- Last Lecture
  - Energy and Momentum of rotation
- Today
  - More about Momentum of rotation
- Important Concepts
  - Kinetic energy of rotation adds a new term to the same energy equation, it does not add a new equation.
  - Momentum of rotation gives an additional equation
    - There is the additional complication that the moment of inertia can change.
    - For particles in orbit, angular momentum gives information about the direction as well as the speed

Angular Momentum for Orbits

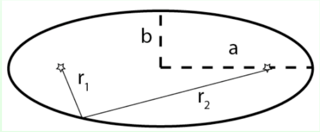
- Conserved because gravity points radially so it does not exert any torque.
- Point particle: $\vec{L} = \vec{r} \times \vec{p} \qquad |L| = |r||p|\sin\phi$
- Conservation of angular momentum is a separate equation from conservation of energy.
- The resulting equations are:

$$r_1 m v_1 \sin\phi_1 = r_2 m v_2 \sin\phi_2$$

$$\frac{-GMm}{r_1} + \tfrac{1}{2} m v_1^2 = \frac{-GMm}{r_2} + \tfrac{1}{2} m v_2^2$$

Connection of L and E to Orbit

- In general, orbits are ellipses: $r_1 + r_2 = 2a$
- The semi-major axis (denoted "a" in drawing) is determined by the total mechanical energy.
- The semi-minor axis (denoted "b" in drawing) is determined by energy and angular momentum.



L and E and a and b

- Basic equations: $r_1 m v_1 \sin\phi_1 = L = r_2 m v_2 \sin\phi_2$

$$\frac{-GMm}{r_1} + \tfrac{1}{2} m v_1^2 = E = \frac{-GMm}{r_2} + \tfrac{1}{2} m v_2^2$$

- Substitute and do algebra to find: $E = \frac{-GMm}{2a}$
- And also: $b = \frac{L}{\sqrt{-2mE}}$ (don't forget that $E<0$)

Torque Checklist

- Make a careful drawing showing ***where*** forces act
  - Clearly indicate what axis you are using
  - Clearly indicate whether CW or CCW is positive
- For each force:
  - If force acts at axis or points to or away from axis, $\tau=0$
  - Draw (imaginary) line from axis to point force acts. If distance and angle are clear from the geometry $\tau=F r\sin(\theta)$
  - Draw (imaginary) line parallel to the force. If distance from axis measured perpendicular to this line (lever arm) is clear, then the torque is the force times this distance
- Don't forget CW versus CCW, is the torque + or −