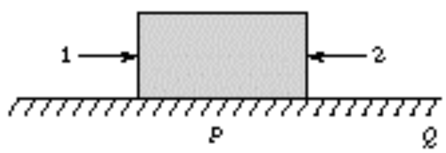
Problem Solving Kinetic Energy and Work
Challenge Problem Solutions

Problem1

Two people push in opposite directions on a block that sits atop a frictionless surface (The soles of their shoes are glued to the frictionless surface). If the block, originally at rest at point P, moves to the right without rotating and ends up at rest at point Q, describe qualitatively how much work is done on the block by person 1 relative to that done by person 2?



Solution: Initially the block is at rest. After the pushing has ended, the block ends at rest, so the change in kinetic energy is zero. From the work-kinetic energy theorem, this implies that the total work done on the block is zero. The total work done on the block is the sum of the work done on the block by each person. Since the block moves to the right, person who pushes the block to the right does a positive work, and the person pushing to the left does negative work. Since the total is zero, the magnitude of the work done by each person is equal.

Problem 2

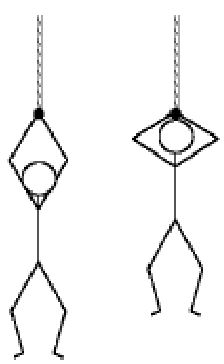
An inextensible rope hangs from the ceiling of a gym. Jamal, who has mass M, grabs the rope and climbs to a height h. Assume that the acceleration of gravity is g. The following questions refer to a time period that begins when Jamal is standing at rest on the ground, and ends when he is hanging motionlessly on the rope at height h. You can write the answers without explanation.

(a) How much work is done on Jamal by gravity?

Answer: Gravity exerts a constant force Mg downward on Jamal, while he moves upward by a distance h. The work done by gravity is therefore $W = Mgh$.

(b) How much work is done on Jamal by the rope?

Answer: This question is tricky, but the answer is that the rope does no work. The question is very similar to the "Energy Conservation Riddle III," which we discussed in L06 on 10/10/08 — you can find the slides posted on the L06 website for that date. The identical problem is discussed in detail in the 8.01 Study Guide (by Busza, Cartwright, and Guth), Problem 4D.1(c). A similar problem is discussed in the Young and Freedman textbook, at Fig. 6.15,which shows a skater pushing himself from a wall. It is tempting to say that the rope does work on Jamal, since the rope exerts an upward force on Jamal, and Jamal moves upward. The error in this point of view, however, is seen by looking closely at the interface between the rope and Jamal — namely, the hand that holds the rope. As Jamal lifts himself on the rope, the rope applies an upward force on Jamal's hand, but the hand holds fastly to the rope and does not move. The basic upward stroke of the climber is shown crudely in the diagram at the right. The muscles in the shoulders and arms cause the joints at the shoulder and elbows to bend, pulling the body upwards. The work is done by these muscles, which exert an upward force on Jamal's torso as it moves upward. For Jamal to do work on himself, as happens in this situation, it is crucial that Jamal is not a rigid object — he changes his shape as he climbs. Note that the energy considerations in this situation are very different from what they would be if Jamal were hanging onto the rope as someone else pulled the rope upward.



Problem 3 Object Sliding on Inclined Plane and Rough Surface

An object of mass $m = 4.0\,\text{kg}$, starting from rest, slides down an inclined plane of length $l = 3.0\,\text{m}$. The plane is inclined by an angle of $\theta = 30^\circ$ to the horizontal. The coefficient of kinetic friction $\mu_k = 0.2$. At the bottom of the plane, the object slides along a rough horizontal surface with a coefficient of kinetic friction $\mu_k = 0.3$ until it comes to rest. The goal of this problem is to find out how far the object slides along the rough surface. You will need to assume that the object does not change speed at the bottom of the plane, when the object begins to more horizontally.

a) Describe how you will model this motion. Include a free body diagram for the object while it is on the inclined plane and while it is sliding along the horizontal surface. Explain whether Newton's Second Law or the Work-Kinetic Energy Theorem provides an easier approach to this problem.

b) What is the work done by the friction force while the mass is sliding down the inclined plane? Is this work positive or negative?

c) What is the work done by the gravitational force while the mass is sliding down the inclined plane? Is this work positive or negative?

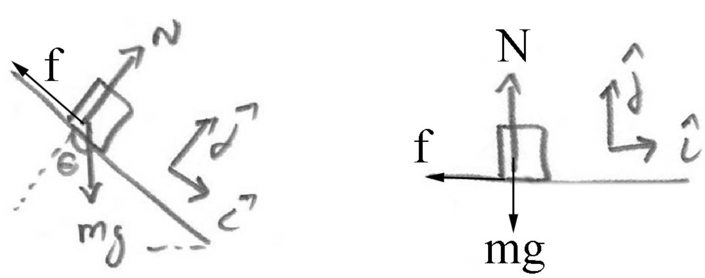
d) What is the kinetic energy of the mass just at the bottom of the inclined plane?

e) What is the work done by the friction force while the mass is sliding along the ground? Is this work positive or negative?

f) How far does the object slide along the rough surface?

Solution:

a) While the object is sliding down the inclined plane the kinetic energy is increasing due to the positive work done on the object by the gravitational force and the negative work (smaller in magnitude) done by friction. As the object slides along the level surface, the (negative) work done by the friction force slows the object down. We will use the work-kinetic energy theorem to calculate the change in kinetic energy for each stage. The free body diagram on the inclined plane and on the level surface are shown below.



b) Choose a coordinate system with the origin at the top of the inclined plane and the positive x-direction pointing down the inclined plane. The magnitude N of the normal force is determined from Newton's Second Law applied to the normal direction to the inclined plane,

$$N - mg\cos\theta = 0\,. \tag{3.1}$$

The work done by the friction force is then

$$W_{\text{friction}} = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{x_0=0}^{x_0=l} F_x dx = -(\mu_{\text{k}})_{\text{incline}} N\,l = -(\mu_{\text{k}})_{\text{incline}} mg\cos\theta\, l < 0\,. \tag{3.2}$$

Using the given numerical values,

$$W_{\text{friction}} = (0.2)(4.0\,\text{kg})\left(9.8\text{m}\cdot\text{s}^{-2}\right)(3.0\,\text{m})\left(\cos 30^{\circ}\right) = 20.4\text{ J} \tag{3.3}$$

or $20\,\text{J}$ to the precision allowed by the given value of the coefficient of kinetic friction.

c) The work done by the gravitational force is product of the vertical component of the gravitational force and the change in vertical height h,

$$\begin{aligned} W_{\text{grav}} &= (-mg)\left(h_f - h_0\right) \\ &= (-mg)(-l\sin\theta) \\ &= mgl\sin\theta > 0. \end{aligned} \tag{3.4}$$

Using the given numerical values,

$$W_{\text{grav}} = (4.0\,\text{kg})\left(9.8\text{ m}\cdot\text{s}^{-2}\right)(3.0\,\text{m})(\sin 30^{\circ}) = 58.8\text{ J} \tag{3.5}$$

or $59\,\text{J}$ to the given precision.

d) The total work is

$$W_{\text{total}} = W_{\text{grav}} + W_{\text{friction}} = mgl\left(\sin\theta - (\mu_{\text{k}})_{\text{incline}}\cos\theta\right). \tag{3.6}$$

Using the given numerical values,

$$W_{\text{total}} = (4.0\,\text{kg})\left(9.8\text{m}\cdot\text{s}^{-2}\right)(3.0\,\text{m})\left(\sin 30^{\circ} - (0.2)\cos 30^{\circ}\right) = 38.4\text{ J} \tag{3.7}$$

where extra significant figures have kept in this intermediate calculation to reduce roundoff error (note that to two figures, the numerical result in (3.7) is not the sum of the results of (3.3) and (3.5)).

The object started from rest, and so the change in kinetic energy is equal to the final kinetic energy at the bottom of the incline,

$$\Delta K_{\text{incline}} = \frac{1}{2} m v_{\text{bottom}}^2 \quad \textbf{(3.8)}$$

The work-kinetic energy theorem $W_{\text{total}} = \Delta K_{\text{incline}}$ becomes $\Delta K_{\text{incline}} = 38.4\,\text{J}$. This would allow determination of the speed v_{bottom} at the bottom of the ramp if parts e) and f) were to be attempted using Newton's Second Law instead of the Work-Kinetic Energy Theorem. This alternate solution will not be presented here.

e) While sliding on the horizontal surface, the work done is the change in kinetic energy. The only work done is that of friction, and the change in kinetic energy is the final kinetic energy of zero minus the kinetic energy $\Delta K_{\text{incline}}$ at the bottom of the ramp, or

$$W_{\text{friction}} = \Delta K_{\text{horiz}} = 0 - \Delta K_{\text{incline}} = -\Delta K_{\text{incline}} = -38.4\,\text{J}\,, \quad \textbf{(3.9)}$$

keeping an extra figure for the intermediate calculation.

f) To use the Work-Kinetic Energy Theorem, choose a coordinate system with the origin at the base of the inclined plane and the positive x-direction in the direction the object moves along the plane and denote the distance the block moves along the surface as Δx. The magnitude N of the normal force on the object is determined from Newton's Second Law applied to the normal direction to the plane,

$$N - mg = 0\,. \quad \textbf{(3.10)}$$

The work done by the friction force is then

$$W_{\text{friction}} = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{x_0=0}^{x_0=\Delta x} F_x\, dx = -\mu_{\text{k}} N \Delta x = -\mu_{\text{k}} mg\, \Delta x < 0 \quad \textbf{(3.11)}$$

where in Equation (3.11), is the coefficient of kinetic friction between the block and the horizontal surface. Solving for Δx,

$$\Delta x = \frac{W_{\text{friction}}}{-\mu_{\text{k}} mg} = \frac{-38.4\,\text{J}}{-(0.3)(4.0\,\text{kg})\left(9.8\,\text{m}\cdot\text{s}^{-2}\right)} = 3.3\,\text{m}\,, \quad \textbf{(3.12)}$$

or $3\,\text{m}$ to the one significant figure allowed from the given precision for μ_{k}.

Note: If we wished to find the answer to part f), the distance the object slides on the horizontal surface, we could consider the entire motion from the release at the top of the inclined plane to coming to rest on the horizontal. The total change in kinetic energy is zero, and

$$0 = \Delta K = W_{\text{total}} = W_{\text{grav}} + (W_{\text{friction}})_{\text{incline}} + (W_{\text{friction}})_{\text{horizontal}} \tag{3.13}$$

$$\begin{aligned} 0 &= mgl\sin\theta - mgl\mu_{\text{k, incline}}\cos\theta - \mu_{\text{k, horiz}}mg\Delta x \\ \Delta x &= \frac{mgl\sin\theta - mgl\mu_{\text{k, incline}}}{\mu_{\text{k, horiz}}mg} = \frac{gl\sin\theta - gl\mu_{\text{k, incline}}}{\mu_{\text{k, horiz}}g}, \end{aligned} \tag{3.14}$$

which is Equation (3.12) in symbolic form.

Problem 4: ***Work-kinetic energy object sliding on inclined plane and rough surface***

An object of mass $m = 4.0\,kg$, starting from rest, slides down an inclined plane of length $l = 3.0\,m$. The plane is inclined by an angle of $\theta = 30^0$ to the ground. The coefficient of kinetic friction $\mu_k = 0.2$. At the bottom of the plane, the mass slides along a rough surface with a coefficient of kinetic friction $\mu_k = 0.3$ until it comes to rest. The goal of this problem is to find out how far the object slides along the rough surface.

a) Describe how you will model this motion. Include a free body diagram for the object while it is on the inclined plane and while it is sliding along the horizontal surface. Explain whether Newton's Second Law or the Work-Kinetic Energy Theorem provides an easier approach to this problem.

b) What is the work done by the friction force while the mass is sliding down the inclined plane? Is this positive or negative?

c) What is the work done by the gravitational force while the mass is sliding down the inclined plane? Is this positive or negative?

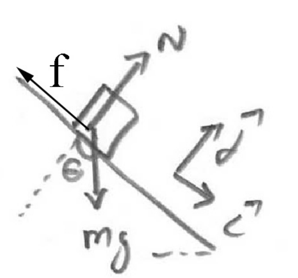
d) What is the kinetic energy of the mass just at the bottom of the inclined plane?

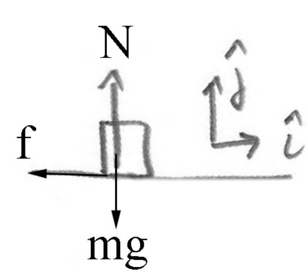
e) What is the work done by the friction force while the mass is sliding along the ground? Is this positive or negative?

f) How far does the object slide along the rough surface?

Solution:

a) While the object is sliding down the inclined plane the kinetic energy is increasing due to the positive work done on the object by the gravitational force and the negative work (smaller in magnitude) done by friction force. As the object slides along the level surface, the (negative) work done by the friction force slows the object down. We will use the work-kinetic energy theorem to calculate the change in kinetic energy for each stage. The free body diagram on the inclined plane and on the level surface are shown below.





b) Choose a coordinate system with the origin at the top of the inclined plane and the positive x-direction pointing down the inclined plane. Then the work done by the friction force is

$$W_{\text{friction}} = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{x_0=0}^{x_0=l} F_x dx = -(\mu_k)_{inclined} Nl = -(\mu_k)_{inclined} mg\cos\theta l < 0$$

$$W_{\text{friction}} = (0.2)(4.0\,\text{kg})(9.8\text{m}\cdot\text{s}^{-2})(3.0\,\text{m})(\cos(30^o)) = 20.4\text{ J}$$

Note that the normal force is determined from Newton's Second Law applied to the normal direction to the inclined plane.

$$N - mg\cos\theta = 0$$

c) the work done by the gravitational force is just

$$W_{grav} = -mg(\text{h}_f - h_0)$$

note that $(h_f - h_0) = \text{-}l\sin\theta$. So the work done by the gravitation force is

$$W_{grav} = mgl\,sin\theta > 0$$

The magnitude of this work is

$$W_{grav} = (4.0\,\text{kg})(9.8\text{ m}\cdot\text{s}^{-2})(3.0\,\text{m})(\sin(30^o) = 58.8\text{ J}$$

d) The total work is

$$W_{\text{total}} = W_{grav} + W_{\text{friction}} = mgl(\sin\theta - (\mu_k)_{inclined}\cos\theta)$$

$$W_{\text{total}} = mgl(\sin\theta - (\mu_k)_{inclined}\cos\theta) = (4.0\,\text{kg})(9.8\text{m}\cdot\text{s}^{-2})(3.0\,\text{m})(\sin(30^o) - (0.2)(\cos(30^o)) = 38.4\text{ J}$$

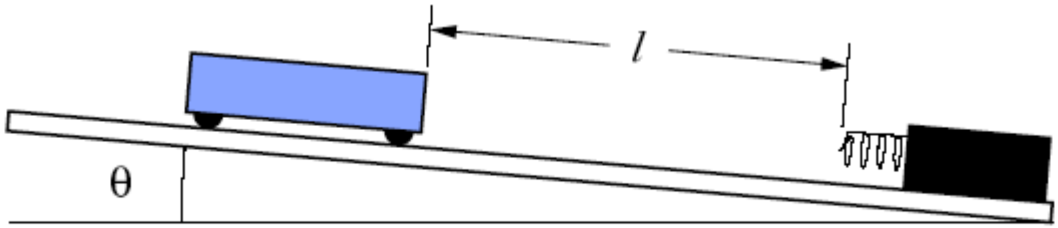
The change in kinetic energy is just equal to the final kinetic energy at the bottom of the incline because the started from rest,

$$\Delta K = \frac{1}{2}mv^2{}_{bottom}$$

So the work-kinetic energy theorem $W_{\text{total}} = \Delta K$ becomes

$$mgl(\sin\theta - (\mu_k)_{inclined}\cos\theta) = \frac{1}{2}mv^2{}_{bottom} = 38.4\text{ J}$$

Problem 5:



A cart of mass M rolls down a track inclined at an angle θ. The cart starts from rest a distance l up the track from a spring, and rolls down to collide with the spring.

1. Assuming no non-conservative work is done, what is the speed of the cart when it first contacts the spring? (Express your answer in terms of the given variables and the gravitational acceleration g.)

2. Suppose the spring has a force constant k. What is the peak force compressing the spring during the collision?

Problem 5 Solutions:

1) The work done on the cart by the force of gravity before it comes in contact with the spring is

$$W = (mg\sin\theta)l$$

and is equal to the change in the kinetic energy of the cart $W = K_f - K_i = mv_f^2/2$. Thus the velocity of the cart when it first contacts the spring is

$$v_c = \sqrt{2gl\sin\theta}\,.$$

2) The peak force is achieved when the spring compression is at maximum, i.e. just before the spring is about to bounce back. Assuming this point is a distance d from the point where the cart first contacts the spring, the work done by the spring on the cart during the collision is $W = -kd^2/2$. The gravitational force does positive work $W = mg\sin\theta d$ on this part of the trajectory. Again, the change of the cart's kinetic energy is equal to the work done by the external forces:

$$W_{total} = mg\sin\theta d - \frac{kd^2}{2} = K_f - K_i = 0 - mv_c^2$$

Solving for d, we obtain

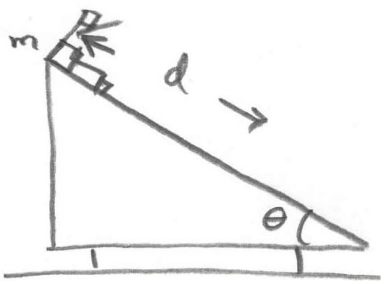
$$d = \frac{mg\sin\theta}{k}\left(1 + \sqrt{1 + \frac{2lk}{mg\sin\theta}}\right)$$

The corresponding force is

$$F_{\max} = kd = mg\sin\theta\left(1 + \sqrt{1 + \frac{2lk}{mg\sin\theta}}\right)$$

Problem 6: slide (Energy, Force, and Kinematics)

A child's playground slide is $d = 5.0m$ in length and is at an angle of $\theta = 2.0\times10^{1} deg$ with respect to the ground. A child of mass $m_b = 2.0\times10^{1} kg$ starts from rest at the top of the slide. The coefficient of sliding friction for the slide is $\mu_k = 0.2$.



a) What is the total work done by the friction force on the child?

b) What is the speed of the child at the bottom of the slide?

c) How long does the child take to slide down the ramp?

Problem 6 Solutions:

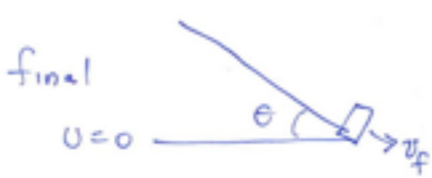
$\hat{i}: mg\sin\theta = f_k = ma_x$

$\hat{j}: N - mg\cos\theta = 0$

$f_k = \mu_k N = \mu_k mg\cos\theta$

$$W_{nc} = -f_k d = \mu_k mg\cos\theta$$

$$W_{nc} = -(0.2)(2.0\times10^{1} kg)\left(\frac{9.8m}{s}\right)(\cos 2\theta)(5.0m) = 1.8\times10^{2} J$$



$U_0 = mgh_0 = mgd\sin\theta$

$k_0 = 0 \;,\; E_0 = mgd\sin\theta$

$U_f = 0$

$k_f = \frac{1}{2}mv_f^2$

$E_f = \frac{1}{2}mv_f^2$

$$W_{nc} = \Delta E = \frac{1}{2}mv_f^2 - mgd\sin\theta$$

$$-f_k d = \frac{1}{2}mv_f^2 - mgd\sin\theta$$

$$v_f = \left(2dg\left(\sin - \mu_k\cos\theta\right)\right)^{\frac{1}{2}}$$

$$v_f = \left((2)(5.0m)\left(9.8\frac{m}{s^2}\right)\left(\sin 20^\circ - (0.2)\left(\cos 20^\circ\right)\right)\right)^{\frac{1}{2}} = 3.9m\cdot s^{-1}$$

$$a_x = \frac{1}{m}\left(mg\sin\theta - f_k\right) = g\left(\sin\theta - \mu_k\cos\theta\right)$$

$$v_f = a_x t \Rightarrow t = \frac{v_f}{a_x} = \frac{\left(2dg\left(\sin\theta - \mu_k\cos\theta\right)\right)^{1/2}}{g\left(\sin\theta - \mu_k\cos\theta\right)}$$

$$t = \left(\frac{2d}{g\left(\sin\theta - \mu_k\cos\theta\right)}\right)^{1/2} = \left(\frac{(2)(5.0m)}{\left(9.8\frac{m}{s^2}\right)\left(\sin 20^\circ - (.1)\left(\cos 20^\circ\right)\right)}\right)^{\frac{1}{2}} = 2.7s$$