

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

Exam 2: Equation Summary

One Dimensional Kinematics:

$$\vec{v} = d\vec{r} / dt, \quad \vec{a} = d\vec{v} / dt$$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$x(t) = x_0 + v_{x,0}(t-t_0) + \frac{1}{2} a_x(t-t_0)^2 \quad v_x(t) = v_{x,0} + a_x(t-t_0)$$
$$y(t) = y_0 + v_{y,0}(t-t_0) + \frac{1}{2} a_y(t-t_0)^2 \quad v_y(t) = v_{y,0} + a_y(t-t_0)$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

Newton's Third Law:

$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$

Force Laws:

Universal Law of Gravity: $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$, attractive

Gravity near surface of earth: $\vec{F}_{grav} = m_{grav} \vec{g}$, towards earth

Contact force: $\vec{F}_{contact} = \vec{N} + \vec{f}$, depends on applied forces

Static Friction: $0 \leq f_s \leq f_{s,max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k|\Delta x|$, restoring

Kinematics Circular Motion: arc length: $s = R\theta$; angular velocity: $\omega = d\theta/dt$
 tangential velocity: $v = R\omega$; angular acceleration: $\alpha = d\omega/dt = d^2\theta/dt^2$; tangential
 acceleration $a_\theta = R\alpha$.

Period: $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$; **frequency:** $f = \frac{1}{T} = \frac{\omega}{2\pi}$,

Radial Acceleration: $|a_r| = R\omega^2$; $|a_r| = \frac{v^2}{R}$; $|a_r| = 4\pi^2 R f^2$; $|a_r| = \frac{4\pi^2 R}{T^2}$

Center of Mass: $\vec{R}_{cm} = \sum_{i=1}^{i=N} m_i \vec{r}_i / m^{total} \rightarrow \int_{body} dm \vec{r} / m^{total}$;

Velocity of Center of Mass: $\vec{V}_{cm} = \sum_{i=1}^{i=N} m_i \vec{v}_i / m^{total} \rightarrow \int_{body} dm \vec{v} / m^{total}$

Torque: $\vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P$ $|\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_\perp F = r F_\perp$

Static Equilibrium: $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}$; $\vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}$.

Kinetic Energy: $K = \frac{1}{2}mv^2$; $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work: $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$; **Work- Kinetic Energy:** $W^{total} = \Delta K$

Power: $P = \vec{F} \cdot \vec{v} = dK/dt$

Potential Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

Potential Energy Functions with Zero Points:

Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

Inverse Square Gravity: $U_{gravity}(r) = -\frac{Gm_1 m_2}{r}$ with $U_{gravity}(r_0 = \infty) = 0$.

Hooke's Law: $U_{spring}(x) = \frac{1}{2}kx^2$ with $U_{spring}(x = 0) = 0$.

Work- Mechanical Energy: $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech} = (K_f + U_f^{total}) - (K_0 + U_0^{total})$,

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Physics 8.01T

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Exam 2

Name _____

Section: Morning Pritchard/Surrow 10 - 12 _____

Afternoon Dourmashkin 1-3 _____

Table and Group: _____

The following exam consists of five problems. Answers without work shown will not be given any credit. Good luck!

Problem 1 (20 points)

Part a) (4 points) _____

Part b) (4 points) _____

Part c) (4 points) _____

Part d) (4 points) _____

Part e) (4 points) _____

Problem 2 (20 points) _____

Problem 3 (20 points) _____

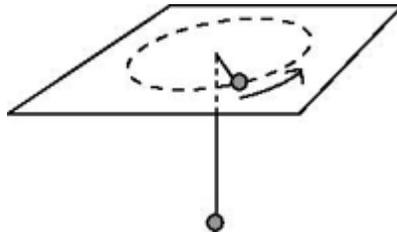
Problem 4 (20 points) _____

Problem 5 (20 points) _____

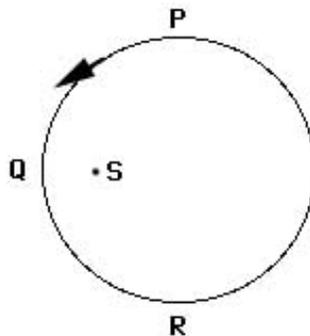
Total (100 points) _____

Problem 1: (20 points) Concept Questions: Circle the correct answer.

- a) **(4 points)** A puck of mass M is moving in a circle at uniform speed on a frictionless table as shown above. It is held by a string which holds a suspended bob, also of mass M , at rest below the table. Half of the length of the string is above the tabletop and half below. What is the centripetal acceleration of the moving puck?

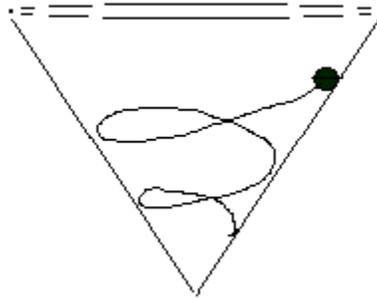


1. less than g
 2. g
 3. greater than g
 4. zero
 5. insufficient information
- b) **(4 points)** An object moves counter-clockwise along the circular path shown below. As it moves along the path its acceleration vector continuously points toward point S . The object



1. speeds up at P , Q , and R .
2. slows down at P , Q , and R .
3. speeds up at P and slows down at R .
4. slows down at P and speeds up at R .
5. speeds up at Q .
6. slows down at Q .
7. No object can execute such a motion.

- c) **(4 points)** A ball is rolling without slipping in a spiral path down the inside of a hollow cone.



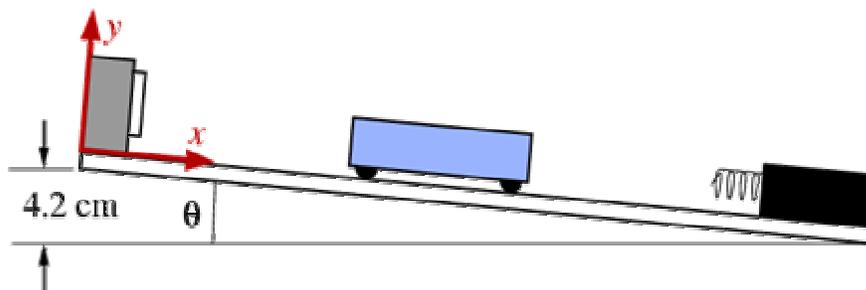
The work done by the inner surface of the cone on the ball is

1. positive
2. zero
3. negative
4. Impossible to determine

d) **(4 points)** Suppose you want to ride your mountain bike up a steep hill. Two paths lead from the base to the top, one twice as long as the other. You start and end at rest. Compared to the average force you would exert if you took the short path, the average force you exert along the longer path is

1. one quarter as much.
2. one third as much.
3. one half as much.
4. the same.
5. undetermined-it depends on the time taken.

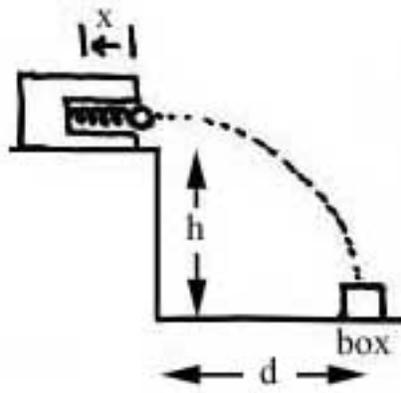
e) **(4 points)** When the cart maximally compresses the spring at the bottom of the track, the cart's



1. velocity and acceleration are zero.
2. velocity is nonzero but its acceleration is zero.
3. acceleration is nonzero, but its velocity is zero.
4. velocity and acceleration are both nonzero.

Part 2: Analytic Questions: Show all your work.

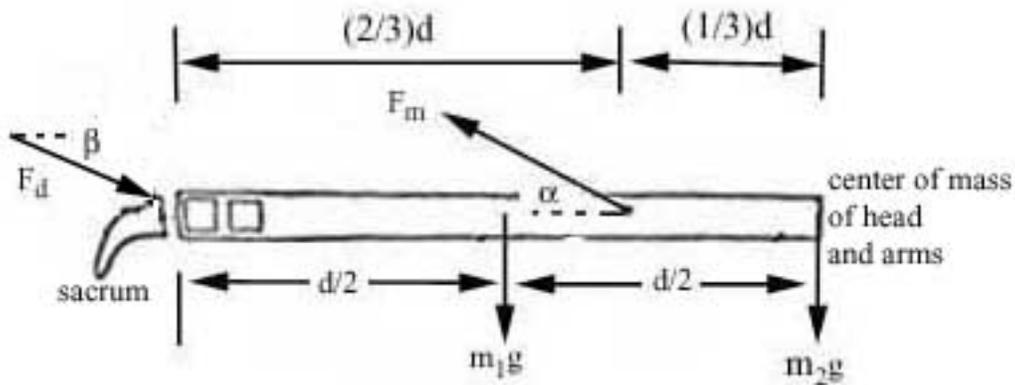
Problem 2: (20 points) A child is trying to shoot a marble of mass m in order to hit the center of a small box using a spring loaded marble gun. The marble gun is fixed on a table and shoots the marble horizontally from the edge of the table. The edge of the table is a height h above the top of the box (the height of which is negligibly small). The center of the box is some horizontal distance d away from the table. The spring has a spring constant k . By what distance x should the child compress the spring so that the marble lands in the center of the box? Let g denote the gravitational constant. The given quantities in this problem are m , k , g , h , and d . Express x in terms of whatever given quantities you may need.



Answer:

Problem 3: (20 points) A Back Bending Exercise

When a person of mass m bends over or lifts an object, the main muscles that lift the back are the erector spinae (sacrospinal muscles). These muscles act approximately at a single point on the vertebral column and exert a force, \vec{F}_m , at a point that is $2/3$ of the distance from base of the spine to the center of mass of the head and arms. The cord applies the force at an angle α relative to the axis of the vertebral column. The sacrum exerts a force, \vec{F}_d , on the spine at an unknown angle β relative to the axis of the vertebral column.



Assume that mass of the head and arms is $m_2 = (1/5)m$ where m is the mass of the person. The center of mass of the trunk is $1/2$ of the distance from the base of the spine to the center of mass of the head and arms. Assume that mass of the trunk is $m_1 = (2/5)m$, where m is the mass of the person. Let g denote the gravitational constant.

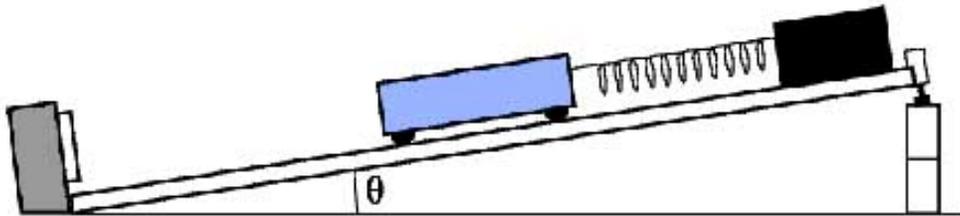
In this problem assume the given quantities are m , α , and g . Express your answers below in terms of these quantities.

- What is the magnitude of the force \vec{F}_m that the sacrospinal muscles exert on the spine?
- What is the angle β that the force of the sacrum on the spine, \vec{F}_d , makes relative to the axis of the vertebral column?
- What is the magnitude of the force \vec{F}_d ?

Answer:

Problem 4: (20 points)

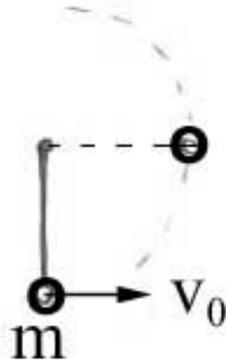
Consider an ideal spring that has an unstretched length l_0 . Assume the spring has a constant k . Suppose the spring is attached to a cart of mass m that lies on a frictionless plane that is inclined by an angle θ from the horizontal. Let g denote the gravitational constant. The given quantities in this problem are l_0 , m , k , θ , and g .



- a) The spring stretches slightly to a new length $l > l_0$ to hold the cart in equilibrium. Find the length l in terms of the given quantities.
- b) Now move the cart up along the ramp so that the spring is compressed a distance x from the unstretched length l_0 . Then the cart is released from rest. What is the velocity of the cart when the spring has first returned to its unstretched length l_0 ?
- c) What is the period of oscillation of the cart?

Problem 5: (20 points)

A ball of negligible radius and mass m hangs from a string of length l . It is hit in such a way that it then travels in a vertical circle (i.e., the tension in the string is always greater than zero). The initial speed of the ball after being struck is v_0 . You may ignore air resistance. Let g denote the gravitational constant.



- a) Find the speed of the ball when the string is horizontal.
- b) Find the tension in the string when the string is horizontal.

Answer: