotted versus applied magnetic field at different temperatures in Fig. 2.15c.

The curves are asymptotic to the line for $T=0$ at low fields. Extrapolation of the measured resistivity data along this line is often done to predict the value of H_{c2} at $T = 0$.

Two other interesting features of the critical state model are phenomena called flux creep and flux jumping. Flux creep is caused by random jumping of flux vortices from one pinning site to another due to thermally induced vibrations. It manifests itself by slow decay of a trapped magnetic field and/or by a measurable resistive voltage. This phenomena has been described by the Anderson-Kim flux creep theory which assumes the vortices move in bundles [-,-]. This creep phenomenon is exceedingly slow resulting in decay times for persistent currents in superconducting loops of many millions of years. However this effect can lead to a potentially more dramatic effect called a flux jump.

The flux jump occurs due to a thermal instability. It is best explained by a series of steps. First assume some flux vortices are set in motion from the pinning site either by flux creep phenomena or due to some other disturbance, such as a change in the external field or transport current or by an increase in temperature. The motion of the vortices dissipates energy which locally increases the temperature. Since the pinning strength is inversely proportional to temperature more vortices can be released from their pinning sites due to the lowered pinning force. This flux motion again leads to more dissipation, higher temperatures, and thus more flux motion in a cascading effect. The total effect is called a flux jump. If the effect is not damped out the result could be a catastrophic flux jump in which the temperature of the sample is rapidly increased above the critical temperature thus creating a normal region which could rapidly propagate and quench the entire winding. Flux creep does not necessarily always lead to flux jumps and flux jumps are not always catastrophic.

Once the underlying physical basis of the phenomena was understood methods of damping this instability were soon devised. These methods are described in the following section on technical superconductors.

3. The Critical State Model Applied to AC Losses

The previous discussion of superconducting phenomena proposed that experimental evidence exists in support of the critical state model for type II superconductors. Acceptance of this model is crucial for it provides a convenient method of computing the loss in a superconductor subjected to a time varying field or transport current. That is, the hysteresis loss in a superconductor can be computed from a succession of quasistatic time steps between equilibrium distributions of magnetic field and current within the specimen. At each equilibrium step the superconductor is in the critical state. The mechanism which allows the field and current distribution to change from one state to the next is the flux flow resistivity.

However, there is no need to determine the flux flow resistivity, ρ_f ,.explicitly in order to calculate the loss. It is sufficient to compute the electric field, E and the current distribution, J from the quasistatic Maxwell equations

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \quad \textbf{and} \quad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}} \tag{8}$$

This condition will hold as long as

$$\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}} << \frac{\boldsymbol{B}_p}{\tau_m}$$

where B_p is the magnetic field required to fully penetrate the filament and τ_m is the time constant for magnetic flux to diffuse through the filament. The diffusive time constant can be computed from the magnetic diffusivity which is given by

$$\boldsymbol{D}_m = \frac{\rho_f}{\mu_o} \tag{9}$$

where ρ_f is the flux flow resistivity. The time constant then is

$$\tau_m = \frac{\boldsymbol{L}^2}{\boldsymbol{D}_m} = \frac{\mu_o \boldsymbol{L}^2}{\rho_f} \tag{10}$$

where the characteristic length, L is equal to the filament radius, d/2. An order-of-magnitude estimate gives values of τ_m of order 10^{-8} sec for a typical NbTi filament diameter of 50μm. As long as the frequency of the external field change does not approach the gigahertz range the quasistatic model should hold. For most practical engineering applications the perturbation field frequency would realistically never exceed ~10^3 hertz, i.e., the kilohertz range.

4. One-Dimensional Flux Penetration

A one-dimensional slab of non-ideal Type II superconductor is often used as a simple example to illustrate the fundamentals of the critical state model. Consider the slab of thickness d in Fig. 3.1a to be of infinite extent in the y and z directions. If a uniform external field, H_e ,oriented parallel to the sides of the slab in the y direction, is increased above H_c , flux vortices will begin to penetrate the specimen. The material will be said to be in the mixed state. The flux will penetrate to the point where the pinning force will just balance the Lorentz force on the flux bundles. This condition is the critical state and the induced current density is called the critical current density, J_c . The field and current distribution will be given by solution of the Maxwell equation $\nabla \times \boldsymbol{H} = \boldsymbol{J}$ with the boundary condition that $H = H_e$ at the surface.

The exact form of the solution depends on whether J_c is considered to be independent of H (Bean model [-]) or J_c is considered to be inversely proportional to H (Kim model [-]). If we choose the former condition the solution is of the form

$$H = H_e + J_c\left(x - d/2\right) \qquad \textbf{(11)}$$

The field penetrates to the point x_p given by

$$x_p = \frac{d}{2} - \frac{H_e}{J_c} \qquad \textbf{(12)}$$

The slab is fully penetrated when $H_e = H_p$ where

$$H_p = \frac{J_c d}{2} \qquad \textbf{(13)}$$

The field and current distributions that correspond to the Bean model are shown in Figs. 3.1a though 3.1c for an increasing external field, and in Figs. 3.1d through 3.1g for a decreasing external field. The hysteresis inherent in the superconductor is shown by the trapped flux remaining in the slab when the external field is reduced to zero (Fig. 3.1e).

The magnetization of the slab is given by

$$M = \frac{1}{d}\int_{-\frac{d}{2}}^{\frac{d}{2}} H dx - H_e \qquad \textbf{(14)}$$

and the magnetization curve for a complete cycle to the upper critical field limit of H_{c2} is shown in Fig. 3.2. The energy loss per cycle per unit volume is easily computed from the area under the magnetization curve and is given by

$$W_h = \int_{volume}\left[\oint_{cycle} \mu_o M dH\right] dV \qquad \textbf{(15)}$$

Carrying out the integration we get for the loss per cycle per unit volume,

$$\frac{W_h}{V} = \frac{2}{3}\mu_o \frac{H_m^3}{H_p} \qquad H_m \le H_p \qquad \textbf{(16a)}$$

and

$$\frac{W_h}{V} = 2\mu_o H_m H_p \left(1 - \frac{2}{3}\frac{H_p}{H_m}\right) \qquad H_m > H_p \tag{16b}$$

If $H_m >> H_p$

$$\frac{W_h}{V} \approx 2\mu_o H_m H_p \tag{16c}$$

These results were originally derived by London [-]. A similar procedure can be followed with the Kim model. In this model the functional dependence of the critical current density on the magnetic field is given by

$$J_c = \frac{J_o H_o}{H + H_o} \tag{17}$$

where $J_o = J_c$ at $H = 0$ and $H_o = H$ at $J_c = J_o/2$.

The field profile then as calculated from Eq. (8) is

$$H = H_o \left\{ \left[\left(1 + \frac{H_e}{H_o}\right)^2 - \frac{2J_o}{H_o}\left(x - \frac{d}{2}\right) \right]^{1/2} - 1 \right\} \quad x > 0 \tag{18}$$

and the field penetrates to

$$x_p = \frac{d}{2} - \frac{H_e}{H_o J_o}\left(H_o + \frac{H_e}{2}\right) \tag{19}$$

Full penetration occurs when $H_e = H_p$ where

$$H_p = H_o \left(1 + \frac{J_o d}{H_o} - d\right) \tag{20}$$

Figures 3.3a though 3.3h show the field and current distributions as given by the Kim model for various values of the external field. The hysteresis loss per cycle can be found from Eq. 15 for the case when H is cycled between $+H_m$ and $-H_m$,

$$\frac{W_h}{V} = \mu_o J_o d H_o \ln\left(\frac{H_o + H_m}{H_o}\right) \tag{21}$$

The purpose of going through this exercise is to show how the critical state model is the basis for computing the hysteresis loss in the superconducting filaments and also to introduce some of the important parameters such as the full penetration field, H_p. These simple slab models serve as a good basis for estimating the hysteresis loss calculations. Solution for a two dimensional geometry is possible but the solutions are not analytic except for the most limiting cases. A 2D solution requires significant non-linear numerical analysis, but the simpler results for 1D models are usually accurate within about a factor of 2 or better.

5. One-Dimensional Effect of Transport Current

The effect of a transport current in the slab specimen can developed by arguments consistent with the critical state model. Consider the one-dimensional slab to be in a uniform external field and carrying a transport current I_t, in the z-direction, that is a fraction of the critical current I_c such that $0<I_t/I_c<1$. The current and field distribution for the slab is shown in Fig, 3.4. Note that the current flows everywhere in the cross-section at the critical current density J_c which has been assumed a constant as per the Bean model. If the external field is cycled by an amount $\pm H_m$ about the value H_e the current and field distributions will be as shown in Fig. 3.4(a-h).

Since one can picture the central region of the slab as being occupied by the transport current, the field required to fully penetrate to this region will now be less than the field required to penetrate to the center of the slab, that is the full penetration field at zero transport current, $H_p(0) = J_c d/2$. The full penetration field now depends upon the fraction of transport current and is given by

$$H_p\left(\frac{I_t}{I_c}\right) = H_p(0)\left(1 - \frac{I_t}{I_c}\right) \tag{22}$$

The magnetization and the hysteresis loss can be computed as before from Eqs. (14) and (15) respectively. The results are given below for the energy loss per unit volume per cycle;

For $H_m \le H_p(i)$

$$\frac{W_h(i)}{V} = \frac{2}{3}\mu_o \frac{H_m^3}{H_p(0)} = \frac{2}{3}\mu_o H_p^2(0)\left[\frac{H_m}{H_p(0)}\right]^3 \tag{23a}$$

For $H_m > H_p(i)$

$$\frac{W_h(i)}{V} = \frac{2}{3}\mu_o \frac{H_p^3(i)}{H_p(0)} + 2\mu_o H_p(0) H_m \left(1 - \frac{H_p(i)}{H_m}\right)\left(1 + i^2\right)$$
$$= \frac{2}{3}\mu_o H_p^2(0)(1-i)^3 + 2\mu_o H_p(0) H_m \left(1 - \frac{H_p(i)}{H_m}\right)\left(1 + i^2\right) \quad \textbf{(23b)}$$

where $i = I_t / I_c$.

The loss is very similar to that given in Eq. (16). If i=0 ,Eq. (23) reduces to (16), the loss with zero transport current. It is interesting to note that if H_m is less then Hp(i) the hysteresis loss is total1y unaffected by the transport current. These results are valid for the two-dimensional hysteresis loss with transport current. If the specimen is carrying the maximum possible transport current (i=1) the loss will be twice the hysteresis loss at zero transport current, i.e.,

$$\frac{W_h(i=1)}{V} = 2\frac{W_h(0)}{V} \quad \textbf{(24)}$$

This condition is specific to the one-dimensional slab model.

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