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8.012 Physics I: Classical Mechanics
Fall 2008

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.012

Fall 2006

Final Exam
Monday, December 18, 2006

NAME: _____

MIT ID number: _____

Instructions:

1. Do all **EIGHT (8)** problems. You have **3 hours**.
2. **SHOW ALL WORK**. Be sure to **circle your final answer**.
3. Read the questions carefully.
4. All work must be done in the blue/white books provided.
5. **NO** books, notes, calculators or computers are permitted. **A list of useful equations is provided on the last page.**

Your Scores

Problem	Maximum	Score	Grader
1	10		
2	15		
3	10		
4	10		
5	15		
6	15		
7	15		
8	10		
Total	100		

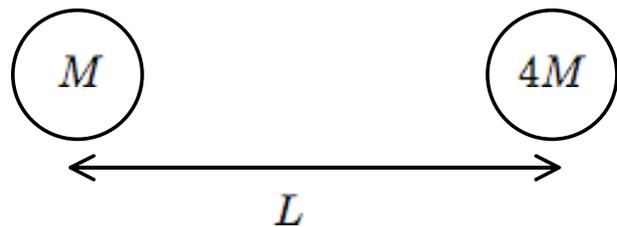
Problem 1: Short Answer Problems [2 pts each]

(a) List Newton's three laws (order is not important).

(b) What are the units of impulse, using the notation $[L]$ = length, $[M]$ = mass and $[T]$ = time?

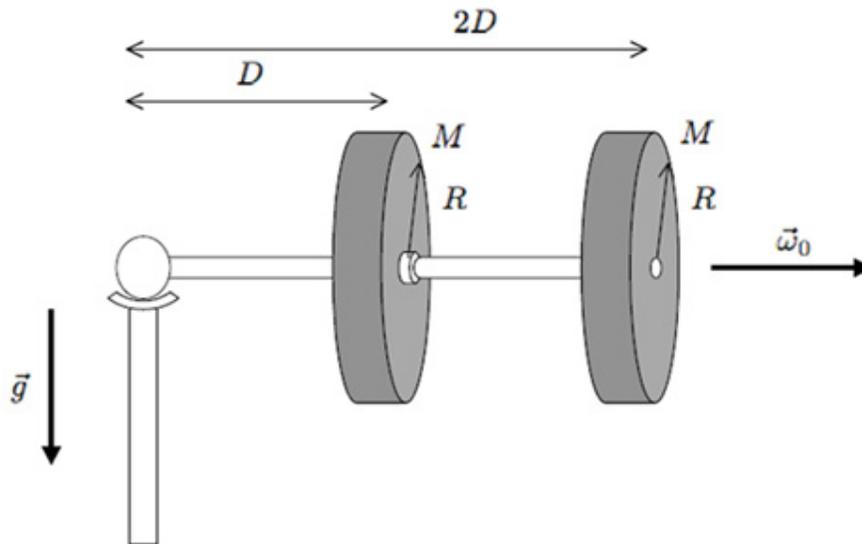
(c) An archer in the northern hemisphere on Earth faces North and shoots an arrow at very high velocity. Will the arrow veer off to the left or the right? Justify your answer (note that the angular velocity vector of the Earth points away from the North Pole).

(d) Referring to the figure to the right, how far from the mass on the left is the point at which a test particle feels equal gravitational force from both spheres? Assume L is much larger than the radii of the spheres



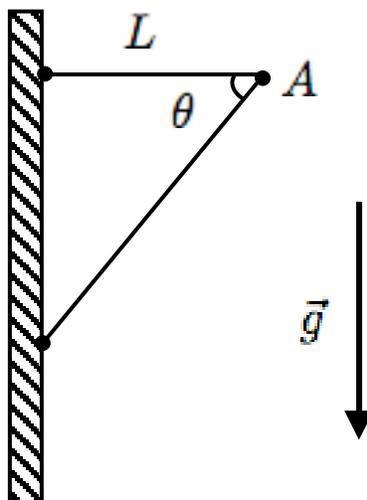
(e) A mass is at rest in an inertial frame at a distance R from the origin. An observer sits at the origin of a rotating frame with angular velocity Ω that is initially aligned with the inertial frame. Describe in words the motion of the object from the observer's perspective, and explain how that motion comes about in terms of fictional forces (there are no real forces acting on the mass).

Problem 2: The Double Gyroscope [15 pts]

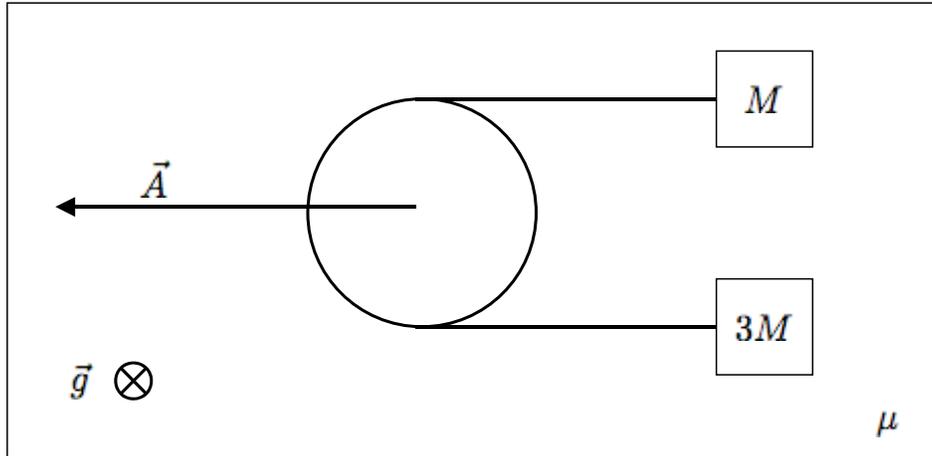


A gyroscope consists of two identical uniform disks with mass M and radius R mounted on a rigid axle with length $2D$. The axle is fixed to the outer disk, while the inner disk is allowed to spin freely but is constrained to remain at a distance D from the pivot (at left) by a collar. The axle spins about the pivot freely on a frictionless mount. The outer disk and axle are initially spun up to an angular frequency ω_0 . Assume that the mass of the axle and pivot mount are negligible. Gravity points downward. Ignore nutation.

- (a) Calculate the precession rate Ω of the gyroscope assuming that the inner disk is not spinning. Assume that ω_0 is very large and the gyroscopic approximation applies.
- (b) Friction between the axle and the disk closest to the pivot causes the latter to spin up. Assuming that a constant torque τ acts at the interface between the axle and inner hole of this disk, calculate the final spin rate ω of both disks and the final precession rate in terms of the initial precession rate.
- (c) How much energy is lost from the gyroscope system during the spinning up of the inner disk?

Problem 3: Balanced Poles [10 pts]

Two sticks are attached with frictionless hinges to each other and to a wall, as shown above. The angle between the sticks is θ . Both sticks have the same constant linear mass density λ , and the horizontal stick has length L . Find the force (both horizontal and vertical components) that the lower sticks applies to the upper one where they connect at point A . Assume gravity points downward.

Problem 4: The Accelerated Atwood Machine [10 pts]

An idealized Atwood machine, consisting of a two blocks of masses M and $3M$ connected via a massless string through a massless pulley, sits on a flat horizontal table. The coefficient of kinetic friction between the block and table surfaces is μ . The pulley is pulled by a string attached to its center and accelerated to the left. Assume that gravity acts with constant acceleration g down through the plane of the table.

- What are the horizontal accelerations of the two masses in the frame of rest of the table?
- What is the maximum acceleration A for which the block of mass $3M$ will remain stationary?

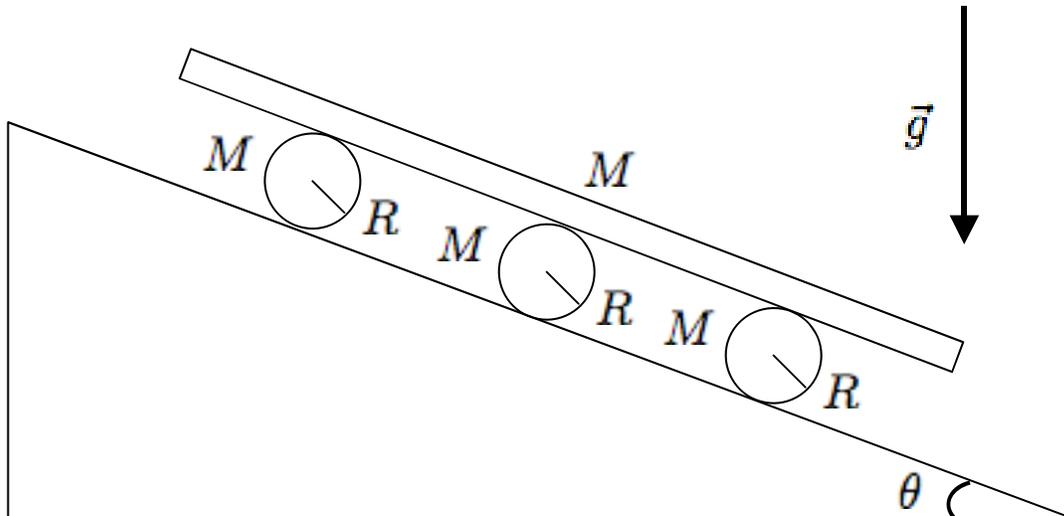
Problem 5: Central Force Potentials [15 pts]

A particle of mass m is inserted into a central force field of the form

$$\vec{F} = -\frac{k}{r^n} \hat{r}$$

where r is the radial distance from the origin, and k and n are constants.

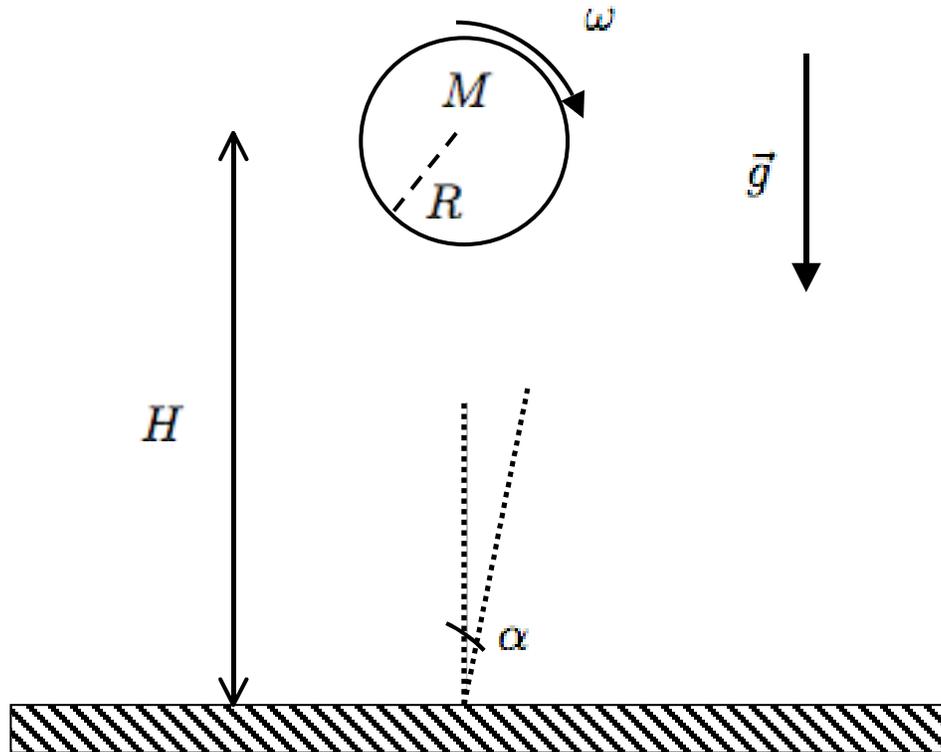
- (a) Show that the total angular momentum \vec{L} about the origin of the system is a constant of the motion.
- (b) Derive the general effective potential for the system, assuming $|\vec{L}| = L$.
- (c) Derive a general expression for the equilibrium point(s) for this potential.
- (d) For what values of n are stable orbits possible; i.e., for which the particle is constrained to a finite range of r ? Show your results in detail (i.e., do not simply state the answer).

Problem 6: Rolling Platform [15 pts]

A platform of mass M and uniform density rests on three solid cylinders, each of mass M , radius R and uniform density. The whole structure is initially at rest on an inclined plane tilted at angle θ , and then released. Assume that there is no slipping between the various surfaces and gravity points downward.

- What is the acceleration of the platform along the incline?
- If the solid cylinders are replaced with hollow cylinders, what is the resulting acceleration of the platform?
- Based on the result of part (b), what are the optimal rollers for moving a heavy platform such as the one shown above: (1) solid uniform cylinders, (2) hollow cylinders, or (3) cylinders with mass concentrated at their centers? Justify your answer (This would have been an engineering consideration for the ancient Egyptians when they transported large blocks of stone to construct the Pyramids.)

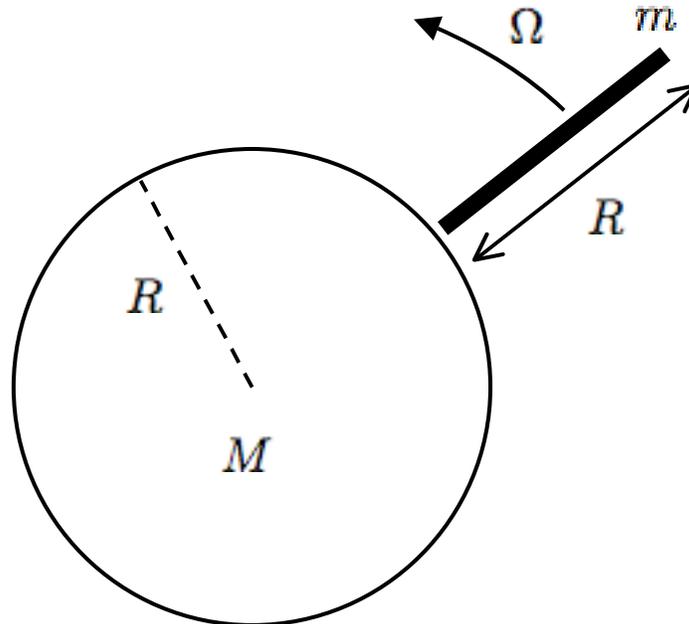
Problem 7: Spinning Bouncing Ball [15 pts]



A uniform sphere of mass M and radius R spinning with angular velocity ω is dropped from a height H . It bounces on the floor and recoils with the same vertical velocity. During the bounce, the surface of the ball slips relative to the surface of the floor (i.e., it does not roll) and in the process the ball is acted upon by a friction force with magnitude $f = \mu N$, where N is the normal contact force between the ball and the floor and μ is a constant. Hence, the ball experiences impulses in both vertical and horizontal directions. Assume that the duration of contact, Δt , is very short.

- What angle α with respect to vertical does the ball recoil?
- What is the final rotation velocity of the ball?
- What value of H results in the ball bouncing off with no spin?

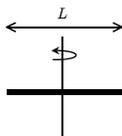
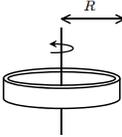
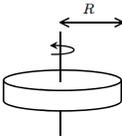
Problem 8: Orbiting Rope [10 pts]

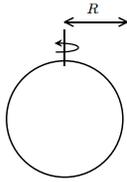


A uniform rope of mass m and length R orbits a spherical planet of mass M and radius R with constant angular velocity Ω , such that the rope remains straight and in a circular orbit by a balance of gravitational and centrifugal forces. For this problem, ignore the effects of air resistance and assume that $M \gg m$.

- (a) Derive an expression for the angular velocity Ω as a function of m , M and R (not all three quantities may be in the expression). Hint: consider the tension in the rope as a function of radius.
- (b) What point on the rope experiences the greatest tension (and is thus most likely to break)?
- (c) An orbiting rope such as this is one concept for a space elevator, allowing material to be transported into orbit without the use of rockets. What restrictions must be made on the angular velocity Ω and the anchor point of the rope on the Earth in this case?

USEFUL EQUATIONS

Trajectory for constant acceleration	$\vec{r}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}(0)t + \vec{r}(0)$
Velocity in polar coordinates	$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$
Acceleration in polar coordinates	$\vec{a} = \ddot{r}\hat{r} + r\ddot{\theta}\hat{\theta} + 2\dot{r}\dot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$
Center of mass of a rigid body	$\vec{R} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{1}{M} \int \rho \vec{r} dV$
Kinetic energy	$K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$
Work	$W = - \int \vec{F} \cdot d\vec{r} = - \int \vec{\tau} d\theta$
Angular momentum	$\vec{L} = \vec{r} \times \vec{p} = I\dot{\theta}$
Torque	$\vec{\tau} = \vec{r} \times \vec{F} = I\ddot{\theta}$
Moment of inertia for a uniform bar	 $I = \frac{1}{12}ML^2$
Moment of inertia for a uniform hoop	 $I = MR^2$
Moment of inertia for a uniform disk	 $I = \frac{1}{2}MR^2$

Moment of inertia for a uniform sphere	 $I = \frac{2}{5}MR^2$
Parallel axis theorem	$I = I_{COM} + MR^2$
Velocity from rotation	$\vec{v} = \vec{\omega} \times \vec{r}$
Moments of inertia tensor (permute x→y→z)	$I_{xx} = \sum_i m_i (y_i^2 + z_i^2)$
Products of inertia tensor (permute x→y→z)	$I_{xy} = - \sum_i m_i x_i y_i$
Euler's equations (permute 1→2→3)	$\tau_1 = I_1 \frac{d\omega_1}{dt} + (I_3 - I_2)\omega_2\omega_3$
Fictitious force in an accelerating frame	$\vec{F}_f = -m\vec{A}$
Fictional forces in a rotating frame	$\vec{F}_f = -2m\vec{\Omega} \times \vec{v}_{rot} - m\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$
Time derivative of an arbitrary vector between inertial and rotating frames	$\frac{d\vec{B}}{dt}_{(in)} = \frac{d\vec{B}}{dt}_{(rot)} + \vec{\Omega} \times \vec{B}$
Reduced mass	$\mu = \frac{m_1 m_2}{m_1 + m_2}$
Effective potential for orbital motion	$U_{eff}(r) = U(r) + \frac{l^2}{2\mu r^2}$