Happy Holidays! Happy New Year! Welcome back!

- Today
  - Intro to angular motion
- Important Concepts
  - Equations for angular motion are mostly identical to those for linear motion with the names of the variables changed.
  - Location where forces are applied is now important.

Important Reminders

- Lectures will be M 11-12, T&W 10-12, F 11-12.
- Check schedule on web for new times and rooms for some recitations (all are still on Thursday).
- Switching of recitations will be permitted if you have a conflict with another IAP activity.
- Contact your tutor about session scheduling
  - Students working with Stephane Essame reassigned.
- Mastering Physics due this Wednesday at 10pm.
- Pset due this Friday at 11am.

Complete Description of Motion

- For an extended object, ***all*** of the equations learned last fall apply ***exactly*** without approximations to the motion of the center of mass.
- This is true whether or not an object is also rotating.
- The two motions (linear position of the center of mass and rotation around the center of mass) can be considered separately, ***except*** for kinetic energy where everything gets lumped into one equation.

Kinematics Variables

Position x	Angle θ
Velocity v	Angular velocity ω
Acceleration a	Angular acceleration α
Force F	Torque τ
Mass M	Moment of Inertia I
Momentum p	Angular Momentum L

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Vector Nature of Angular Motion

- Most of the problems we will consider will involve only motion about a single axis, effectively 1-D
  - 1-D is still a vector (up/down or left/right, for example).
  - For the two directions of rotation, use clockwise (CW) and counter-clockwise (CCW).
  - The choice of which one is positive is arbitrary but you need to be consistent throughout a problem.
- However, CW and CCW do not a really specify a unique direction, so where is the vector?

First Strange Feature of Angular Motion

- What is the vector associated with angular motion?
- The only unique direction in the problem is the axis, but again, there are two directions along the axis.
- The vector for any angular quantity (θ, ω, α, τ, J) points along the axis with the direction given by a right-hand-rule.
  - Fingers curl in direction of θ, ω, α, τ, J, thumb points in the direction of the vector

Kinematics of Angular Motion

- All equations are exactly identical to those for linear motion but with the variables renamed.
  - If a is constant: $v = v_0 + at \qquad x = x_0 + v_0 t + \frac{1}{2}at^2$
  - If α is constant: $\omega = \omega_0 + \alpha t \qquad \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

Constraints/Connections between Linear and Angular Motion

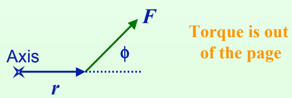
- For all points on a rigid object and for any axis considered, the value of ω and α are identical.
- For a point a distance R from the axis of a rotating object, the path traveled and the angle are related by $S=R\theta$, which implies that $v=R\omega$, and $a=R\alpha$.
- Tangential acceleration of a point a distance R from the axis of a rotating object is $R\alpha$.
- Radial acceleration of a point a distance R from the axis of a rotating object is v^2/R.

Rolling Without Slipping

- For an object rolling without slipping, the point in contact with the ground has a speed of exactly zero so that it is not sliding along the ground.
- Use relative velocity concepts to show that the center of mass of the wheel must be moving with a velocity given by $v=R\omega$ and an acceleration $a=R\alpha$.
- The same constraints apply for a string on a pulley that is not slipping. In this case, v and a are for the end of the string and ω and α are for the pulley.

Torque

- How do you make something rotate? Very intuitive!
  - Larger force clearly gives more “twist”.
  - Force needs to be in the right direction (perpendicular to a line to the axis is ideal).
  - The “twist” is bigger if the force is applied farther away from the axis (bigger lever arm).
- In math-speak: $\vec{\tau} = \vec{r} \times \vec{F} \qquad |\tau| = |r||F|\sin(\phi)$



More Ways to Think of Torque

- Magnitude of the force times the component of the distance perpendicular to the force (aka lever arm).
- Magnitude of the radial distance times the component of the force perpendicular to the radius.
- Direction from Right-Hand-Rule for cross-products and can also be thought of as clockwise (CW) or counter-clockwise (CCW).
- For torque, gravity acts at the center of mass.

Conditions for Equilibrium

- Same as before: $\Sigma\vec{F} = 0$
  - It’s totally irrelevant where the forces are applied to an object, only their direction and magnitude matters.
  - This gives one independent equation per dimension.
- Additional condition: $\Sigma\vec{\tau} = 0$
  - This is true for ***any*** axis. However, if all of the forces are in the same plane (the only type of problem we will consider in this class), you only get ***one*** additional independent equation by considering rotation.