

OK, now we go to the next problem, which is 7.3.2.

I, Walter Lewin, pull a mass up on a slope. The slope is at an angle θ . The mass that I'm pulling up is m and the kinetic friction coefficient is always a little less than the static friction coefficient is μ_k . And here it that object. And I'm pulling it and the speed along this line is constant. Let's call this the direction y perpendicular to the slope and let's call this x the direction along the slope.

I'm going to make free body diagram. This is gravity, which is mg . I'm going to decompose gravity in two directions. Let me use blue for that. One which is perpendicular to the slope, which is mg times the cosine of θ and one which is parallel to the slope, which is mg times the sine of θ .

Well then, there is the contact force from the slope onto this object. We normally decompose that into two. As I just mentioned already N and there is friction, which of course, the friction must be in this direction F friction. The friction reaches a maximum value because it's moving at constant speed upwards. That's a given. And here is the force by Walter Lewin. And to repeat myself, the actual contact force, the force by which the slope pushes onto this object is the vectorial sum between N and the friction.

All right, let's now look in the y direction. In the y direction there is no acceleration, so you see immediately that N equals mg times the cosine of θ .

Now I'm going to look in the x direction. The sum of all forces in the x directions I want to be 0 because I know that the speed along this slope-- up the slope is a constant. Well, let's first evaluate what the maximum friction is. We know it's maximum because it is sliding. So it must have reached the maximum value. So the magnitude of the maximum friction equals μ_k because it's moving, so it has the kinetic coefficient times N . And that is mg cosine θ . And so there has to be equilibrium now in forces along the x direction. So the force of Walter Lewin must be-- there are two in this direction. There's is mg sine θ downhill so to speak and there's downhill the frictional force, which is μ_k times mg times cosine θ . And so here you see the required force uphill. The magnitude of this plus the magnitude of this is the force that I have to apply in order to give this object a constant speed.

I hope you notice that when you make μ_k 0, that you find an answer, which is completely trivial. Which is an obvious answer that the force that I have to apply would be exactly mg sine θ , which is a result of course, that you must have seen many times in the past.

Now how much work do I have to do if I displace this object over a vertical distance h ? So I pull it along the slope, but my vertical displacement is h . So here is this slope. Here is the angle θ . And I displace it from here to here. I call that s for historical reasons and the vertical displacement I call h . And you see immediately that h divided by s equals the sine of θ .

The work that I would have to do is the integral from where I begin to where I end of the force by Walter Lewin with the dot product of ds . In this case, the force and ds are in the same direction. And the force is constant. And, as I said, since they're in the same direction, I can leave the dot off. So I can immediately replace this by the force of Walter Lewin times the whole displacement s . Since the force is not changing as a function of s , it is simply this product.

Well, what is s ? s is h divided by sine θ . So the work that Walter Lewin will have to do is the force by Walter Lewin, which we already calculated. Which is mg . Times the sine of θ plus μ_k times the cosine of θ . That's my force and s equals h divided by sine θ . So this is the total work that I have to do. And again, it's always nice to check that you get a trivial answer.

If you take all the friction out, if you take the friction out and you make this 0, then you get as an answer that the work that Walter Lewin has to do equals $mg h$. The famous equation, Massachusetts General Hospital. That's always the work you have to do if there is no friction.

Friction is a non-conservative force. It takes energy out of the system, it converts it to heat. Gravity itself is not a conservative force. Excuse me. Gravity is a conservative force, so it doesn't matter-- the routing that you choose does not matter. And so you always get that the work that you have to do is $mg h$ regardless of where you begin and where you end, as long as the vertical displacement is h . But it is because of the friction that you add a non-conservative force, and that's exactly what you see here. And the work that you have to do now would depend on the fact that you are losing energy. You're losing heat due to friction.