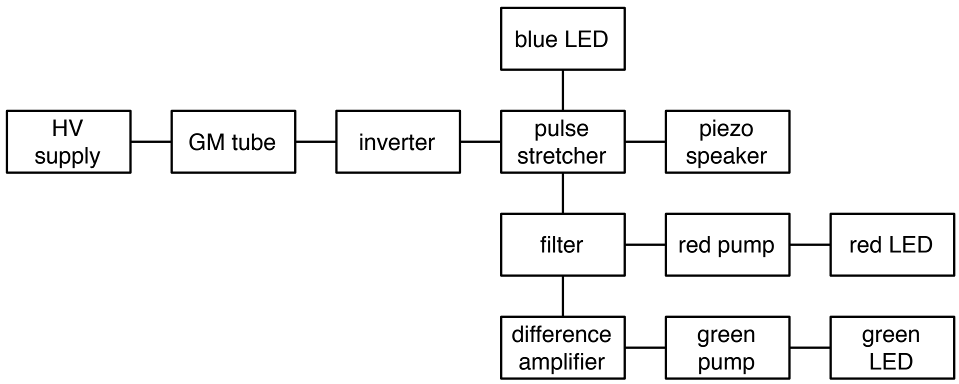
Geiger-Müller counter circuit theory

1: Basic structure of the Geiger-Müller (GM) counter



A block diagram for the GM counter you will be building is given above. The purposes of these blocks are as follows:

HV supply Converts the 9 V battery voltage to the 400 V needed by the GM tube.

GM tube Detects ionizing radiation: emits a current pulse whenever an ionization event occurs inside the tube.

inverter Converts the current from the GM tube into an inverted voltage pulse.

pulse stretcher Converts the very short pulse from the inverter into a 1.5 ms pulse.

blue LED Flashes every time a pulse occurs.

piezo speaker Produces a click every time a pulse occurs.

filter Accumulates charge from the pulses to produce a voltage roughly proportional to the count rate.

red/green LED Displays a visual indication of the count rate that continuously fades from green to orange to red as the rate increases.

red pump Drives the red LED with a current proportional to the voltage from the filter.

difference amplifier Subtracts the filter voltage from a reference voltage to produce a signal that is goes down as the count rate goes up.

green pump Drives the green LED with a current proportional to the voltage from the difference amplifier.

The full schematic is given in Figure 1. By the end of this lecture you will understand the basics of how each of these blocks work. You will be equipped to understand the full details shown in the schematic by the end of 22.071.

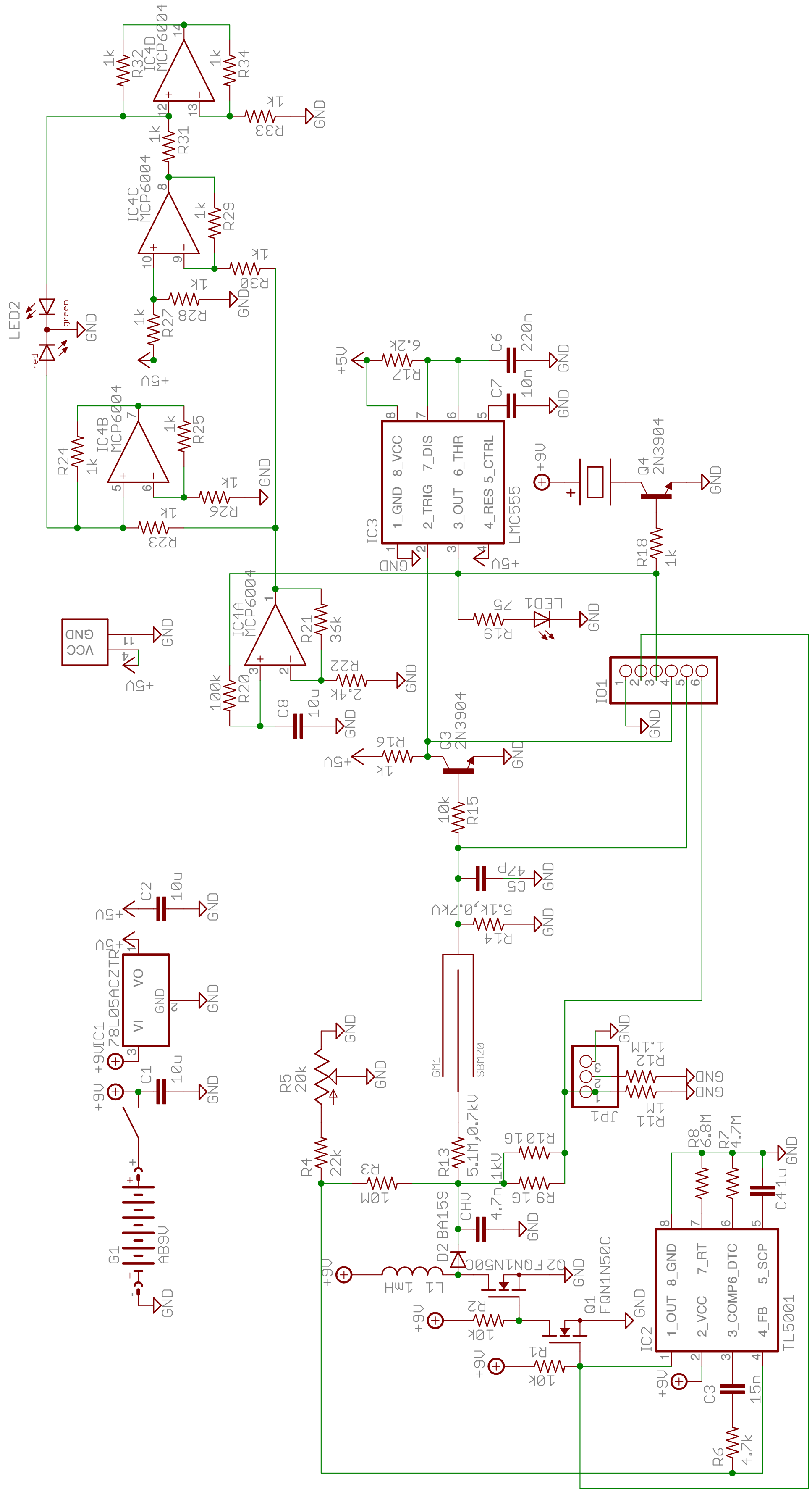


Figure 1: Full schematic for the GM counter.

2: Basic circuit concepts

There are three quantities of interest in a circuit:

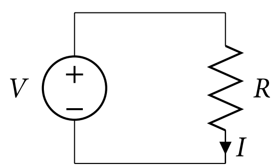
charge A fundamental property of the particles that make up matter (protons, electrons, etc.). Charge is measured in units of coulombs (C).

current The flow of charge through a wire or component. Current is measured in amperes (1 A = 1 C/s). Recall that an electron has a charge of -1.6×10^{-19} C: a current of 1 A corresponds to *many* electrons flowing per second!

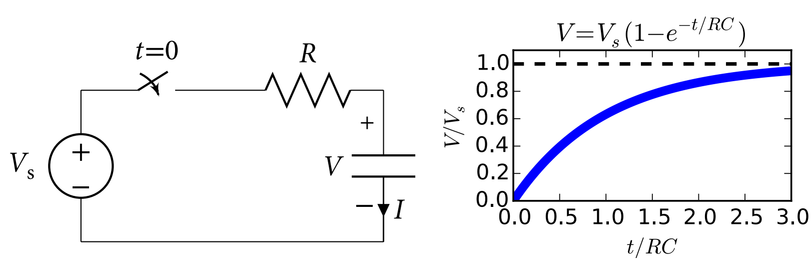
voltage The potential energy per unit charge. The units are volts (1 V = 1 J/C).

Circuits are built up of components that manipulate the current and voltage in the desired manner. The most fundamental three components are:

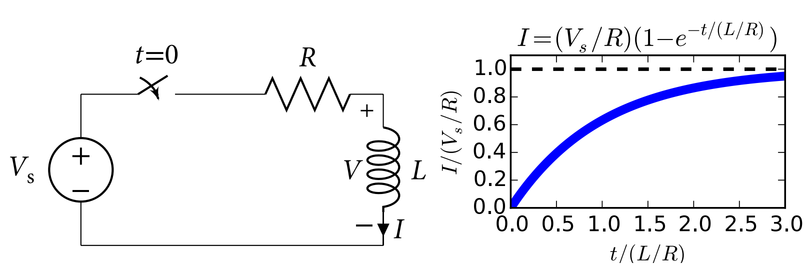
resistors Resist the flow of current: they dissipate the energy carried by the current. The voltage across a resistor is given by **Ohm's law**: $V = IR$, where the resistance R is measured in units of ohms (volts per ampere, $1\ \Omega = 1$ V/A).



capacitors Store charge: the charge stored on a capacitor is $Q = CV$, where C is the capacitance and is given in units of farads (1 F = 1 C/V). The time derivative of charge stored is the current through the capacitor: $I = C\,\mathrm{d}V/\mathrm{d}t$. Note that the voltage across a capacitor must be continuous, otherwise an infinite current would be required!

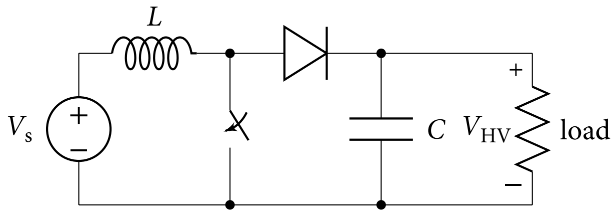


inductors Store magnetic flux: the flux in an inductor is $\Phi = LI$, where L is the inductance and is given in units of henries (1 H = 1 Wb/A = 1 V s/A). The voltage across the inductor is the time derivative of the flux: $V = \mathrm{d}\Phi/\mathrm{d}t = L\,\mathrm{d}I/\mathrm{d}t$. Note that the current through an inductor must be continuous, otherwise an infinite voltage would be present!



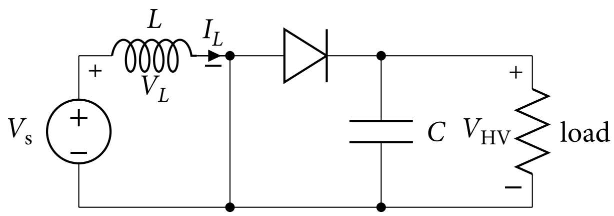
3: HV supply: the boost converter

The boost converter takes the 9 V battery voltage and steps it up to 400 V. This is accomplished using an inductor and another very important component: the **diode**, a component which only permits current to flow in one direction. A basic boost converter looks like this:



The switch turns on and off rapidly, staying on for duration t_{on} and off for duration t_{off} each time.

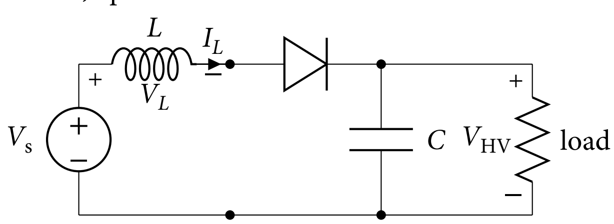
Consider the case when the switch is closed:



There can be no current through the diode in this case, so the capacitor is what provides current to the load. The voltage across the inductor is equal to the supply voltage: $V_L = V_s$. So, the current in the inductor is increasing according to

$$\frac{\mathrm{d}I_L}{\mathrm{d}t} = \frac{V_\mathrm{s}}{L} \rightarrow \Delta I_{L,\text{on}} = \frac{t_\text{on} V_\mathrm{s}}{L}$$

Now, open the switch:



Recall that there can be no discontinuity in the inductor current – the current now flows through the diode to both charge the capacitor and power the load. But, the voltage is free to change instantly: it is now given by $V_L = V_\mathrm{s} - V_\text{HV}$. During this period the current in the inductor therefore drops according to

$$\frac{\mathrm{d}I_L}{\mathrm{d}t} = \frac{V_\mathrm{s} - V_\text{HV}}{L} \rightarrow \Delta I_{L,\text{off}} = \frac{t_\text{off}(V_\mathrm{s} - V_\text{HV})}{L}$$

For the circuit to operate in steady-state, the current must come back to the same value after each cycle:

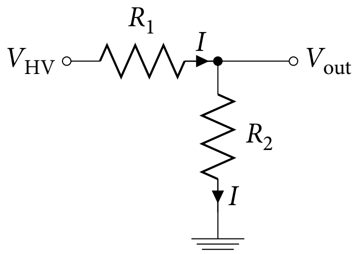
$$\Delta I_{L,\text{on}} = -\Delta I_{L,\text{off}} \rightarrow \frac{t_\text{on} V_\mathrm{s}}{L} = -\frac{t_\text{off}(V_\mathrm{s} - V_\text{HV})}{L} \rightarrow \frac{t_\text{on} + t_\text{off}}{t_\text{off}} = \frac{V_\text{HV}}{V_\mathrm{s}} = \frac{1}{1 - D},$$

where $D = t_\text{on}/(t_\text{on} + t_\text{off})$ is the **duty cycle**. Since $0 \leq D \leq 1$, the output voltage is always greater than or equal to the input voltage.

The actual circuit uses a chip called the TL5001 to actively control D in order to maintain the desired output voltage.

4: Side note: measuring the high voltage

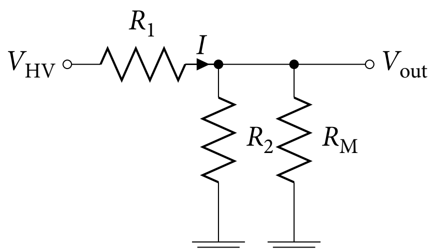
For energy to be conserved, the current to the load has to be smaller than the current drawn from the battery. But, measuring devices such as multimeters and oscilloscopes have large (but finite) input impedances and can draw too much current, thus causing the output voltage to drop. Therefore, to get an accurate measurement of the voltage applied to the GM tube it is necessary to use a **voltage divider**:



$$V_{HV} = I(R_1 + R_2) \rightarrow I = \frac{V_{HV}}{R_1 + R_2}$$

$$V_{out} = IR_2 = \frac{R_2}{R_1 + R_2} V_{HV}$$

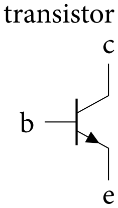
As long as the total resistance, $R_1 + R_2$, is sufficient to not load down the boost converter, an arbitrary division can be obtained. But, the meter itself has a resistance R_M, so the circuit actually looks like this:



The analysis proceeds as before, but R_2 and R_M must first be replaced with their equivalent resistance, $R_{eq} = R_2 R_M/(R_2 + R_M)$. Multimeters typically have an input resistance of 10 MΩ, while oscilloscopes typically are only 1 MΩ. Therefore, the GM counter includes a jumper to select which measuring device you are using.

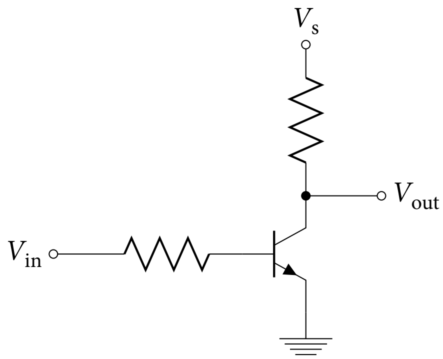
5: Inverter

The GM tube emits a *current* pulse, but the subsequent circuits require a *voltage* pulse. This conversion is accomplished simply by putting the current from the GM tube through a 5.1 kΩ resistor. But, as will be discussed in the next section, the pulse stretcher actually needs the inverse of this pulse – a signal that sits at $V_s = 5$ V when nothing is happening, then temporarily drops to 0 V when a pulse happens. This conversion is accomplished using a **transistor**. In more advanced settings you can use a transistor as an amplifier, but we will simply be using it as a switch. The transistor has three leads: the **emitter** (e), **collector** (c) and **base** (b):



When $V_b > V_e$, current can flow from c to e. But, when $V_b < V_e$ no current can flow.

Therefore, an inverter can be constructed like this:



When V_{in} is high, current can flow through the transistor and V_{out} is pulled low. When V_{in} is low, current cannot flow and V_{out} is pulled to V_s.

6: Pulse stretcher

The output from the GM tube (and hence the inverter) is very brief – if it were used to drive the piezo speaker and blue LED directly, the effect would be nearly imperceptible. The pulse stretcher uses a very useful component known as a 555 timer to turn a short input pulse (of arbitrary shape and duration) into a square pulse 1.5 ms long.

The 555 timer is a circuit building block that brings together several related functions in one convenient device. For those familiar with comparators and flip-flops, a simplified internal schematic is given in Figure 2a. Refer to Figure 2b and Table 1 for the locations and functions of each pin. Refer to Franco [1], Horowitz and Hill [2], and manufacturer data sheets such as [3] for more details on this.

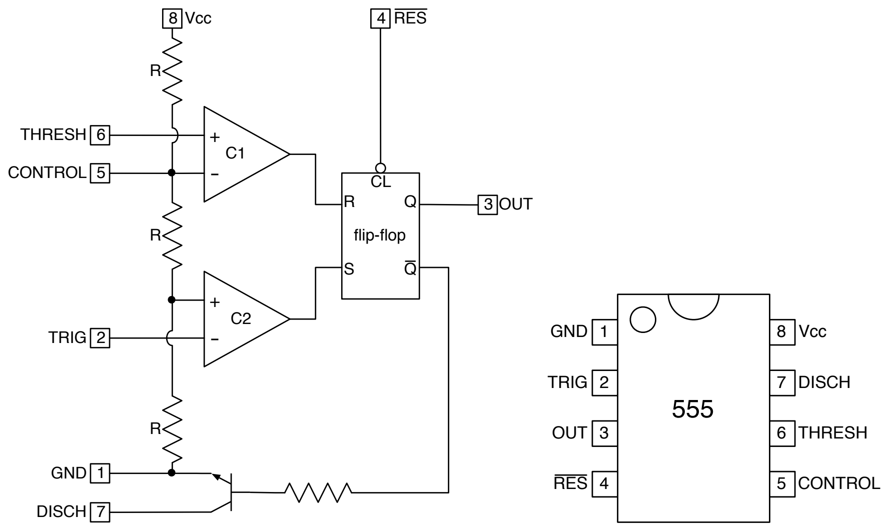
The pulse stretcher is formally known as a monostable multivibrator, the implementation is shown in Figure 3. Because of the structure of the timer, the pulse input to TRIG must be falling – i.e., the signal starts out above 1/3 V_{CC}, then drops below that level to trigger a pulse. This is why the inverter is necessary. To understand how the circuit works, first consider the state before the pulse arrives:

- There is no current from the tube, so the inverter puts V_s at TRIG.
- The timer has not yet been triggered, so OUT is at 0 V.
- DISCH is conducting, and hence the capacitor C is drained of all charge.

Once the pulse arrives:

- The voltage at TRIG briefly drops below 1/3 V_{CC}.
- This causes OUT to be set to V_{CC}.
- This also causes DISCH to stop conducting.
- Hence, the capacitor C starts to charge through resistor R.
- After $t_{on} = 1.1RC$, capacitor C is charged to the point that THRESH is above 2/3 V_{CC}.
- This causes OUT to be set to 0 V and DISCH to become conducting, thus rapidly draining the capacitor and getting the circuit ready for the next pulse.

Sample voltage histories for a single triggering of this circuit are given in Figure 4.



(a) internal schematic

(b) pinout

Figure 2: (a) Simplified internal schematic of the 555 timer. Figure inspired by [1].

(b) Pinout of the 555 timer in an 8-pin DIP package.

Table 1: 555 Pin Descriptions

Pin	Name	Function
1	GND	Ground connection.
2	TRIG	Trigger: when the voltage at this pin drops below 1/3 V_{CC}, the flip-flop is set, which sends OUT *high*.
3	OUT	Output: turns *high* when TRIG drops below 1/3 V_{CC}, turns *low* when THRESH rises above 2/3 V_{CC}.
4	$\overline{\text{RES}}$	Reset: when the voltage at this pin goes *low*, the flip-flop is reset, therefore setting OUT to *low* regardless of the conditions at TRIG or THRESH.
5	CONTROL	Control voltage: accesses the internal voltage divider, allowing you to modify the thresholds to some extent.
6	THRESH	Threshold: when the voltage at this pin rises above 2/3 V_{CC}, the flip-flop is reset, which sends OUT *low*.
7	DISCH	Discharge: when the flip-flop is on (OUT is *high*), there is no base current and the transistor does *not* conduct. When the flip-flop is off (OUT is *low*), the transistor is turned *on* (saturated), thus allowing an external capacitor to be discharged to ground.
8	V_{CC}	Power supply.

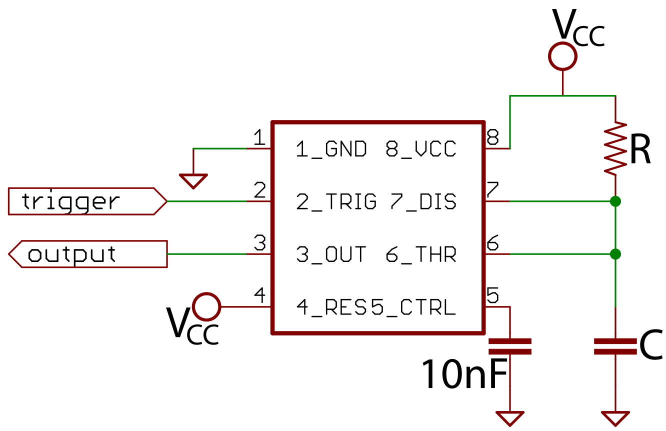


Figure 3: Monostable multivibrator (one-shot pulse stretcher) implemented with a 555 timer.

Pin 4 ($\overline{\mathrm{RES}}$) is held to V_{CC} in order to keep the flip-flop enabled.

Pin 5 (CONTROL) is connected to ground through a 10 nF capacitor in order to reduce the impact of noise in the circuit on the threshold voltage.

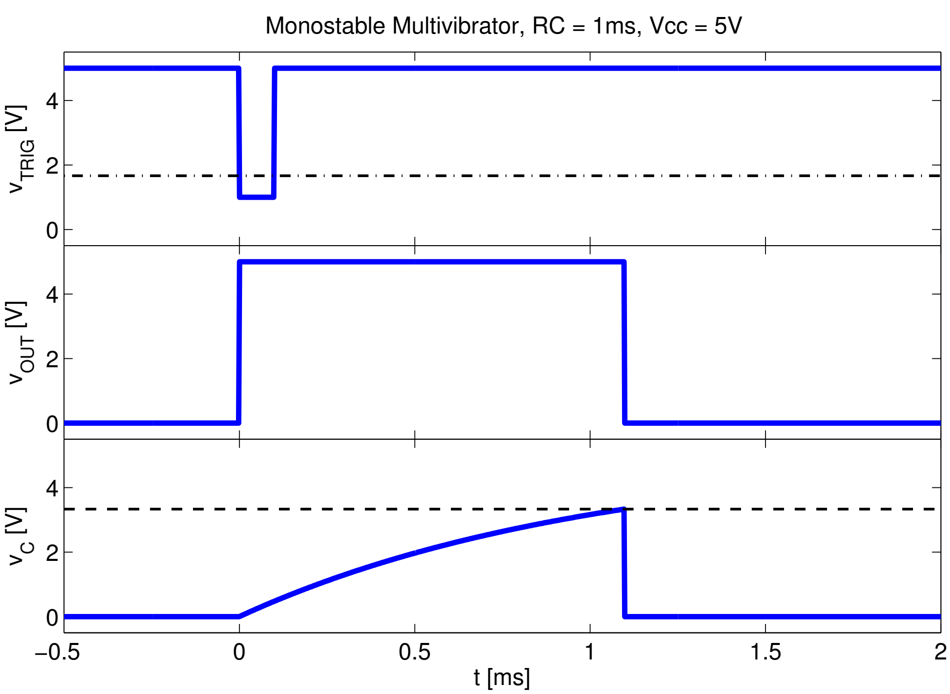


Figure 4: Voltage histories for the monostable circuit.

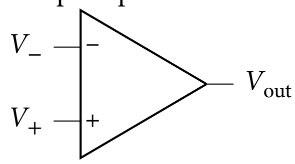
The top plot shows the voltage at pin 2 (TRIG). Notice that the pulse does not need to drop all the way to zero, nor does it even need to have a particularly "square" shape – as soon as v_{TRIG} drops below $1/3\ V_{\mathrm{CC}}$ the cycle starts.

The middle plot shows the voltage at pin 3 (OUT). The pulse starts as soon as the voltage at pin 2 (TRIG) drops below $1/3\ V_{\mathrm{CC}}$ and continues for a duration $1.1RC$.

The bottom plot shows the voltage across the capacitor (and hence at pin 6 (THRESH)). The pulse ends and the capacitor is rapidly discharged through pin 7 (DISCH) as soon as v_{C} reaches $2/3\ V_{\mathrm{CC}}$.

7: Op-amps

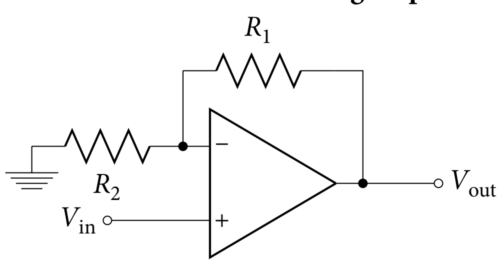
The **op-amp** (short for "operational amplifier") is an exceedingly versatile component – it can be used to construct many types of filters and amplifiers, among other things. The op-amp has three terminals of interest:



There are two simple rules that enable you to work out what many op-amp circuits do:

1. There is no current in or out of the "+" and "−" inputs.
2. *When negative feedback is present*, the op-amp will adjust the voltage at the output terminal to ensure that $V_+ = V_-$.

Consider the **non-inverting amplifier**:

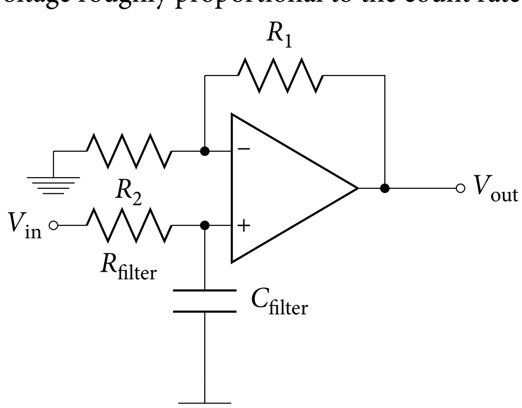


From rule (2), we know that $V_- = V_+ = V_{\text{in}}$. From rule (1), we know that all of the current goes from the output to ground, since it cannot go into the "−" input. Notice that R_1 and R_2 form a voltage divider, so

$$V_{in} = \frac{R_2}{R_1 + R_2} V_{\text{out}} \rightarrow V_{\text{out}} = \left(1 + \frac{R_1}{R_2}\right) V_{\text{in}}$$

8: The filter

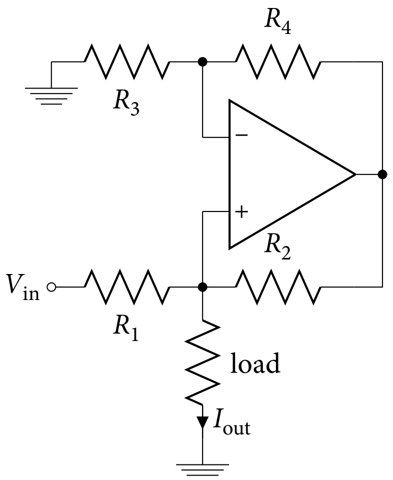
The filter acts to smooth out the pulses from the GM tube in order to produce a voltage roughly proportional to the count rate. It is of a very simple type:



This is simply the non-inverting amplifier from above with a filter composed of R_{filter} and C_{filter} added to shunt the high-frequency components off to ground.

9: The current pump

The brightness of the LEDs is determined by the current. But, the filter described above puts out a voltage. A circuit that converts a voltage to a current is called a **transconductance amplifier**. The GM counter uses a particular implementation known as the **Howland current pump**:



As long as

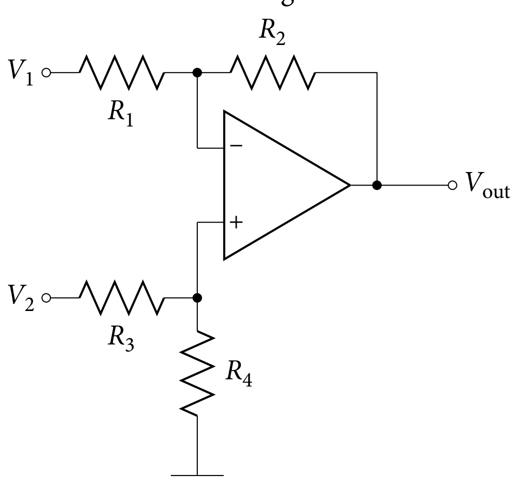
$$\frac{R_4}{R_3} = \frac{R_2}{R_1},$$

the current delivered to the load will be

$$I_{out} = \frac{V_{in}}{R_1}$$

10: The difference amplifier

The filter and one current pump are sufficient to drive a current through the red LED which is (roughly) proportional to the count rate. But, to have the desired effect the green LED must fade out as the count rate increases. A **difference amplifier** is used to subtract the filter voltage from a reference voltage:



If $R_3 = R_1$ and $R_4 = R_2$, then the output voltage will be

$$V_{out} = \frac{R_2}{R_1}(V_2 - V_1)$$

References

[1] S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 3rd ed. McGraw-Hill, 2002, ch. 10.3.

[2] P. Horowitz and W. Hill, *The Art of Electronics*, 2nd ed. Cambridge University Press, 1989, ch. 5.14.

[3] Texas Instruments, “LM555 Timer Datasheet,” SNAS548B.