- Last Lecture
  - Conclusion of Angular Momentum
- Today
  - Final Exam Review
- Suggestions
  - Focus on basic procedures, not final answers.
  - Make sure you understand all of the equation sheet.
  - Look over the checklists and understand them.
  - Work on practice problems without help or books.
  - Get a good night's sleep.

Important Reminders

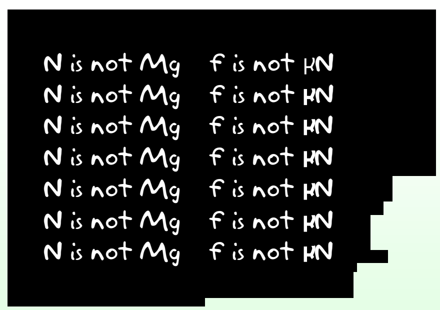
- Sorry about the last minute Mastering Physics problems.
- Final Exam is next Monday: 9am - noon.
- Question & Answer Review Sunday 1-4pm
  - 1-2pm
  - 2-4pm
- Sadly no extra office hours, would not be healthy for you or for me
- If you missed the course evaluations and diagnostic exam on Wed, they are available today

Problem-Solving Strategy 4-steps

- Don't try to see your way to the final answer
  - Focus on the physical situation, not the specific question
- Think through the techniques to see which one (or ones) apply to all or part of the situation
  - Focus on the conditions under which techniques work
- Think carefully about the geometry
  - Here is the one place where lots of practice can help
- Make sure you are efficient in applying techniques
  - Here is one place where memorization can help

Helpful Hints

- Don't memorize special cases (N=mg, for example).
- Think about why things you write are true
  - For example, never write $f=\mu N$ without thinking (or preferably writing down) why that is true
- Draw a careful picture.
- Think about special cases ($\theta=0$, for example) to check that you have the geometry correct.
- Watch out for missing minus signs.



- Problem Solving Tool: Setting up
  - Make a careful drawing
  - Think carefully about all of the forces
  - Chose an axis, put it on your drawing
  - Think carefully about the angles
- Problem Solving Tool: Component checklist
  - Loop through vectors:
    - Is there a component?
    - Is there an angle factor
    - Is it sine or cosine?
    - Is it positive or negative?

Key Kinematics Concepts

- Change=slope=derivative

$$v_x = \frac{dx}{dt} \qquad a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

  - velocity is the slope of position vs t, acceleration is the slope of velocity vs t and the curvature of position vs t
- Even in simple 1D motion, you must understand the vector nature of these quantities
- Initial conditions
- All formulas have assumptions

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Circular Motion Summary

- Motion in a circle with constant speed and radius is accelerated motion.
- The velocity is constant in magnitude but changes direction. It points tangentially.
- The acceleration is constant in magnitude but changes direction. It points radially inward.
- The magnitude of the acceleration is given by:

$$|\vec{a}| = \frac{v^2}{R}$$

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Newton's Three Laws

1) If $\vec{v}$ is constant, then $\Sigma\vec{F}$ must be zero and if $\Sigma\vec{F}=0$, then $\vec{v}$ must be constant.

2) $\sum \vec{F} = m\vec{a}$

3) Force due to object A on object B is always exactly equal in magnitude and always exactly opposite in direction to the force due to object B on object A.

Some Advice

- Your instincts are often wrong. Be careful!
- $\sum \vec{F} = m\vec{a}$ is your friend. Trust what it tells you.

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- Problem Solving Tool:(Revised)Free-Body Checklist
  - Draw a clear diagram of (each) object
  - Think carefully about all of the forces on (each) object
  - Think carefully about the angles of the forces
  - Chose an axis, put it on your drawing
  - Think carefully about the acceleration and put what you know on your drawing
  - Calculate components: $\sum F_x = ma_x \quad \sum F_y = ma_y \quad \ldots$
  - Solve…

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Properties of Friction - Magnitude

- Not slipping: The magnitude of the friction force can only be calculated from $\sum \vec{F} = m\vec{a}$. However, it has a maximum value of $|f| \le \mu_s N$
- Just about to slip: $|f| = \mu_s N$ where N is the Normal force and μ_s is the coefficient of static friction which is a constant that depends on the surfaces
- Slipping: $|f| = \mu_k N$ where N is the Normal force and μ_k is the coefficient of kinetic friction which is a constant that depends on the surfaces
- Note: $\mu_s \ge \mu_k$

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Properties of Spring Force

- The direction is always unambiguous!
  - In for stretched spring, out for compressed spring.
- The magnitude is always unambiguous!
  - $|F|=k(\ell-\ell_0)$
- Two possibilities for confusion.
  - Double negative: Using F=−kx where it doesn't belong
  - Forgetting the "unstretched length", ℓ_0

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Work done by a Force

- Not a vector quantity (but vector concepts needed to calculate its value).
- Depends on both the direction of the force and the direction of the motion.
- Four ways of saying the same thing
  - Force times component of motion along the force.
  - Distance times the component of force along the motion.
  - W=Σ|F||d|cos(θ) where θ is the angle between F and d.
  - $W = \int \vec{F} \bullet d\vec{s}$ where the "s" vector is along the path

Checklist to use Work/Energy

- Clearly define what is "inside" your system.
- Clearly define the initial and final conditions, which include the location and speed of all object(s)
- Think carefully about all forces acting on all objects
- All forces must be considered in the Work term or in the Potential Energy term, but never in both.

$$W = \Delta E = E_{Final} - E_{Initial}$$
$$= (KE_{Final} + PE_{Final}) - (KE_{Initial} + PE_{Initial})$$

Work/Energy Summary

- $W = \Delta E = E_F - E_I \quad E = PE + KE \quad KE = \frac{1}{2}mv^2$
- $PE_{gravity} = mgy \quad PE_{spring} = +\frac{1}{2}k\left(L - l_0\right)^2$
- $W = \int \vec{F} \bullet d\vec{s} \quad |W| = |F||ds|\cos(\theta)$
- Every force goes in the work term or in the PE
- Minima and maxima of the PE correspond to F=0, which are equilibrium points. PE minima are stable equilibrium points, maxima are unstable.

Momentum

- Very simple formula: $\vec{p}_{Tot} = \Sigma\left(m_i \vec{v}_i\right)$
  - Note the vector addition!
- Momentum of a system is conserved only if:
  - No net external forces acting on the system.
  - Or, study the system only over a very short time span.

$$\Delta\vec{p}_{Tot} = \int \vec{F} dt$$

Simple Harmonic Motion - Summary

- Basics: $F_x = -kx = m\, d^2x / dt^2$
- General solution: $x = A\cos(\omega t + \phi) \qquad \omega = \sqrt{k/m}$
- Practical solutions:
  - t=0 when position is maximum and therefore v=0 $\quad \phi = 0$
    - $x = A\cos(\omega t)$
    - $v_x = -A\omega\sin(\omega t)$
    - $a_x = -A\omega^2\cos(\omega t)$
  - t=0 when speed is maximum and therefore a=0 and therefore x=0 $\quad \phi = \frac{\pi}{2}$
    - $x = A\sin(\omega t)$
    - $v_x = A\omega\cos(\omega t)$
    - $a_x = -A\omega^2\sin(\omega t)$

Gravity Summary

- Numerical constant: $G = 6.673 \times 10^{-11} \frac{Nm^2}{kg^2}$
- Force: $F_G = -\frac{GM_1M_2}{r^2}\hat{r}$
- Energy: $PE(r) = -\frac{GM_1M_2}{r}$
- Escape velocity: $E_{Total} = KE + PE = 0$

Some Derived Results

- Found from applied F=ma
- Pressure versus height (if no flow):

  $P_2 - P_1 = -\rho g (y_2 - y_1)$ y is positive upward

  $P = P_0 + \rho g h$
- Buoyancy forces (causes things to float):

  $F_B = \rho_{fluid} g V_{disp}$ V_{disp} is the volume of fluid displaced

  $\frac{V_{submerged}}{V_{object}} = \frac{\rho_{object}}{\rho_{fluid}}$

Ideal Gas law

- Physicist's version: $PV = NkT$
  - N=number of molecules or separate atoms
  - Boltzman constant: $k = 1.38 \times 10^{-23}\ Joule/{}^{\circ}K\ \ per\ \ molecule$
- Chemist's version: $PV = nRT$
  - n=number of moles
    - Avogadro's number: $1\ mole = 6.0 \times 10^{23}\ \ atoms\ or\ molecules$
  - Different constant: $R = 8.3\ Joules/{}^{\circ}K\ \ per\ \ mole$

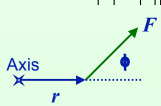
Kinematics Variables

- Position x
- Velocity v
- Acceleration a
- Force F
- Mass M
- Momentum p
- Angle θ
- Angular velocity ω
- Angular acceleration α
- Torque τ
- Moment of Inertia I
- Angular Momentum L

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Torque

- How do you make something rotate? Very intuitive!
  - Larger force clearly gives more "twist".
  - Force needs to be in the right direction (perpendicular to a line to the axis is ideal).
  - The "twist" is bigger if the force is applied farther away from the axis (bigger lever arm).
- In math-speak: $\vec{\tau} = \vec{r} \times \vec{F}$ $|\tau| = |r||F|\sin(\phi)$



Torque is out of the page

Torque Checklist

- Make a careful drawing showing ***where*** forces act
  - Clearly indicate what axis you are using
  - Clearly indicate whether CW or CCW is positive
- For each force:
  - If force acts at axis or points to or away from axis, τ=0
  - Draw (imaginary) line from axis to point force acts. If distance and angle are clear from the geometry τ=Frsin(θ)
  - Draw (imaginary) line parallel to the force. If distance from axis measured perpendicular to this line (lever arm) is clear, then the torque is the force times this distance
- Don't forget CW versus CCW, is the torque + or −

Right Hand Rules

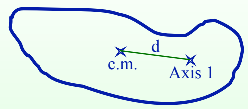
- For angular quantities: θ, ω, τ
  - Curl the fingers of your right hand in the direction of the motion or acceleration or torque and your thumb points in the direction of the vector quantity.
    - The vector direction for "clockwise" quantities is "into the page" and "counterclockwise" is "out of the page"
- Vector cross-products (torque, angular momentum of point particle) generally A×B
  - Point the fingers of your right hand along the first vector, curt your fingers to point along second vector, your thumb points in the direction of the resulting vector

Moment of Inertia

- Most easily derived by considering Kinetic Energy (to be discussed next week).
- $I = \Sigma m_i r_i^2 = \int r^2 dm$
- Some simple cases are given in the textbook on page 342, you should be able to derive those below except for the sphere. Will be on formula sheet.
  - Hoop (all mass at same radius) $I=MR^2$
  - Solid cylinder or disk $I=(1/2)MR^2$
  - Rod around end $I=(1/3)ML^2$
  - Rod around center $I=(1/12)ML^2$
  - Sphere $I=(2/5)MR^2$

Parallel Axis Theorum

- Very simple way to find moment of inertia for a large number of strange axis locations.



- $I_1 = I_{c.m.} + Md^2$ where M is the total mass.

Everything you need to know for Linear & Rotational Dynamics

- $\Sigma \vec{F} = M\vec{a}$
- $\Sigma \vec{\tau} = I\vec{\alpha}$
  - This is true for ***any fixed*** axis and for an axis through the center of mass, even if the object moves or accelerates.
- Rolling ***without*** slipping: $v = R\omega \quad a = R\alpha \quad f \neq \mu N$
  - Friction does NOT do work!
- Rolling ***with*** slipping: $v \neq R\omega \quad a \neq R\alpha \quad f = \mu N$
  - Friction does work, usually negative.
  - Rarely solvable without using force and torque equations!

Kinetic Energy with Rotation

- Adds a new term not a new equation!
- Rotation around any fixed pivot: $KE = \frac{1}{2} I_{pivot}\omega^2$
- Moving and rotating: $KE = \frac{1}{2} I_{CM}\omega^2 + \frac{1}{2} M_{Tot} v_{CM}^2$

Pendulums

- Simple pendulum: Small mass at the end of a string
  - Period is $T = 2\pi\sqrt{l/g}$ where l is the length from the pivot to the center of the object.
- Physical pendulum: More complex object rotating about any pivot
  - Period is $T = 2\pi\sqrt{I/Mgl}$ where l is the distance from the pivot to the center of mass of the object, M is the total mass, and I is the moment of inertia around the pivot.

Angular Momentum

- Conserved when external torques are zero or when you look over a very short period of time.
  - True for any fixed axis and for the center of mass
- Formula we will use is simple: $\vec{L} = I\vec{\omega}$
  - Vector nature (CW or CCW) is still important
- Point particle: $\vec{L} = \vec{r} \times \vec{p}$
- Conservation of angular momentum is a separate equation from conservation of linear momentum
- Angular impulse: $\vec{\tau} = \frac{d\vec{L}}{dt} \qquad \Delta\vec{L} = \int \vec{\tau}\,dt$