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8.012 Physics I: Classical Mechanics
Fall 2008

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.012

Fall 2008

Exam 1

NAME: _____

Instructions:

1. Do all **FIVE (5)** problems. You have **90 minutes**.
2. **SHOW ALL WORK.** Be sure to **circle your final answer**.
3. Read the questions carefully.
4. All work must be done in this booklet in workspace provided. Extra blank pages are provided at the end if needed.
5. **NO** books, notes, calculators or computers are permitted. **A sheet of useful equations is provided on the last page.**

Your Scores

Problem	Maximum	Score	Grader
1	10		
2	30		
3	20		
4	20		
5	20		
Total	100		

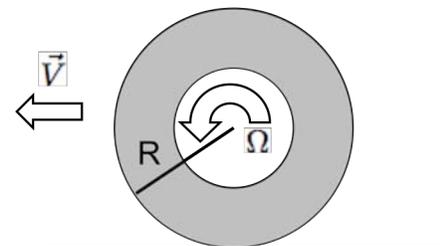
Problem 1: Quick Multiple Choice Questions [10 pts]

For each of the following questions circle the correct answer. You do not need to show any work.

(a) Which of the following is **not** a valid force law?

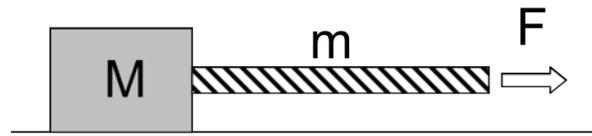
$\vec{F} = b \frac{m_1 + m_2}{m_1 - m_2} \hat{r}_{12}$ <p>where b has units of Newtons</p>	$\vec{F} = 2Gmv^2 \hat{x}$ <p>where G is the gravitational force constant</p>	$\vec{F} = b\ddot{r}\hat{\theta} + m\frac{v^2}{r}\hat{r}$ <p>where b has units of kg</p>	$\vec{F} = b\frac{mx}{t^2}\hat{z}$ <p>where b has units of radians</p>
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(b) A tire rolls on a flat surface with constant angular velocity Ω and velocity \vec{V} as shown in the diagram to the right. If $V > \Omega R$, in which direction does friction from the road act on the tire?



Toward the left	Toward the right	Friction does not act on the tire
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(c) You pull a brick of mass M sitting on a flat table using a thick rope of mass m . Friction between the surface of the table and the brick is sufficient so that the brick remains at rest. Compared to the force with which you pull on one end of the rope, the force that the other end of the rope exerts on the brick is:



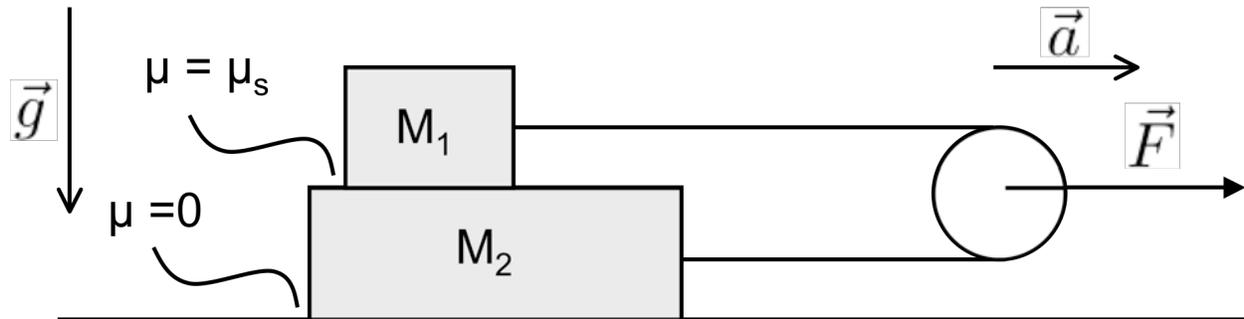
Less	Greater	The same	Zero
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(d) As a swinging pendulum passes through its lowest point, in which direction does the total net force act?

Only in an angular direction	Only in a radial direction	In both angular and radial directions	There is zero net force at the equilibrium point
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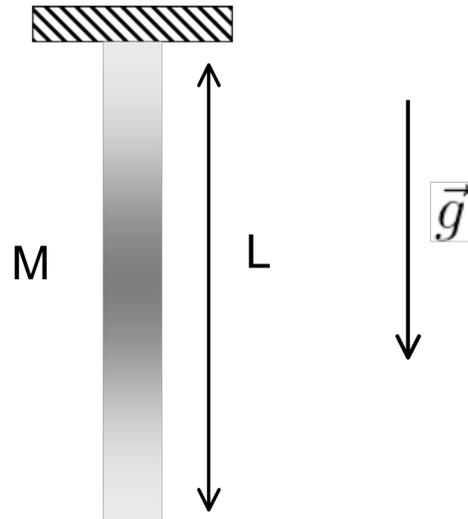
(e) A pendulum with mass M and length L is released as a small angle off of vertical and oscillates with period P . If we double the mass and halve the length of the pendulum then the new period is

$2P$	$\sqrt{2}P$	P	$P/\sqrt{2}$	$P/2$
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Problem 2: The Accelerated Atwood Machine [30 pts]

Two blocks of masses M_1 and M_2 ($M_2 > M_1$) are stacked on top of each other and start at rest on the surface of a frictionless table. The masses are connected via an ideal pulley (massless string and nearly massless pulley wheel), and the coefficient of static friction (assumed equal to the coefficient of kinetic friction) between the block surfaces is μ_s . The pulley is accelerated to the right by a force \vec{F} , resulting in an acceleration of the pulley wheel of \vec{a} . Assume that gravity acts with constant acceleration g downward.

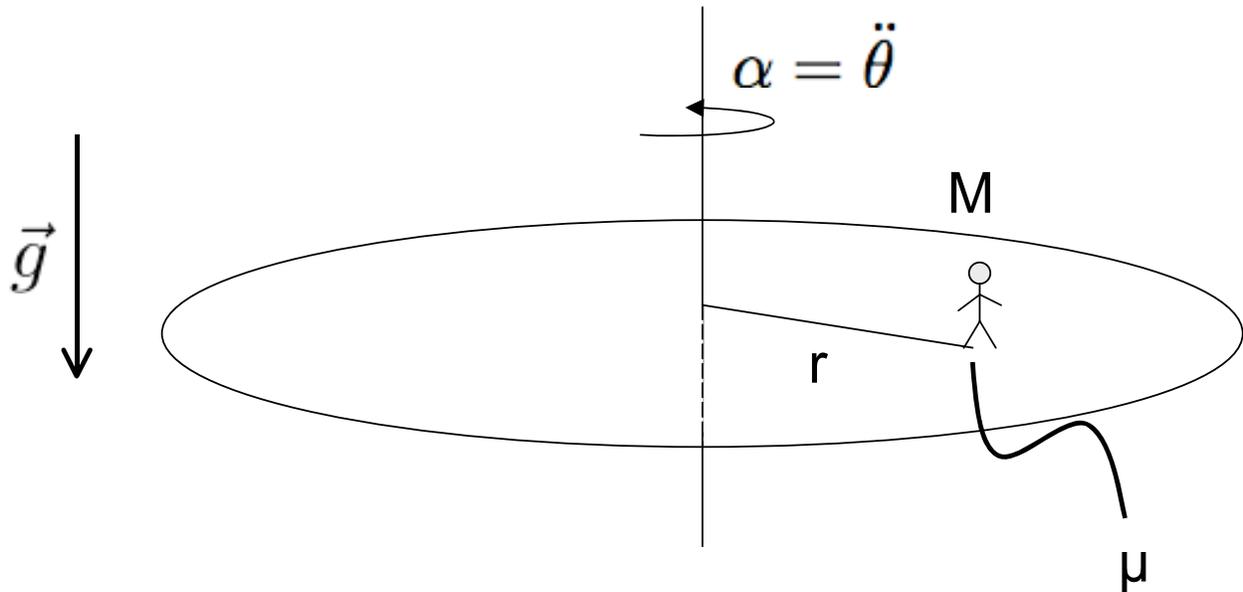
- (a) [5 pts] Draw force diagrams for each of the blocks and the pulley wheel, clearly indicating all horizontal and vertical forces acting on them.
- (b) [5 pts] If the blocks do not slip relative to each other, what are their accelerations?
- (c) [10 pts] Assume that the blocks do slip relative to each other. Determine each block's horizontal acceleration as a function of the parameters specified above (i.e., M_1 , M_2 , μ_s , g , a and F). Which block has a higher acceleration? Be sure to work in an inertial reference frame!
- (d) [10 pts] What is the minimum force F required to cause one block to slip relative to the other? Assume that the mass of the pulley is negligible compared to those of the blocks.

Problem 3: Hanging Rope [20 pts]

Consider a rope of total mass M and length L suspended at rest from a fixed mount. The rope has a linear mass density that varies with height as $\lambda(z) = \lambda_0 \sin(\pi z/L)$ where λ_0 is a constant. Constant gravitational acceleration g acts downward.

- (a) [5 pts] Determine the constant λ_0 .
- (b) [5 pts] What is the tension force at the free (bottom) end of the rope?
- (c) [10 pts] Calculate the tension along the rope as a function of distance z below the mount.

Problem 4: Don't Slip! [20 pts]



An 8.012 student of mass M stands on a rigid disk at a distance r from the center axis. Assume that the coefficient of friction between the student's shoes and the disk surface is μ . At time $t = 0$, the disk begins to rotate with a constant angular acceleration rate $\alpha = \ddot{\theta}$. Assume that gravity acts with constant acceleration g downward.

- (a) [5 pts] What is the maximum value of angular acceleration rate (α_{\max}) such that the student does not immediately slip?
- (b) [10 pts] Assuming that $\alpha < \alpha_{\max}$, what is the total friction force acting on the student as a function of time (prior to slipping)? Write your answer as a vector in polar coordinates.
- (c) [5 pts] Assuming that $\alpha < \alpha_{\max}$, how long after the disk starts rotating will the student slip?

Problem 5: Will the LHC Destroy the World? [20 pts]

There has been some speculation that the Large Hadron Collider (or LHC), a particle accelerator experiment that will smash protons together at incredible energies, might create a small black hole that could devour the Earth. Let's imagine that the LHC does create such a black hole of mass M (assume initially that $M \ll M_{\text{Earth}}$). Once it has formed (initially at rest), the black hole will immediately fall under the influence of gravity toward the center of the Earth. Assume for this problem that the Earth has a constant volume mass density ρ , and that the black hole doesn't eat that much of the Earth as it passes through it.

(a) [5 pts] Using dimensional analysis, come up with an order-of-magnitude expression for the oscillation period of the black hole as it passes through the Earth based on the relevant parameters of this problem.

(b) [10 pts] Now show that the black hole undergoes simple harmonic motion (similar to spring motion) by determining its acceleration as a function of time, and derive an expression for the oscillation period. How does this compare with your answer in part (a)?

(c) [5 pts] The Earth does not have a constant mass density but rather gets denser closer to its center. Assuming that it is still spherically symmetric, would this fact cause the oscillation period of the black hole to be shorter or longer? Justify your answer.

(d) [10 pts **BONUS**] Assume that the particle collision that creates the black hole converts the equivalent of an entire year's worth of global energy production (about 5×10^{20} Newton-meters) into a single black hole at rest. Also, the radius of a black hole is the distance at which the gravitational escape velocity exceeds the speed of light. Based on these assumptions, use dimensional analysis to estimate the mass and radius of the black hole to within an order of magnitude. Based on these values, should we worry about an LHC black hole? Some important numbers for this question are the gravitational constant, $G \approx 7 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$; the speed of light, $c \approx 3 \times 10^8 \text{ m/s}$; and, for comparison, the radius of a proton, $r_{\text{proton}} \approx 10^{-15} \text{ m}$, and the radius of the Earth, $R_{\text{Earth}} \approx 6 \times 10^6 \text{ m}$.

USEFUL EQUATIONS

Trajectory for constant acceleration \vec{a}

$$\vec{r}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}(0)t + \vec{r}(0)$$

Velocity in polar coordinates

$$\frac{d\vec{r}}{dt} = \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

Acceleration in polar coordinates

$$\frac{d^2\vec{r}}{dt^2} = \vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

Taylor expansion of function $f(x)$:

$$f(x) = f(a) + \frac{1}{1!}\frac{df}{dx}\Big|_a(x-a) + \frac{1}{2!}\frac{d^2f}{dx^2}\Big|_a(x-a)^2 + \dots$$

Newton's Gravitational Law

$$\vec{F}_{12} = -G\frac{m_1m_2}{r_{12}^2}\hat{r}_{12}$$