- Last Lecture
  - Intro to angular motion
- Today
  - Statics and dynamics of rotational motion
- Important Concepts
  - Equations for angular motion are mostly identical to those for linear motion with the names of the variables changed.
  - Location where forces are applied is now important.
  - Rotational inertia or moment of inertia (rotational equivalent of mass) depends on how the material is distributed relative to the axis.

Important Reminders

- Lectures will be M 11-12, T&W 10-12, F 11-12.
- Check schedule on web for new times and rooms for some recitations (all are still on Thursday).
- Switching of recitations will be permitted if you have a conflict with another IAP activity.
- Contact your tutor about session scheduling
  - Students working with Stephane Essame reassigned.
- Mastering Physics due this Wednesday at 10pm.
- Pset due this Friday at 11am.

Complete Description of Motion

- For an extended object, ***all*** of the equations learned last fall apply ***exactly*** without approximations to the motion of the center of mass.
- This is true whether or not an object is also rotating.
- The two motions (linear position of the center of mass and rotation around the center of mass) can be considered separately, ***except*** for kinetic energy where everything gets lumped into one equation.

Kinematics Variables

- Position x
- Velocity v
- Acceleration a
- Force F
- Mass M
- Momentum p

- Angle θ
- Angular velocity ω
- Angular acceleration α
- Torque τ
- Moment of Inertia I
- Angular Momentum L

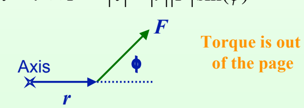
$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Vector Nature Angular Motion

- The vector for any angular quantity (θ, ω, α, τ, J) points along the axis with the direction given by a right-hand-rule.
  - Fingers curl in direction of θ, ω, α, τ, J, thumb points in the direction of the vector
- For most problems, all variables can be considered either clockwise (CW) or counter-clockwise (CCW).

Torque

- How do you make something rotate? Very intuitive!
  - Larger force clearly gives more "twist".
  - Force needs to be in the right direction (perpendicular to a line to the axis is ideal).
  - The "twist" is bigger if the force is applied farther away from the axis (bigger lever arm).
- In math-speak: $\vec{\tau} = \vec{r} \times \vec{F} \qquad |\tau| = |r||F|\sin(\phi)$



More Ways to Think of Torque

- Magnitude of the force times the component of the distance perpendicular to the force (aka lever arm).
- Magnitude of the radial distance times the component of the force perpendicular to the radius.
- Direction from Right-Hand-Rule for cross-products and can also be thought of as clockwise (CW) or counter-clockwise (CCW).
- For torque, gravity acts at the center of mass.

Conditions for Equilibrium

- Same as before: $\Sigma\vec{F} = 0$
  - It's totally irrelevant where the forces are applied to an object, only their direction and magnitude matters.
  - This gives one independent equation per dimension.
- Additional condition: $\Sigma\vec{\tau} = 0$
  - This is true for ***any*** axis. However, if all of the forces are in the same plane (the only type of problem we will consider in this class), you only get ***one*** additional independent equation by considering rotation.

Equations for Dynamics

- Same as before: $\Sigma\vec{F} = M\vec{a}$
  - Only the direction and magnitude of the forces matter.
  - This gives one independent equation per dimension.
- Additional condition: $\Sigma\vec{\tau} = I\vec{\alpha}$
  - This is true for ***any fixed*** axis (for example, a pulley).
  - In addition, this equation holds for an axis through the center of mass, even if the object moves or accelerates.
  - As for statics, if all of the forces are in the same plane, you only get ***one*** additional independent equation by considering rotation.

Inertia in Rotation

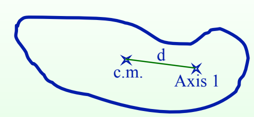
- Depends linearly on the total mass
- Depends on how the mass is distributed. Mass farther from the axis is harder to rotate.
- The same object could have a different moment of inertia depending on the choice of axis.
- In the equation: $\Sigma\vec{\tau} = I\vec{\alpha}$ all three quantities need to be calculated using the same axis (either a fixed axle or the center of mass).

Moment of Inertia

- Most easily derived by considering Kinetic Energy (to be discussed next week).
- $I = \Sigma m_i r_i^2 = \int r^2 dm$
- Some simple cases are given in the textbook on page 342, you should be able to derive those below except for the sphere. Will be on formula sheet.
  - Hoop (all mass at same radius) $I = MR^2$
  - Solid cylinder or disk $I = (1/2)MR^2$
  - Rod around end $I = (1/3)ML^2$
  - Rod around center $I = (1/12)ML^2$
  - Sphere $I = (2/5)MR^2$

Parallel Axis Theorum

- Very simple way to find moment of inertia for a large number of strange axis locations.



- $I_1 = I_{c.m.} + Md^2$ where M is the total mass.