

## ↻ Last Lecture

- ↻ Statics and dynamics of rotational motion

## ↻ Today

- ↻ Everything you need to know about dynamics of rotation

## ↻ Important Concepts

- ↻ Equations for angular motion are mostly identical to those for linear motion with the names of the variables changed.
- ↻ Location where forces are applied is now important.
- ↻ Rotational inertia or moment of inertia (rotational equivalent of mass) depends on how the material is distributed relative to the axis.

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## Important Reminders

- ↻ Lectures will be M 11-12, T&W 10-12, F 11-12.
- ↻ Check schedule on web for new times and rooms for some recitations (all are still on Thursday).
- ↻ Switching of recitations will be permitted if you have a conflict with another IAP activity.
- ↻ Contact your tutor about session scheduling
  - ↻ Students working with Stephane Essame reassigned.
- ↻ Mastering Physics due today at 10pm.
- ↻ Pset due this Friday at 11am.

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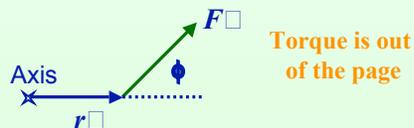
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## Torque

### ↻ How do you make something rotate? Very intuitive!

- ↻ Larger force clearly give more “twist”.
- ↻ Force needs to be in the right direction (perpendicular to a line to the axis).
- ↻ The “twist” is bigger if the force is applied farther away from the axis (bigger lever arm).

↻ In math-speak:  $\vec{\tau} = \vec{r} \times \vec{F}$      $|\tau| = |r||F|\sin(\phi)$



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## More Ways to Think of Torque

- ↻ Magnitude of the force times the component of the distance perpendicular to the force (aka lever arm).
- ↻ Magnitude of the radial distance times the component of the force perpendicular to the radius.
- ↻ Direction from Right-Hand-Rule for cross-products and can also be thought of as clockwise (CW) or counter-clockwise (CCW).
- ↻ For torque, gravity acts at the center of mass.

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## Equations for Dynamics

- Same as before:  $\Sigma \vec{F} = M\vec{a}$ 
  - Only the direction and magnitude of the forces matter.
  - This gives one independent equation per dimension.
- Additional condition:  $\Sigma \vec{\tau} = I\vec{\alpha}$ 
  - This is true for **any fixed** axis (for example, a pulley).
  - In addition, this equation holds for an axis through the center of mass, even if the object moves or accelerates.
  - As for statics, if all of the forces are in the same plane, you only get **one** additional independent equation by considering rotation.

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## Moment of Inertia

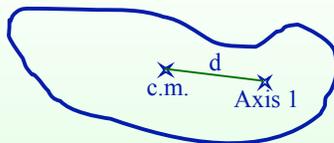
- $I = \Sigma m_i r_i^2 = \int r^2 dm$ 
  - Hoop (all mass at same radius)  $I=MR^2$
  - Solid cylinder or disk  $I=(1/2)MR^2$
  - Rod around end  $I=(1/3)ML^2$
  - Rod around center  $I=(1/12)ML^2$
  - Sphere  $I=(2/5)MR^2$
- The same object could have a different moment of inertia depending on the choice of axis.
- In the equation:  $\Sigma \vec{\tau} = I\vec{\alpha}$  all three quantities need to be calculated using the same axis.

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## Parallel Axis Theorem

- Very simple way to find moment of inertia for a large number of strange axis locations.



- $I_1 = I_{c.m.} + Md^2$  where  $M$  is the total mass.

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## Everything you need to know for Linear & Rotational Dynamics

- $\Sigma \vec{F} = M\vec{a}$ 
  - This gives one independent equation per dimension.
- $\Sigma \vec{\tau} = I\vec{\alpha}$ 
  - This is true for **any fixed** axis and for an axis through the center of mass, even if the object moves or accelerates.
  - For problems in 8.01, you only get **one** additional independent equation by considering rotation.
- Rolling **without** slipping:  $v = R\omega$   $a = R\alpha$   $f \neq \mu N$
- Rolling **with** slipping:  $v \neq R\omega$   $a \neq R\alpha$   $f = \mu N$

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