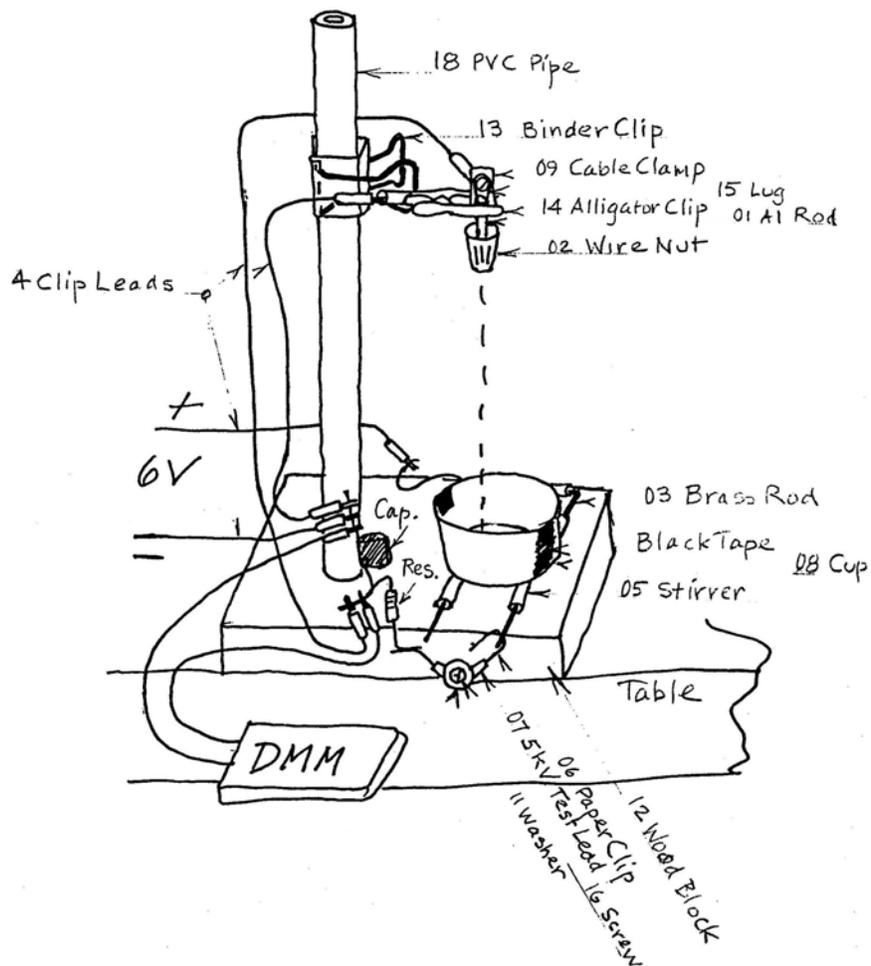


# Mechanics

## Experiments from Kits



Peter Dourmashkin  
John G. King

# Mechanics Experiments from Kits

**Peter Dourmashkin**  
**Senior Lecturer in Physics**  
**Massachusetts Institute of Technology**

**John King**  
**Francis Freidman**  
**Professor of Physics**  
**Massachusetts Institute of Technology**

## **Table of Contents**

About the Experiments

Experiment ES—Estimating a Second

Making Clip Leads—CLK—About Soldering

Exercise MM—About the Multimeter

Building the LVPS—Low Voltage Power Supply

Experiment FO—Falling Object

Experiment FM—Force Between Magnets

Experiment CF—Centripetal Force

Experiment ET—Energy Transformations

Experiment VS—Vibrating Systems

Experiment AM—Angular Momentum

Experiment FL—Flow

Red Box and Tool Kit Contents

Kit Parts List

# About Experiments

## 1) Partners

Each of you should have one partner. During the first week, each partnership will receive a Red Box containing returnable parts used in the experiments, and parts kits for each experiment.

## 2) Tool Kits

Each student will buy a special Tool Kit designed for these experiments.

## 3) Red Boxes

Each pair of partners should have one Red Box to be returned at the end of the semester. Each experiment has its own plastic bag of parts in the Red Box. In addition, each experiment will make use of a selection of parts from the Red Box. Your instructions for the individual experiments will identify those items. Do not throw things away. Items that are used in one experiment may be re-used in a later experiment. We shall ask you to return the Red Box at the end of the semester, with all the major items present (of course this does not include obvious disposable items such as bits of wire).

## 4) Experiment Write-ups

Each experiment has a write-up that describes the experiment, apparatus, and measurements you will make. Carefully read these write-ups.

## 5) Safety

You will be working sometimes with sharp-edged tools and materials, soldering irons, and apparatus, which connects to the 120 Volt, 60 Hertz (60 cycles per second) line supply through a wall transformer. Use them carefully to avoid accidents. Always think about what you're doing and what would happen if the hand or the tool or the work slips, or if the soldering iron falls on flammable stuff, or if electrical things get wet. The items you will be using are fairly harmless, but be sure to tell us about any worries you or your friends have about safety.

# Experiment ES Estimating a Second

## Introduction: About Estimating

Before measuring and calculating comes estimation; a chance to exercise ingenuity. Often it's a matter of organizing your rough knowledge and experience in quantitative form, or making a simplified model so as to answer questions about numbers, such as: how many hairs are on your head? how much energy does an AA cell store? how many gallons of gasoline are burnt annually in the US? how many piano tuners are there in Chicago? etc. You might say: who wants such a rough answer, perhaps only good to an order of magnitude? Well if you aren't near a reliable reference source or able to measure, it's far better to have some estimate than nothing. A good estimate enables you to consider whether a new idea for an experiment or even a business is possible.

The question about piano tuners was an example from the physicist Enrico Fermi, and we often call these estimations "Fermi problems".

Consider the estimating of amounts, sizes and duration. These correspond to the physical quantities mass, length and time, the fundamental quantities of classical mechanics. One can develop a feel for some of these by using standards based on common objects (for instance a penny, dated 1982 and later has a mass about 2.5 g; earlier pennies have a mass about 3 g. A penny's diameter is about 2 cm. One's pulse, with a period of about 1 second, is a rather variable time standard that you always have around. But there is another way of estimating time which we'll explore here.

## Estimating Time

Some people have a good idea of what time of day it is without looking at a watch, or can feel when an hour has passed; but how do they do it? It often turns out that the estimator has practiced this skill for many years. However, when it's just a matter of estimating a few seconds, most people mark time by counting a word that takes one second to pronounce like: one one thousand, two one thousand, three one thousand, etc., or: one Mississippi, two Mississippi, three Mississippi, etc., To see how well this works we have developed a simple electrical timer which you and your partner can assemble and use to test how accurately you count seconds.

## Parts

**DPDT knife switch:** This "double-pole double-throw" (DPDT) knife switch was chosen because it's obvious how it works and it's easy to connect to. Knife switches, so-called because the motion of the swinging arm is like that of a knife blade, are rarely used now but acronyms like

SPST, SPDT, DPST, DPDT (S for "single") still apply. The words "throw" and "pole" are old usages: "throw the switch", meaning to turn on or off, close or open the circuit, whose conducting wires are connected to the switch terminals, or "poles". See Fig.1 for the schematic of these switches.

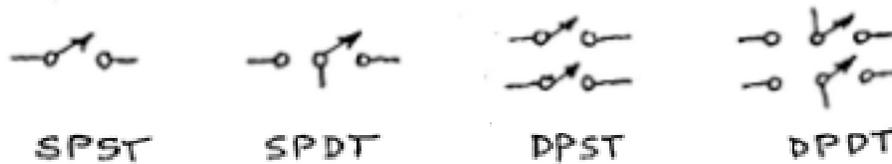


Figure 1: Various switches; SPST, SPDT, DPST, DPDT

Notice that in attaching leads to the screw terminals of the switch you should bend the lead clockwise around the screw so that it is drawn in as you tighten the conventional right-hand screw.

**Two 10 megohm resistors, one 20 ohm resistor:** Resistors are color coded by colored bands read from left to right when you hold them so that the gold band is on the right. Two of the three resistors in your kit have at the left a brown band meaning 1, for the first digit; a black band meaning 0, for the second digit; a blue band meaning 6 zeros-hence

$$1-0-000000 \text{ ohms } (\Omega) = 107 \Omega = 10 \text{ megohm } (M\Omega).$$

The gold band on the right means that the resistor is within  $\pm 5\%$  of the rated value. The third resistor is a  $20 \Omega$  resistor having at the left a red band meaning 2, for the first digit; a black band meaning 0, for the second digit; a black band meaning 0 zeros-hence  $2-0- = 20 \Omega$ .

**1.0  $\mu\text{F}$  capacitor:** The capacitor has a brown molded plastic case and the relevant part of the label is 105K. This is a code that has the following meaning. The "5" means place five zeros after the 10 and read the capacitance in picofarads (pF). A picofarad is  $10^{-12}$  F. Thus

$$105 = 1 - 0 - 00000 \text{ picofarads} = 10^6 \text{ pF} = 10^{-6} \text{ F} = 1\mu\text{F} = 1 \text{ microfarad}$$

The "K" means  $\pm 10\%$ . Some capacitors have leads labelled plus or minus, but this one doesn't.

**AA cell and AA cell holder:** The kit contains an AA cell and a holder with a red lead (plus,+) and a black lead (minus,-). These leads are insulated but you will need to remove more of the insulation so that about 1/2 inch , 12 mm, of bare wire are exposed at the end of each wire. Do this with the wire strippers in your toolkit, set so that they just grab a paper clip; too large and they won't cut the insulation or too small and they will cut the wire.

**Three Wire Connectors:** These hollow plastic cones containing conical springs are often called "wiring nuts" and are widely used in 120/240 volt wiring in buildings. They come in various sizes and what you have is about the smallest. Even so the leads are so small, (#22 and #25 AWG: American Wire Gauge) that you'll have to twist them together, bend them over and squeeze them with pliers before screwing on the wiring nut (see Fig. 2). In any case the leads should seem quite firmly connected when pulled or wiggled.

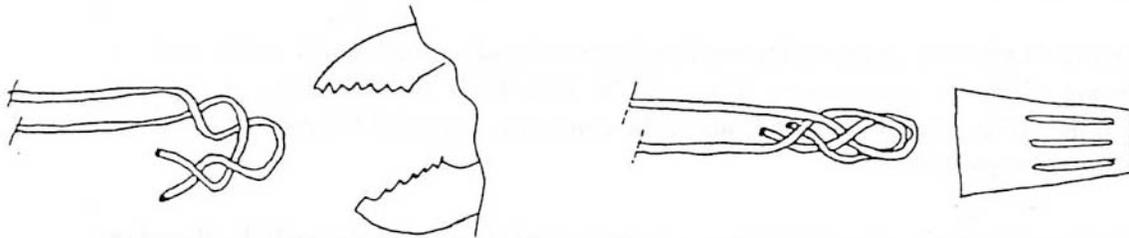


Figure 2: Wiring Nuts

Connections are made by attaching leads to screw terminals on the switch and using the wiring nuts to connect other leads. Alternatively you can use your soldering iron after tinning it. Assemble the circuit according to the sketch of the circuit, Fig. 3, and the circuit diagram, Fig. 4. For this experiment, a single-pole double-throw (SPDT) would do so, but the DPDT provides more mechanical stability when switching.

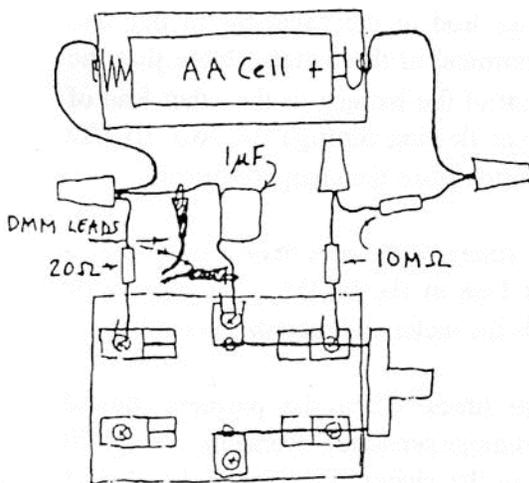


Figure 3: Sketch of circuit

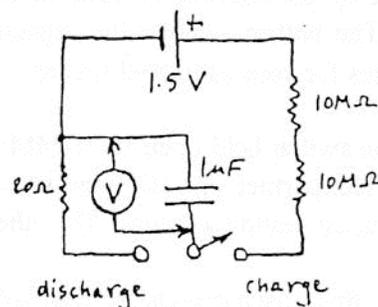


Figure 4: Circuit diagram

## Experiment

In this experiment you will estimate a second by closing a switch for about a second in a circuit that is charging a capacitor. A charged capacitor has a voltage difference between its two leads. Once you open the switch the charging stops and the voltage difference across the

capacitor has a fixed value. The voltage difference is related to the charging time and for our device is nearly linearly proportional. After each trial the capacitor must be discharged until there is zero voltage difference across the leads so that a new trial can begin.

Operate the switch by grasping the small cylindrical handle; don't touch metal parts to avoid introducing electrical disturbances. Connect the two leads of the digital multimeter (DMM), set on the DCV range to the two leads of the capacitor. The DMM displays the voltage difference across the capacitor.

You first need to make sure the capacitor is discharged. Start with the switch, closed on the side that connects the  $1\ \mu\text{F}$  capacitor to the  $20\ \Omega$  resistor. This is the discharge side. Notice that the battery is not connected when the switch is in this position. The  $1\ \mu\text{F}$  capacitor is discharged in a short time by current flowing through the  $20\ \Omega$  resistor until the digital multimeter (DMM), reads zero. Once the capacitor is discharged you can put the switch in the open position.

We can now begin the charging. You will charge up the capacitor put throwing the switch to the other position for you consider to be one second and then opening the switch and reading the voltage difference across the capacitor.

When the switch is closed, you have connecting one lead of the capacitor to the two  $10\ \text{M}\Omega$  resistors, which are in turn connected to the positive terminal of the battery. Note that the circuit is closed by the connection from the negative terminal of the battery to the other lead of the capacitor. The battery charges the capacitor by a current flowing through the two  $10\ \text{M}\Omega$  resistors in series for your estimated second. Opening the switch stops the charging process.

With the switch held open the DMM displays the voltage difference across the leads of the capacitor. The partner operating the switch should not look at the DMM so as not to be influenced when estimating a second. The other partner reads the meter and records the reading.

Repeat this discharge-charge-read-voltage cycle 36 times. Then the partners should exchange tasks. These 72 readings (keep each partner's 36 readings separate), averaging around 70 millivolts ( $1\ \text{mV} = 10^{-3}\ \text{V}$ ) are measures of the time the switch was closed in the charge position, and hence of your estimates of a second.

## **Data**

Use your calculator or your favorite software program to compute the mean and standard deviation of the DMM readings corresponding to your estimate of one second.

Make a histogram of your data as follows. On a sheet of graph paper, draw a line along the long side about 1 inch from the bottom. Label every fifth space, starting at the left so as to

accommodate the range of DMM readings you have made, say, from 40 to 100. Make an X in each space corresponding to each of your 36 values of DMM reading rounded off to the nearest integer. Count the number of Xs in each interval of 5 (eg. 45 through 49) and represent that number by a horizontal line above that interval. Connect the ends of these lines with vertical lines. You now have a bar graph (histogram) of your data.

## Determining the Time Constant $RC$

To interpret the DMM readings in millivolts in terms of time it's necessary to determine the time constant,  $\tau$ , of the resistance-capacitance combination.

Consider the charging circuit in Experiment ES that is shown in figure 4. The DMM is measuring the voltage across the capacitor,  $V_{cap}$ . The rate of change of the voltage,  $dV_{cap}/dt$ , across the capacitor is proportional to two terms. The first is the voltage of the cell,  $V_{cell}$ . This is a constant factor. The second is the negative of the voltage,  $V_{cap}$ , that is already present across the capacitor.

$$\frac{dV_{cap}}{dt} = \frac{1}{\tau}(V_{cell} - V_{cap}).$$

The constant of proportionality  $1/\tau$  has the dimensions of inverse time. Hence  $\tau$  is called the time constant. The rate of change of the voltage is inversely proportional to the resistance,  $R$  measured in ohms, in the circuit. The greater a resistance to the flow of current and hence charge, the slower the rate of change of the voltage across the capacitor. The rate of change is also inversely proportional to the amount of charge the capacitor is capable of storing. This property of the capacitor is called the capacitance and is denoted by  $C$ , measured in farads. So the rate of change of voltage across the capacitor is

$$\frac{dV_{cap}}{dt} = \frac{1}{RC}(V_{cell} - V_{cap}).$$

Thus we can conclude that the time constant  $\tau = RC$ . For the charging circuit,  $R = 20 \text{ M}\Omega$  and  $C = 10^{-6} \text{ F}$  so if the parts were ideal we would have for the charging process

$$\tau = RC = (20 \text{ M}\Omega)(10^{-6} \text{ F}) = 20 \text{ s}.$$

To convert DMM readings to time, measure the AA cell voltage,  $V_{cell}$  and a particular DMM reading is  $V_{cap}(t)$  corresponding to a time  $t$ . The voltage across the capacitor for a charging circuit is given by the relation

$$V_{cap}(t) = V_{cell}(1 - e^{-t/RC}).$$

The power series expansion for the exponential function is given by

$$e^x = \sum_{n=0}^{n=\infty} \frac{x^n}{n!} = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \dots$$

In particular we have that

$$e^{-t/RC} = \sum_{n=0}^{n=\infty} \frac{(-t/RC)^n}{n!} = 1 - \frac{t}{RC} + \frac{1}{2}\left(\frac{t}{RC}\right)^2 - \dots$$

Notice that when  $t \ll RC$ , we can ignore second order terms like  $(t/RC)^2$  so the voltage across the capacitor grows linearly with time for  $t \ll RC$ .

$$V_{cap}(t) = V_{cell}(1 - e^{-t/RC}) \cong V_{cell}\left(1 - \left(1 - \frac{t}{RC}\right)\right) = V_{cell} \frac{t}{RC} = V_{cell} \frac{t}{\tau}.$$

Therefore the time can be related to the voltage across the capacitor according to

$$t = \frac{V_{cap}(t)}{V_{cell}} \tau.$$

**Example:** Suppose  $t \ll \tau = 20$  s,  $V_{cell} = 1.645$  V, and  $V_{cap}(t) = 80$  mV. So

$$t = \frac{V_{cap}(t)}{V_{cell}} \tau = \frac{(.08 \text{ V})}{(1.645 \text{ V})} (20 \text{ s}) = (1.0 \text{ s}).$$

### Experiment ES: Parts

- 1 DPDT knife switch
- 1 AA cell
- 1 AA cell holder
- 3 wiring nuts
- 2 10 M $\Omega$  resistors
- 1 20  $\Omega$  resistor
- 1 1  $\mu$ F capacitor

## Making Clip Leads ---CLK--- About Soldering

You will solder together wires and electronic components to make an adjustable regulated direct current power supply and other devices. You will also use your magnetic multi-meter (MMM) to measure voltage, current and resistance.

Before you build the power supply and start doing the experiments, you and your partner will do three things involving soldering, namely: making two clip leads, putting alligator clips on the multimeter leads, and putting alligator clips on the wall transformer leads.

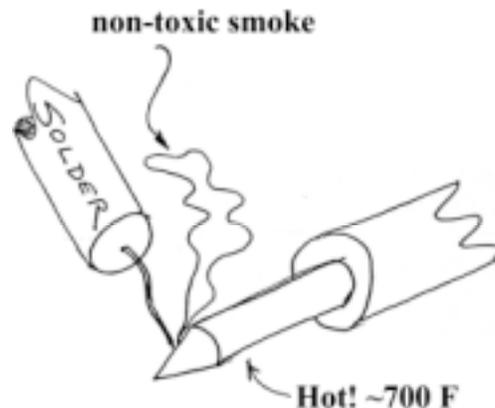
You will also use your MMM to measure the resistance of a resistor, the voltage difference across the terminals of a battery, and the current that flows in a simple circuit consisting of the battery and the resistor.

### About Soldering:

This is a way of joining metals with solder, an alloy of 60% tin and 40% lead that melts at about 180 C (360 F). The hollow core of the wire solder in your tool kit contains a rosin flux that cuts through crud films on the surfaces of the metals so that the molten solder can wet and bond them.

### Soldering Iron:

Your tool kit contains a 25 W (watt) soldering iron. Plug it in and after 2 to 3 minutes, rub its conical tip with solder to tin it; that means cover the tip with a film of molten solder.

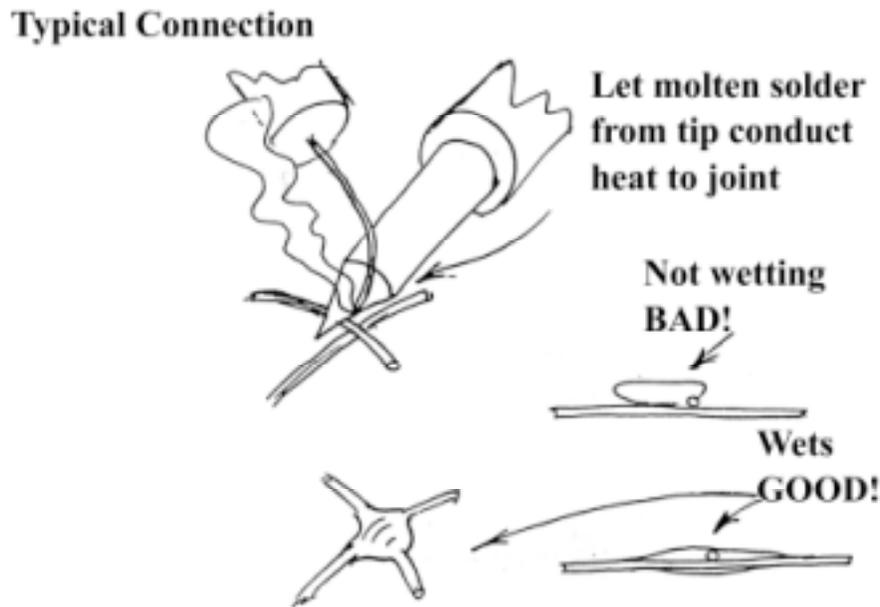


**Figure 1: Tinning the soldering iron**

The soldering iron in your toolkit requires some care. Once you have tinned the iron, you can clean the tip with a quick pass with a paper towel or emery cloth. It is important to keep your iron tinned otherwise crud will pile up on the tip of the iron and it will not properly conduct heat. You should not leave your soldering iron plugged in when not in regular use. Keep the soldering iron away from flammable materials. The tip of the iron gets very hot; be careful not to inadvertently touch it.

## Soldering Wires:

To solder, put the iron up against the wires to be joined, letting the iron heat up the wires for a few seconds. Feed solder to the iron tip near the wires so that the molten solder can conduct heat to them and wet them. Smoke and vapors from this procedure are not harmful. Remove the iron and let the solder solidify before moving the wires. The resulting joint should not come apart except with a very strong pull.



**Figure 2: Soldering Connections**

You should practice stripping (removing) the vinyl insulation from the ends of the stranded wire (in the clip lead kit—use the red as there's some extra), twisting the strands and tinning them with molten solder using your soldering iron. Then solder the two ends of the wires together. Try tugging them apart.

## Making Clip Leads

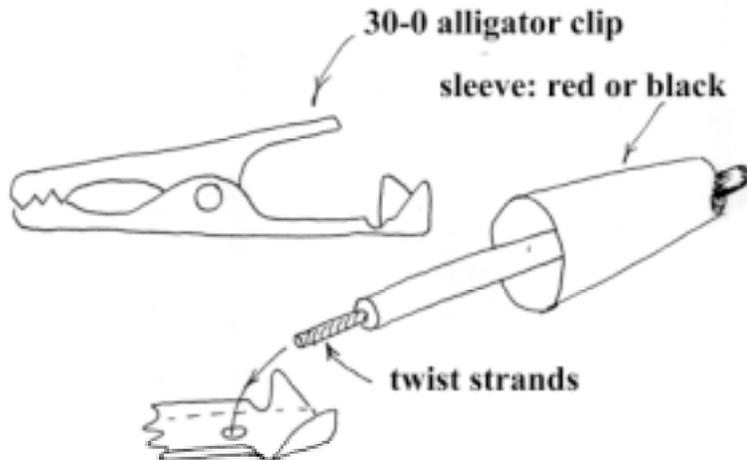
Cut two pieces of vinyl-insulated stranded wire, about 250 mm (10 inch) long, one red, one black. You'll find wire in the Clip Lead kit. You can use the wire cutter part of the long-nose pliers, or the wire strippers or scissors. Loosen the adjusting nut of the stripper, and set it so that the notch in the jaws just allows a regular paper clip to slide through. Wire is most easily stripped of its insulation by holding the wire with pliers with the part to be stripped protruding and then levering the wire strippers against the plier jaws. Try not to cut any of the strands. Remove

about 4 mm (1/4 inch) of insulation from each end of each of the wires, and twist the strands into a compact bundle.



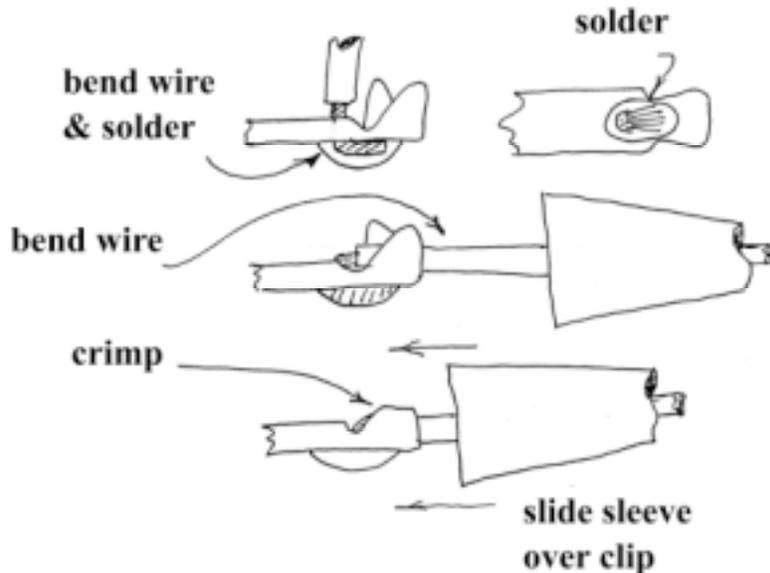
**Figure 3: Stripping wires**

Tin all the ends, which means put solder on them with your iron so that the strands are bound together. Put two sleeves of the same color as the wire insulation on the wires, large end facing the wire ends. Make a right angle bend in the middle of the tinned part and put that through the hole in the end of the alligator clip from above (the side where there is a round pad to put your thumb when opening the jaws of the clip).



**Figure 4: Alligator clip**

Solder the tinned wire to the bottom side of the clip. You may find that it helps to hold the clip in your pliers using a rubber band around the handle of the pliers to give a firm grip. Let it all cool. Bend the insulated part back and crimp the end of the clip around it. Work the sleeve over the clip. **Repeat three more times to make two clip leads.**



**Figure 5: Soldering clip leads**

**Alligator Clips on Clip Leads:** The analog or magnetic multimeter (MMM) come with test prods that have been removed. These prods would be useful in making quick measurements from point to point in some devices. They are not good for our purposes where we want to leave the meter connected for extended times. Put red and black sleeves on the appropriate leads. **Solder on alligator clips as you did for clip leads.**

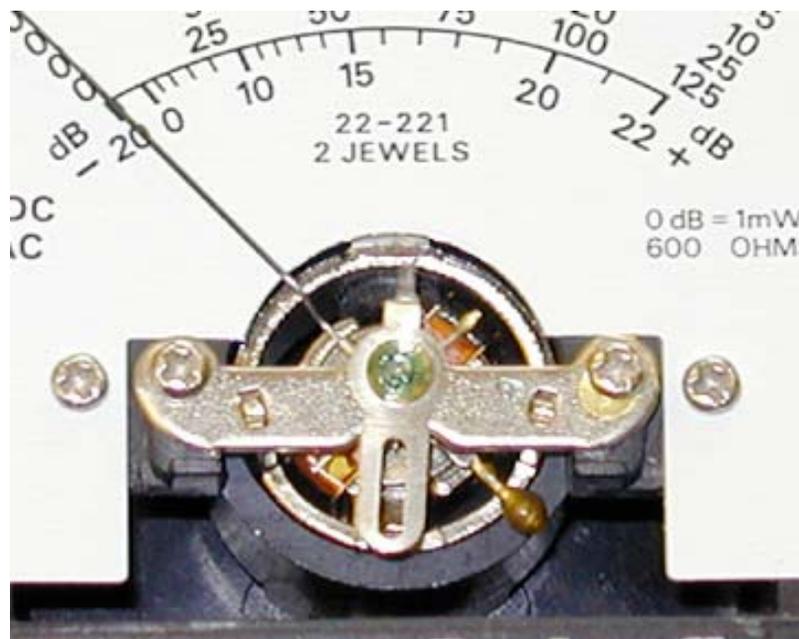
**Alligator Clips on Transformer Leads:** In your Red Box you will find a black Class 2 Transformer, that reduces the input 120 V ac, 60 Hz (30 Watt) from the line to a safe and convenient 12 V ac 1000 mA output with only moderate loss of power (heating the transformer). It is IMPORTANT that the leads of the transformer should be of unequal length to reduce the likelihood of short circuits. If they are not, cut one of them so that it is about 50 mm (2 inches) shorter than the other. Put a black sleeve on each of the leads. Strip about 4 mm (1/4 inch) of insulation from each end of each of the leads. **Solder on alligator clips as before.**

**Putting Alligator clips on the Multimeter Leads:** Whenever you want to make a measurement, you will use the red and black test leads. Your analog multi-meter comes with test leads that have been removed. These leads would be useful in making quick measurements from point to point in some devices. They are not good for our purposes where we want to leave the meter connected for extended times. Put red and black sleeves on the appropriate leads. **Solder on alligator clips as you did for clip leads.**

## Exercise MM—About the Multimeter

### Introduction

Our world is filled with devices that contain electrical circuits in which various voltage sources cause currents to flow. Electrical currents generate heat, light, and magnetic fields, and produce chemical effects. Any of these phenomena can be used to measure current. One of the simplest ways is to let the current flow through a coil of wire that is in a magnetic field and to measure the resulting torque on the coil by observing the deflection of a torsion spring. This is how your multimeter works, which we call a magnetic multimeter (MMM), in contrast to the standard term `analog`. Look at the meter itself. You can see the copper colored coil and one of the two spiral torsion springs (the other is at the back; they also lead current in and out of the moving coil). The MMM is a current meter with a range selector switch, so that with appropriate resistors and other parts it can measure voltages and resistances.



**Figure 1: Coil and torsion spring**

A meter has a needle that moves clockwise in proportion to the current flowing through the meter. The needle goes over various scales above a reflecting mirror intended to reduce parallax error—move your head so that you see the needle just above its reflection, and you'll be looking straight down onto the scale and be able to read the right number.

In this exercise you will use your MMM to measure the resistance of a resistor, the voltage difference between the terminals of a AA cell (battery), and the current that flows in a simple circuit consisting of the battery and a resistor.

The MM Kit contains one AA cell and one battery holder with a red lead (plus,+) and a black lead (minus,-). You may need to remove some of the insulation on the leads so that about  $1/2$  in, 12 mm, of bare wire is exposed. Do this with the wire strippers in your toolkit. The kit

also contains two  $20\Omega$  resistors. You will also need one of the clip leads that you have just made.

There is a separate package in the top tray of the Red Box containing two AA cells, and four  $500\text{ mA}$  fuses. You should first open the back of the MMM and put in a  $1.5\text{ V}$  AA cell into the holder at the top of the MMM. Make sure the battery is placed with the + terminal connecting to the red wire. While the multimeter is open, notice that there is a fuse. If your multimeter is not working then there are two likely reasons. The first is that your test leads are broken or not making a good connection. The second reason is that the fuse may have blown. You have four spare fuses in your Red Box. The fuses may blow if you make a measurement with an inappropriate range selector setting, in particular the 250 DCA and Rx1 ranges.

## **Measuring Voltage, Resistance and Current with the Multimeter**

### **Checking the MMM zero:**

With the MMM lying flat on a table or desk and with nothing connected look down so as to line up the needle and its image in the scale mirror. The needle should bisect the 4 black zeros. Tap or swing the meter; the needle should still show zero. If not, your  $5\text{ mm}$  flat screw driver will just about fit the adjusting screw in the lower center of the meter. Turn carefully until the needle sits on zero.

### **Measure the resistance of the $20\Omega$ resistor**

There are 3 resistance ranges, RX1, RX10, and RX1K (1K=1000). The ohmmeter operation depends on the  $1.5\text{ V}$  AA cell that is inside the meter. Essentially, current flows through the meter in inverse proportion to the resistance in the circuit. This accounts for the markedly non-linear green scale at the top of the meter.

In order to zero the meter before measuring resistance, short the test leads by connecting them together. Then adjust the OHMS ADJUST knob (located to the left of center of the MMM) so that the meter reads 0 ohms; the needle is then at its maximum deflection. When the test leads are not connected, (an open circuit), no current flows and the needle sits on the infinite resistance mark,  $\infty$ , all the way on the left side of the scale.

Set the range selector switch on the MMM to the RX1 range. Connect the test leads to the resistor. Measure the resistance. You may want to make other resistance measurements. For example make a thick line with a lead pencil and measure the resistance of the mark. Grasp the clips firmly and see what your resistance is. Touch the clips to your tongue.

### **Measure the voltage of the AA cell**

Set the range selector switch on the MMM to the 5 DCV range. Place the AA cell in the battery holder. Connect the test leads to the leads from the holder. Measure the voltage.

### **Measure the current in a simple circuit**

First set the range selector switch on the MMM to the 250m DCA range. Make a simple circuit consisting of the  $20\Omega$  resistor, the AA cell, and the MMM. You can do this by connecting the red lead of the MMM to the red lead (plus, +) of the AA cell holder. Use a clip lead to connect the black lead (minus, -) of the AA cell holder to one end of the  $20\Omega$  resistor. Connect the black lead of the MMM to the other end of the  $20\Omega$  resistor. Measure the current in the circuit. What effect do you think the MMM has on the circuit?

## About the Magnetic Multimeter

### Introduction

The MMM will be one of your most important tools in this course. Please read the following explanation of the MMM. You may not be completely familiar with all the terminology. As the course develops, you will learn all the physical principles necessary to understand the MMM. So please keep on referring to the reading below if you have any questions about your MMM.

Your analog multimeter (we call them MMM—magnetic multimeters) is a “moving coil meter” with a needle whose deflection shows and measures the torque on a current loop placed in a magnetic field. That torque is proportional to the current, and a device that measures current is called an *ammeter*.

The multimeter consists of a cylindrical magnet (magnetized across a diameter) arranged coaxially with a cylindrical magnetic return path as shown in Figure 2.



**Figure 2: Multimeter Coil**

In the gap is a pivoted rectangular coil; you can see the top of it if you look down into the meter. Spiral springs, top and bottom, lead current in and out of the coil and also provide a restoring torque. Jeweled bearings provide a low-friction mounting as in some watches.

Current in the coil interacts with the radial magnetic field to generate tangential forces, and hence torques about the axis of rotation. These turn the coil until the magnetic torque is balanced by the torque of the spiral springs. The meter has a pointer or needle, which moves clockwise in proportion to the current flowing through the meter. The needle goes over various scales above a reflecting mirror intended to reduce parallax error—move your head so that you see the needle just above its reflection, and you’ll be looking straight down onto the scale and be able to read the right number.

Any instrument that measure current will disturb the circuit under observation. (The coil itself has resistance.) There will be some voltage drop due to the resistance of the flow of current through the ammeter. An ideal ammeter has zero resistance, but a  $0.1 - 0.2V$  drop is tolerable in our applications.

The range of an ammeter can be extended to measure higher currents by placing a resistor (called a shunt resistor) of resistance,  $R_s$ , generally lower than the coil resistance, *across* the meter coil. When connected in a circuit with flowing current  $I$ , the meter will read a fraction of that current say  $0.1 I$ , with  $0.9 I$  passing through the shunt. The meter scale can be calibrated so that it reads 10 times its original range.

To convert an ammeter into a voltmeter, a resistor (called a multiplier resistor) of resistance,  $R_m$ , generally higher than the coil resistance, is put in *series* with the meter coil. Suppose a current  $I$  through the meter coil produces a full-scale (FS) reading, that is FS deflection of the needle. The coil resistance is  $R_c$ , so the voltage across it,  $V = I R_c$ . Putting a multiplier resistor  $R_m = 9R_c$  in series with the coil means that it will take  $10V$  to produce a FS reading, so we now have another range and can calibrate and label the scale accordingly.

The ideal voltmeter should draw no current, corresponding to the zero voltage drop across the ideal ammeter. But in any moving coil meter currents produce torques which deflect springs and keep them deflected. Electrical power deflects the springs during the short time that the needle is moving and is also dissipated in the coil resistance as long as the needle is deflected.

## Test Leads

The test leads are generally placed into the two pin jacks on the lower left of the MMM; black into  $-COM$  and red into  $+V-\Omega-A$ . Note the warning label that the inputs for these cannot exceed the maximum values of 500V DC, 1000V AC, 250mA DC (Figure 8). When you want to measure DC voltages up to 1000  $V$ , put the positive test lead into the pin jack labeled DC 1000V while leaving the black lead in  $-COM$ .

## Range Selector Switch

The meter can measure current, dc voltage, resistance, or ac voltage depending on the setting of the range selector switch. There are four types of positions: DCA for dc current, DCV for dc voltage, OHMS for resistance, and ACV for ac voltage. Each position has several ranges; for example OHMS has three ranges: RX1, RX10, and RX 1K.



For example, if you set your range selector switch to 25 DCV then each large division corresponds to  $2.5V$ . When the needle points to full scale deflection, the voltage is  $25V$ . (Notice that there is no  $10V$  full scale switch setting.) When the switch is set to the 500 & 1K setting, each large division corresponds to  $50V$  or  $100V$  depending on which pin jack the positive lead is inserted into. The non-linear bottom scale (also in black) measures decibels (dB), a logarithmic unit associated with sound level. This scale has the zero setting at  $-20\text{ dB}$  and then ranges from 0 to  $22\text{ dB}$  moving from left to right.

## Current-Voltage Measurements

### DC current Ranges (DCA)

First we'll consider the dc current ranges. DCA, ('DC' stands for direct current), 'A' stands for amperes so DCA means direct current amperage). There are two DCA current ranges,  $250\text{m}$  and  $50\mu$  ( $250\text{ mV}$ ). The more sensitive  $50\mu$  ( $250\text{ mV}$ ) range can also be used to measure voltage. When the dial is set to  $50\mu$  ( $250\text{ mV}$ ) the resistance of the meter is  $5000\Omega$ . If the needle deflects to full scale, then  $50\mu\text{A}$  flows through the meter. This corresponds to a voltage difference

$$V = IR = (50\mu\text{A})(5000\Omega) = 2.5 \times 10^{-1}V = 250\text{mV}.$$

So this setting can measure voltages between  $0V$  and  $250\text{mV}$ .

Besides this most sensitive range, there is a  $250\text{mA}$  range marked  $250\text{mDCA}$ . This puts in a  $1\Omega$  shunt resistor in parallel across the  $5000\Omega$  of the meter itself. Thus when current causes  $250\text{mV}$  to appear across the shunt and the  $5\text{k}\Omega$  meter resistance, a current of  $250\text{mA}$  passes through the shunt while  $50\mu\text{A}$  passes through the meter giving full-scale deflection. Many of the MMM's have a fuse to protect the meter from overload on this range, otherwise the  $1\Omega$  resistor will burn out inside the meter.

### DC voltage Ranges (DCV)

There are 4 DCV range switch positions 5, 25, 125, and 500 & 1K, selected by turning the range selector switch. DC stands for direct current, so DCV means direct current voltage. Selecting the various DCV ranges introduces more resistance in series. The resistance of the meter on any DCV range is always the full scale reading in volts times 20,000 ohms/volt  $[\Omega]/[V]$ , a number that characterizes this meter as a dc voltmeter. For example, on the  $25V$  setting, the resistance is

$$R_{25V} = (25V)(20,000\Omega/V) = 500\text{k}\Omega = 5.0 \times 10^5\Omega.$$

Table 1 shows the full scale value, the resistance of the meter on that range, and the power dissipated in watts through the meter for the DCV and DCA ranges.

**Table One: resistance and power characteristics of DCV and DCA ranges on MMM**

Range	Resistance in ohms [ $\Omega$ ]	Power in milliwatts for full scale deflection [ $mW$ ]
5V	100 <i>k</i>	0.25
25V	500 <i>k</i>	1.25
125V	2.5 <i>M</i>	6.25
500V	10 <i>M</i>	25.0
1000V	20 <i>M</i>	50.0
250mV (50 $\mu$ A)	5 <i>k</i>	0.0125
250mA	1	62.5

The DCV range 500 & 1K is one range selector setting. When the positive test lead is in the +V- $\Omega$ -A, the full scale deflection corresponds to 500 *V*. When the positive test lead is in the pin jack labeled DC 1000V, the full scale deflection corresponds to 1000V.

### AC voltage ranges (ACV)

Suppose the ac input voltage is  $V(t) = V_0 \sin(2\pi f t)$  where  $V_0$  is the amplitude. A half-wave rectifier is inserted in series with the various resistors so that the ac has a dc component. The meter is insensitive to the fast variation of the output voltage across a load, so it will read the time averaged dc voltage  $\langle V \rangle$ . Each ac scale is then calibrated by various resistors to indicate the root-mean square value  $V_{rms} = V_0 / \sqrt{2}$ .

For non-sinusoidal waveforms, or for ac superimposed on dc (average in time not zero), the readings of the meter will most likely not be meaningful.

For ac you read the scales whose divisions and associated numbers are printed in red. Full-scale deflection corresponds to your choice of range for the root mean square voltage indicated on the rotary setting. At low voltages, the diode is not linear, (this is due to the small forward drop voltage) as can be seen from the small displacement of the red ac marks at the low end of the scale, from the corresponding black dc ones directly below.

The resistance of the meter on any ACV range is always the full scale reading in volts time 10,000 ohms/volt, a number that characterizes this meter as an ac voltmeter.

### Resistance-Ranges (RX)

There are 3 resistance ranges, RX1, RX10, and RX1K. The ohmmeter operation depends on the 1.5V AA cell that is inside the meter case. Essentially, current flows through the meter in inverse proportion to the resistance in the circuit. This accounts for the markedly non-linear green scale at the top of the meter.

In order to zero the meter, short the test leads by connecting them together. Then adjust the OHMS ADJUST knob (located to the left of center of the MMM) so that the meter reads 0 ohms; the needle is then at its maximum deflection. When the test leads are not connected, an open circuit, no current flows and the needle sits on the infinite resistance mark  $\infty$  all the way on the left side of the scale.

Half scale readings (that is with the needle pointing straight up parallel to the edge of the case) are:  $24\Omega$ ,  $240\Omega$  and  $24,000\Omega$  on the RX1, RX10, and RX1K ranges, respectively.

Note also that on the resistance ranges the meter puts substantial current through the resistor being measured. Maximum currents are  $0.05A$ ,  $5mA$ , and  $50\mu A$  on the RX1, RX10, and RX1000 ranges, respectively. You can check out some of this by making measurements of one meter with another.

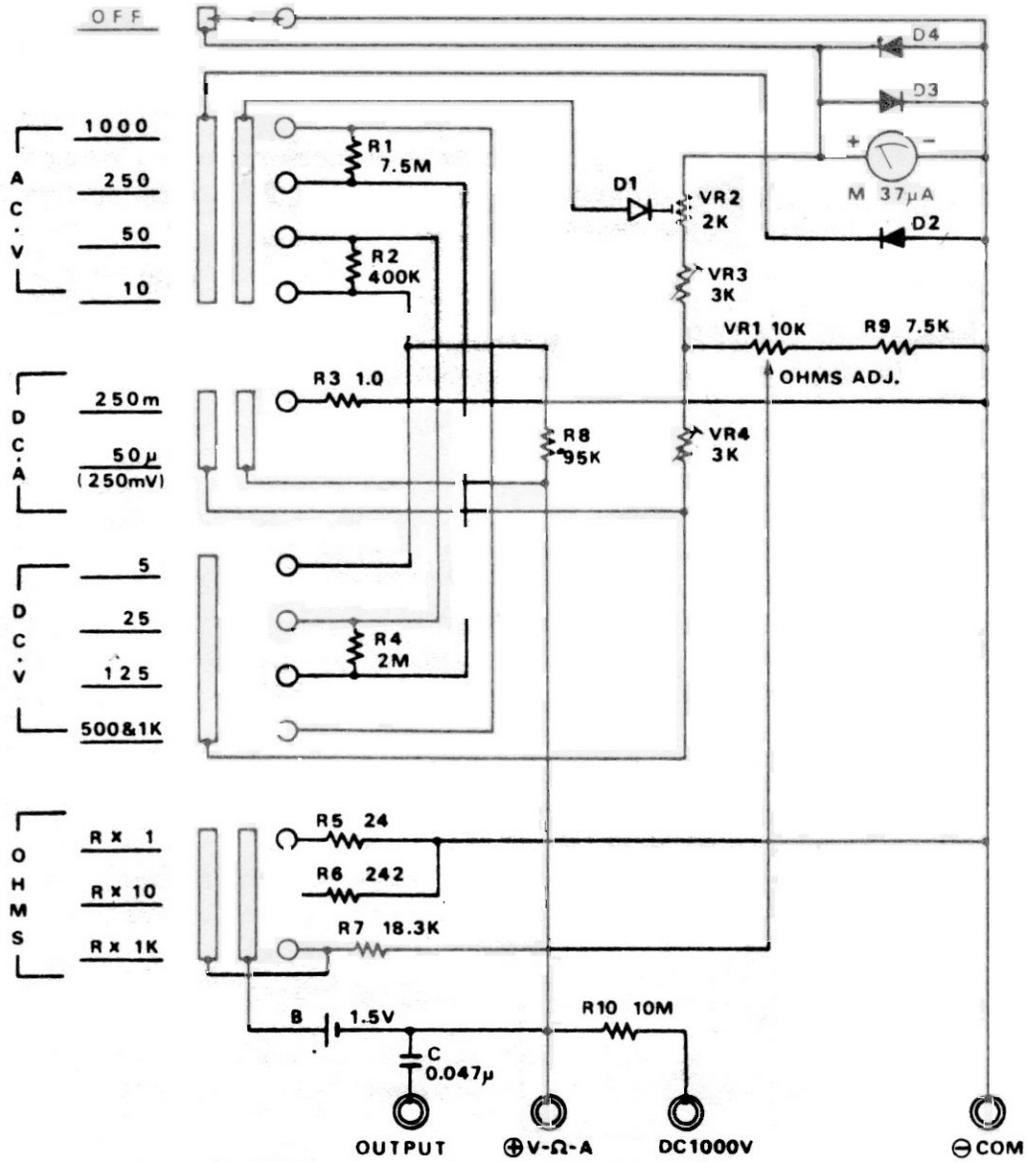
### **Meter Damage**

Avoid dropping the meter. Keep its range switch on zero when it is not in use. This damps the motion of the coil and needle—you can see this by rotating the case back and forth in a horizontal plane and comparing the needle motion with the switch on a voltage scale and on OFF. Or: set the meter on the RX1K, connect the test leads and note the time that the needle takes to return to zero when the leads are disconnected. Compare that time with the time it takes the needle to return to zero when the leads stay connected but the range is switched from RX1K to OFF.

Besides the fuse, the moving coil is protected by resistors and a pair of back-to-back diodes across its windings. This means that it's hard to damage the meter coil except for the ranges with low resistance where resistors can be damaged by excessive currents (RX1 and 250mA ranges). However, it's good practice to start with high ranges, and not to measure the resistances of components that are wired into circuits, especially if power is on.

Figure 6 shows the circuit diagram for multimeter. Notice that when the meter range selected is  $50\mu(250\text{ mV})$ , it takes  $50\mu A$  in the external circuit to produce full scale deflection, but only  $37\mu A$  flows through the coil of the meter.

# SCHEMATIC DIAGRAM



**NOTE:** (1) ALL RESISTANCE VALUES ARE INDICATED IN "OHM" (K = 10<sup>3</sup> OHM, M = 10<sup>6</sup> OHM)  
 (2) ALL CAPACITANCE VALUES ARE INDICATED IN "µF" (P = 10<sup>-12</sup> µF).

Schematic subject to change without notice. For most accurate Schematic (and parts) contact Radio Shack, National Parts Dept., Fort Worth, TX 76101

In UK, contact Tandy Electronics, National Parts Dept., Bilston Road Wednesbury West Midlands WS10 7 JN

In Australia contact Tandy Australia Limited, National Parts Dept., 91 Kurrajong Avenue, Mount Druitt, N.S.W. 2770

**Figure 6: Circuit diagram for MMM**

## ***Measuring Voltage, Resistance and Current with the Multimeter***

- Measure the resistance of the  $20\Omega$  resistor
- Measure the voltage of the AA cell
- Measure the current in a simple circuit

**Measure the resistance of the  $20\Omega$  resistor:** In order to zero the meter, short the test leads by connecting them together. Then adjust the OHMS ADJUST knob (located to the left of center of the MMM) so that the meter reads 0 ohms; the needle is then at its maximum deflection. Set the range selector switch on the MMM to the RX1 range. Connect the test leads to the resistor. Measure the resistance. You may want to make other resistance measurements. For example make a thick line with a #2 pencil and measure the resistance of the mark.

**Measure the voltage of the AA cell:** Set the range selector switch on the MMM to the 5 DCV range. Place the AA cell in the battery holder. Connect the test leads to the leads from the holder. Measure the voltage.

**Measure the current in a simple circuit:** First set the range selector switch on the MMM to the 250mDCA range. Make a simple circuit consisting of the  $20\Omega$  resistor, the AA cell, and the MMM. You can do this by connecting the red lead of the MMM to the red lead (plus, +) of the AA cell holder. Use a clip lead to connect the black lead (minus, -) of the AA cell holder to one end of the  $20\Omega$  resistor. Connect the black lead of the MMM to the other end of the  $20\Omega$  resistor. Measure the current in the circuit. What effect do you think the MMM has on the circuit?

# Building the LVPS—Low Voltage Power Supply

## Introduction

Low voltage is one of those relative terms—up to 25 volts [ $V$ ] dc is low, and most people would call 1000  $V$  high. Power supplies provide energy from many different kinds of sources and at widely varying rates: gigawatts ( $10^9 W$ ) from nuclear plants to microwatts ( $10^{-6} W$ ) from watch batteries. Sources of energy for power supplies include nuclear fission, burning of coal, oil, gas or wood, chemicals reacting, and sunlight, wind and tides. Power is delivered in electrical form as alternating or direct current (ac or dc) and in many combinations of current and voltage. Electrical power supplies in a narrow sense are really converters from one voltage/current combination to another—with, one hopes, only small power losses.

## Project LVPS

In this project, you'll build a power supply that takes power at 120  $V$ , 60 hertz [ $Hz$ ] ac from a wall outlet and converts it to dc. The power supply is adjustable between 2  $V$  to 12  $V$  and can supply currents up to 1 ampere ( $A$ ).

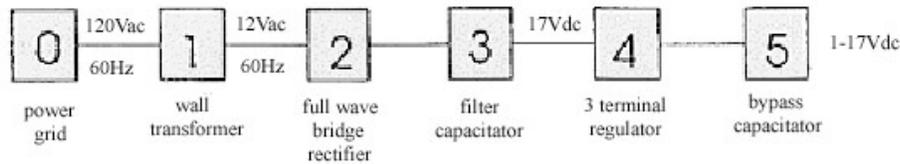


Figure 1: Block diagram of LVPS

## Background

The circuit diagram for the LVPS looks like

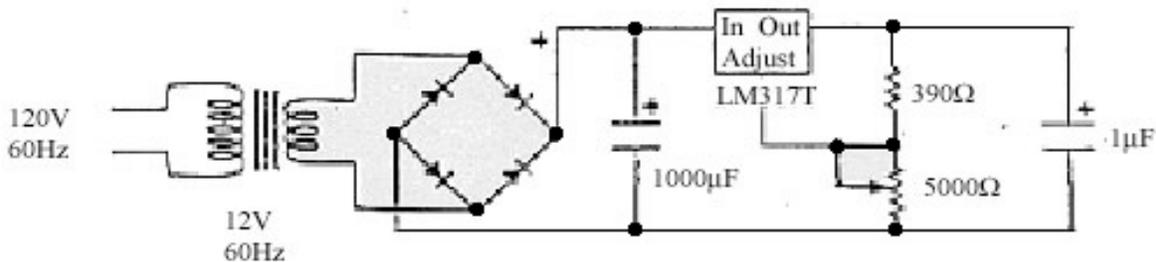


Figure 2: Circuit diagram for LVPS

## Wall Transformer

The LVPS starts with your wall transformer, which reduces the  $120V$  ac from the line to a safe and convenient nominal  $12V$  ac sine wave voltage with only moderate loss of power (heating the transformer). A sine wave voltage varies in time and can be described mathematically by the function

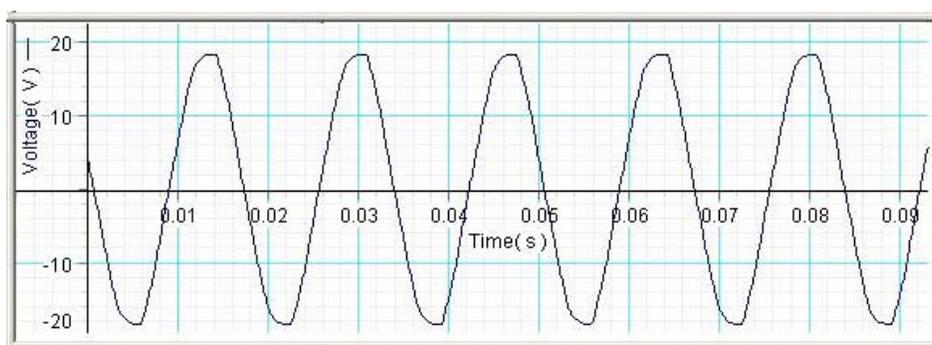
$$V(t) = V_0 \sin(2\pi t/T + \phi) = V_0 \sin(2\pi f t + \phi)$$

where  $V_0$  is called the *amplitude* (maximum value). The voltage varies between  $V_0 = 17V$  and  $-V_0 = -17V$  since a sine function varies between +1 and -1.



**Figure 3: Wall transformer**

The  $12V$  ac refers to the root mean square (rms) amplitude defined by  $V_{rms} = V_0/\sqrt{2}$ . The sine function is periodic in time. This means that the value of the voltage at time  $t$  will be exactly the same at a later time  $t' = t + T$  where  $T$  is the *period*. The *frequency*  $f$  is defined to be  $f = 1/T$ . The units of frequency are inverse seconds [ $\text{sec}^{-1}$ ] which are called hertz [ $\text{Hz}$ ]. A graph of the sine wave voltage vs. time looks like



**Figure 4: Wall transformer output voltage**

# Bridge Rectifier

Next comes a full-wave bridge rectifier consisting of four half-wave rectifiers that act as diodes. A half-wave rectifier allows current to flow through it in only one direction, as shown by the arrow in the symbol for it.

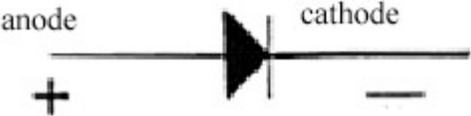


Figure 5: Half-wave rectifier

If an alternating sine-wave voltage is applied to a rectifier, it transmits only the positive half-waves as shown in the sketch below.

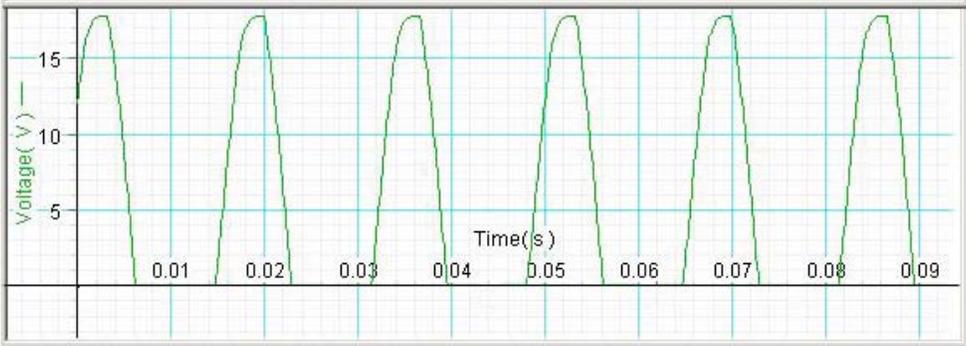


Figure 6: Rectifier sine wave after passing through half-wave rectifier

Four half-wave rectifiers connected as shown in Figure 7 form a bridge rectifier.

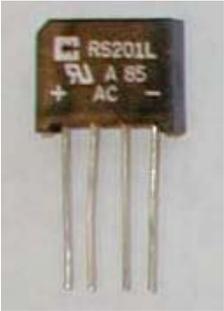
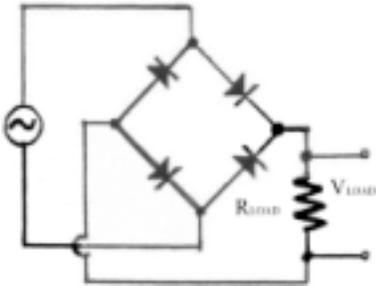
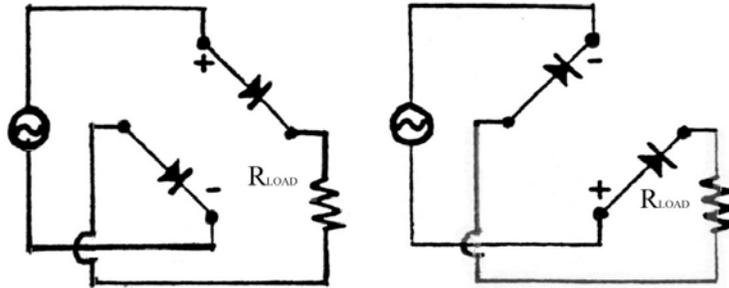


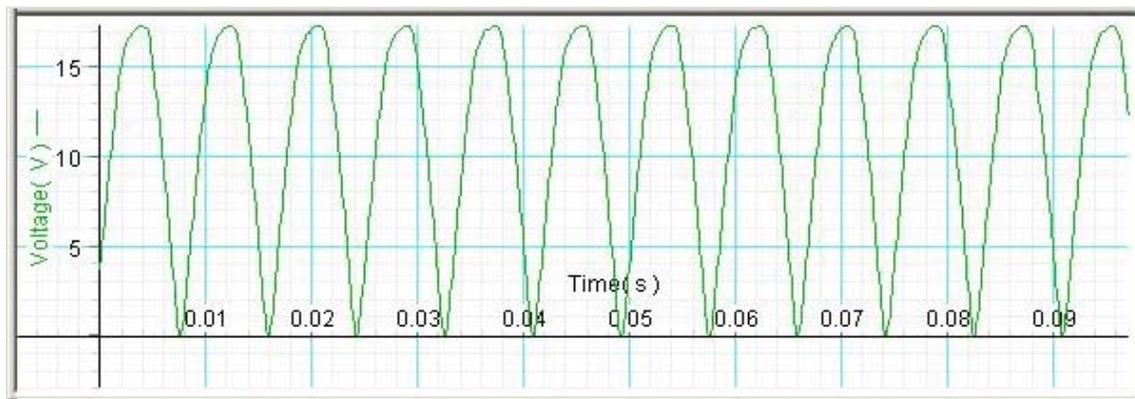
Figure 7: Bridge rectifier

In the next two sketches below, the four half-wave rectifiers act as switches that connect the upper or lower lead on the left, when either is positive, to the right-hand output lead, and to the left-hand output lead when either is negative (convince yourself of this).



**Figure 8: Bridge rectifier in action**

In this way the wiggly ac is made to flow in only one direction—i.e., it is straightened out or rectified. This is shown in the next sketch.



**Figure 9: Voltage output from the bridge rectifier**

## Capacitors

Capacitors are circuit elements that store electric charge  $Q$  according to

$$Q = CV$$

where  $V$  is the voltage across the capacitor and  $C$  is the constant of proportionality called the capacitance. The unit of capacitance is the farad [ $F$ ] and is defined by  $[1F] = [1C]/[1V]$ .

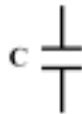
Capacitors come in many shapes and sizes but the basic idea is two conductors separated by a spacing which may be filled with an insulating material (dielectric). One conductor has charge  $+Q$  and the other conductor has charge  $-Q$ . The conductor with positive charge is at a

higher voltage  $V$  than the conductor with negative charge. Most capacitors are in the picofarad [ $pF$ ] to millifarad range,  $1000\mu F$ .

Capacitors can do many things in both ac circuits and dc circuits.

- Capacitors store energy
- Capacitors when coupled with resistors can delay voltage changes
- Capacitors can be used to filter unwanted frequency signals
- Capacitors are needed to make resonant circuits
- Capacitors and resistors can be combined to make frequency dependent and independent voltage dividers

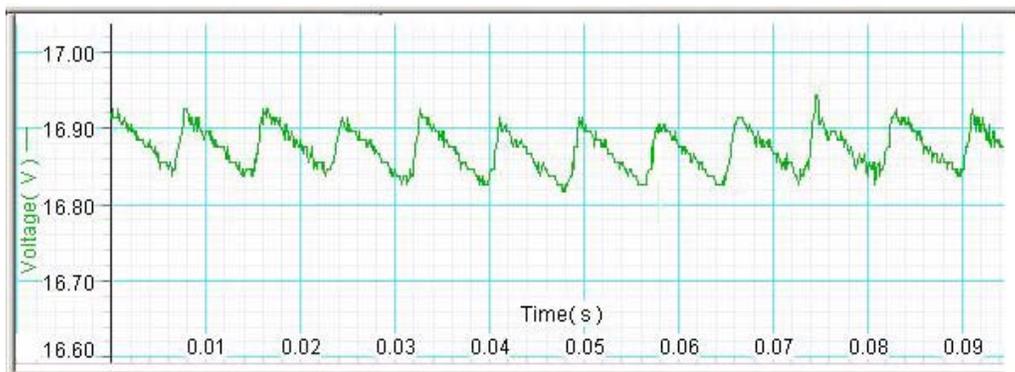
We denote capacitors in circuits by the symbol



**Figure 10: Capacitor symbol**

### Smoothing Out the Rectifier Output

A  $1000\mu F$  capacitor then smoothes out the rectifier output.



**Figure 11: Smoothed out voltage due to  $1000\mu F$**

### Voltage Regulator

Next comes the LM317T three-terminal integrated circuit (IC), containing 26 transistors and various resistors and capacitors. It keeps the output voltage constant with respect to an internal reference voltage, using feedback—i.e., it is a ‘voltage regulator’. It also protects itself against overload (too much current) and is compensated for changes in temperature.



Figure 12: Heat sink, LM317T voltage regulator, and socket

## Potentiometer

A resistor network—one variable resistor (a  $5000\ \Omega$  potentiometer, or "5k pot") and one fixed resistor ( $390\ \Omega$ ,  $1/2W$ ) serves to adjust the output voltage. Notice that the pot, here used as a variable resistance, has the slider and one end connected. This guarantees that some part of the pot resistance will be in the circuit, even if there is an uncertain contact inside the pot.

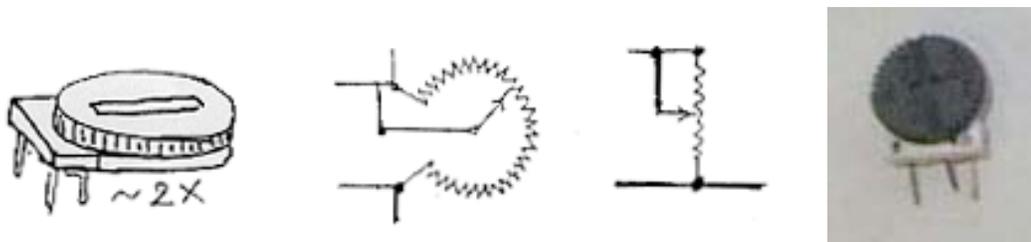


Figure 13: potentiometer and 'pot' circuit diagram

## High Frequency Filter

Finally, a  $1\ \mu F$  capacitor across the output bypasses high-frequency disturbances from either direction—from the ac supply line or from the load.

## Building the Low Voltage Power Supply

The circuit diagram for the LVPS tells us how the various parts are connected but we will place the parts on the perfboard in order to minimize the number of wires and solders. So in the following instructions try to understand the layout in terms of the circuit diagram. This will help you find any missed or incorrect connections.

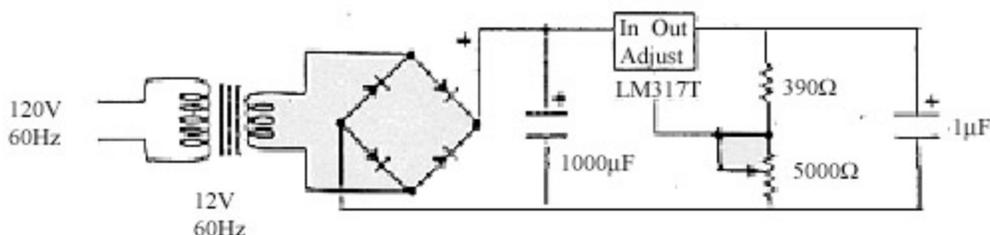
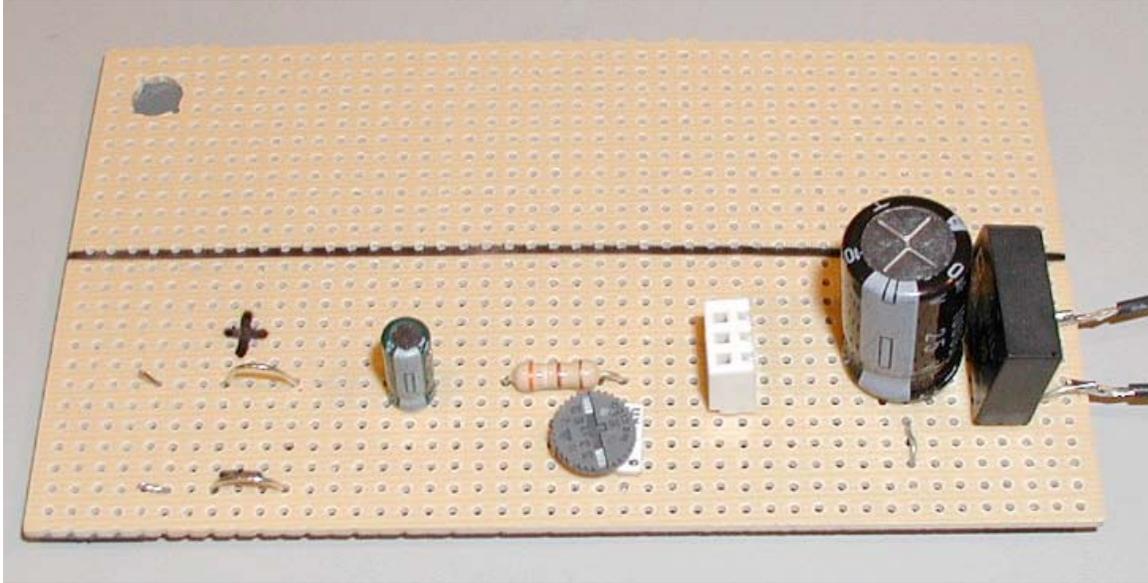
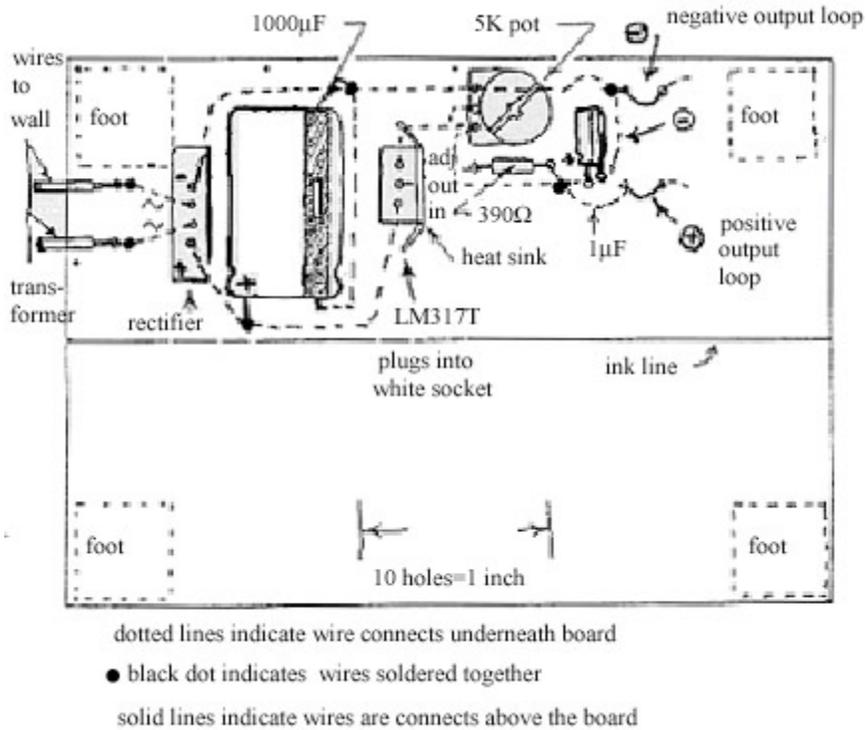


Figure 14: Circuit diagram for LVPS



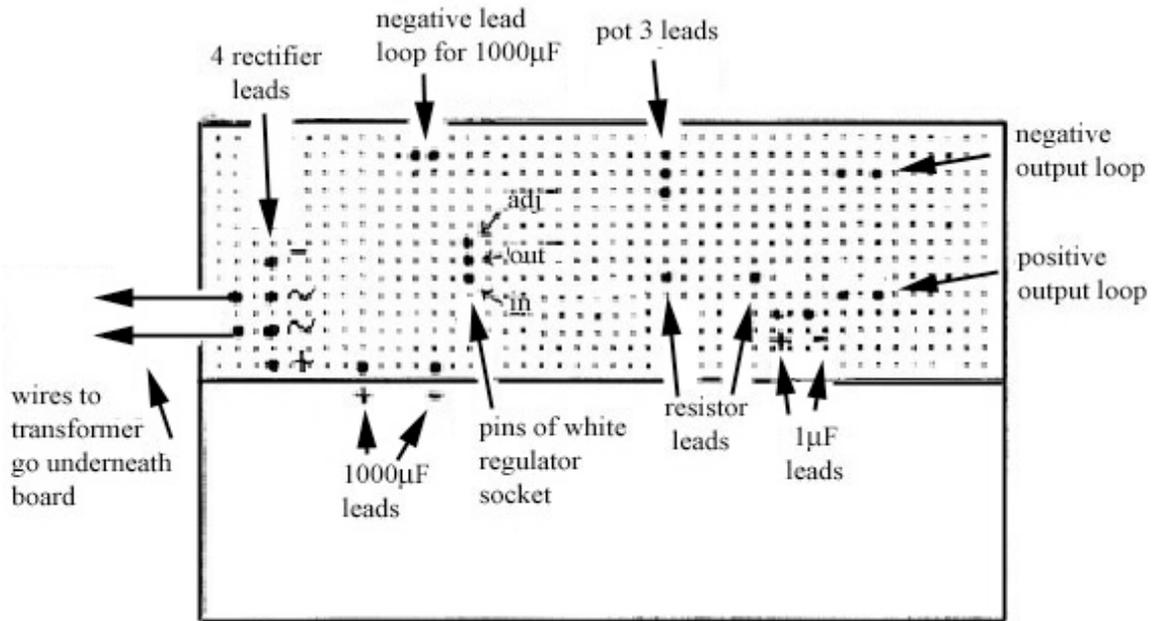
**Figure 15: Top view of LVPS (transformer leads on right)**

There are many ways to assemble the LVPS, but we will give you detailed step-by-step instructions to guarantee success. It takes up less than half the space on the perfboard, **leaving room to build other things later**. The top view of the LVPS will help in placing the parts.



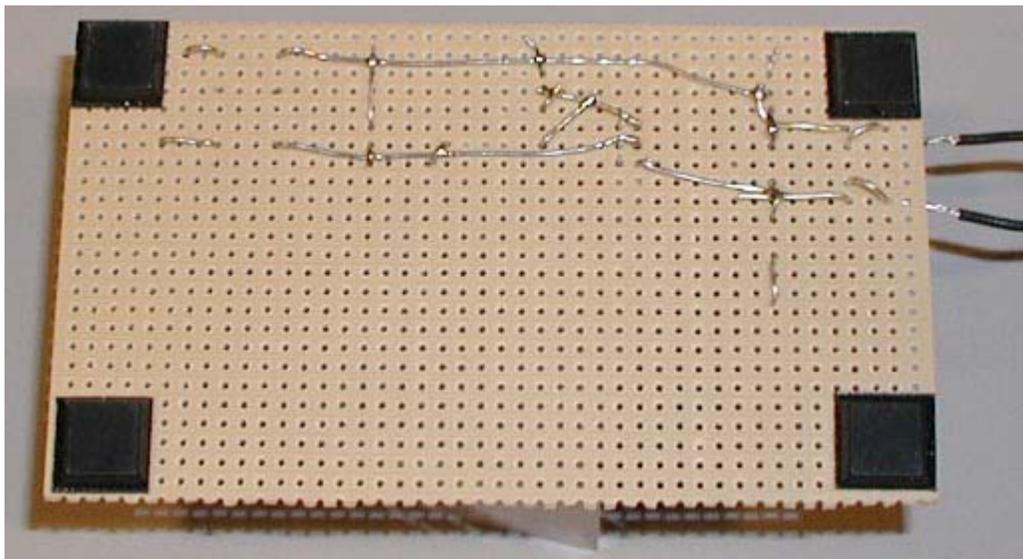
**Figure 16 Top view of layout of LVPS (transformer leads on left)**

Here is a template (top view) to help place the parts on the perfboard.

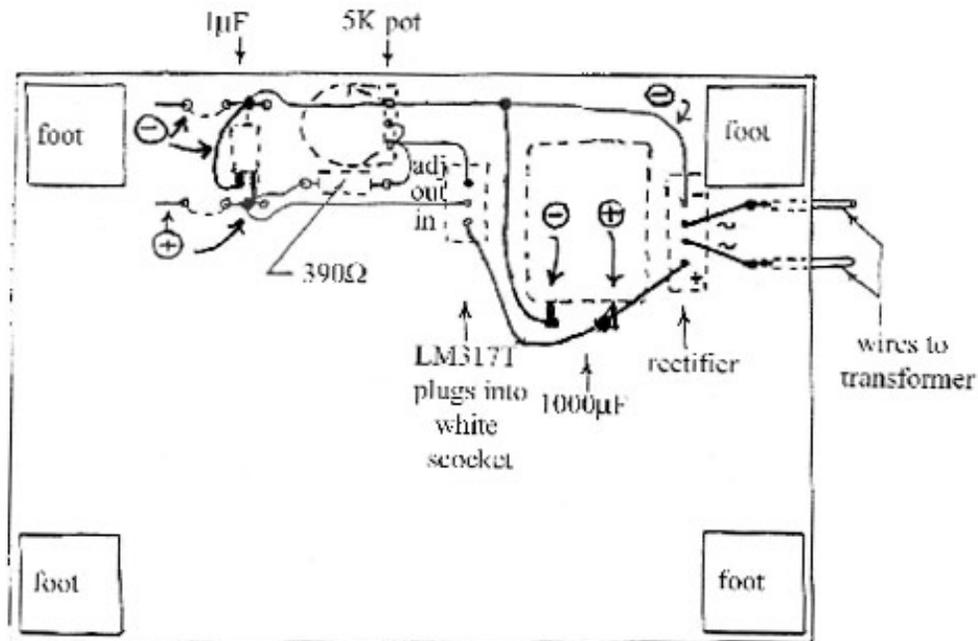


**Figure 17 Template LVPS (top view)**

The bottom view of the LVPS shows the wiring.



**Figure 18: Bottom view of LVPS**



**Figure 19: Wiring on bottom side of LVPS**

## Construction Steps

1. Find and identify parts in the plastic bag.
2. Draw a line with a pen lengthwise along the center of the perfboard.
3. Stick 4 feet on the corners of the bottom side as close to the edge as possible.
4. Place the parts according to the top view of perfboard. Bend the white socket's short leads carefully while installing. (The black regulator's three leads will fit into the socket. You will only solder the socket's leads so that the regulator can be easily removed). Identify on your perfboard which socket leads will correspond to the ADJ, OUT, and IN leads of the regulator.
5. Bend the leads of the rectifier, capacitors, and resistors as shown on the bottom view of perfboard.
6. Measure, cut, and solder a piece of the bare #22 wire to the minus lead (-) of the rectifier. Extend this wire across the board, and then form a loop on the top side. This will be the minus (-) output loop.

7. Loop the end of the minus lead (-) of the large capacitor (the band points to the minus lead) through the perfboard at the bare wire from step 6. (This will help hold the capacitor to the perfboard). Solder the minus lead (-) of the large capacitor to the bare wire from step 6.
8. Solder the pot lead nearest the edge of the perfboard to the bare wire of step 6. Be sure the pot is oriented as shown in the top view.
9. Solder the minus lead (-) of the small capacitor (the band points to the minus lead) the bare wire of step 6.
10. Solder the plus lead (+) of the rectifier to the plus lead (+) of the large capacitor.
11. Solder the plus lead (+) of the large capacitor to the IN lead of the socket. (See step 4).
12. Measure, cut, and solder another piece of the bare #22 wire to the OUT lead of the socket. Extend this wire across the perfboard, and then form a loop on the top side. This will be the plus (+) output loop.
13. Solder the plus lead (+) of the small capacitor to the bare wire of the previous step 12.
14. Solder one lead of the resistor to the bare wire of step 12.
15. Solder the other lead of the resistor to the two other leads of the pot, thus connecting those two leads of the pot together.
16. Measure, cut, and solder another piece of the bare #22 wire to the ADJ lead of the socket to either of the connected pot leads of the previous step 15.
17. Remove about 6 mm of the insulation from two different lengths, 50 mm and 100 mm (2 in and 4 in) of black stranded wire. Tin all four ends and solder one length to each of the ac leads of the rectifier.

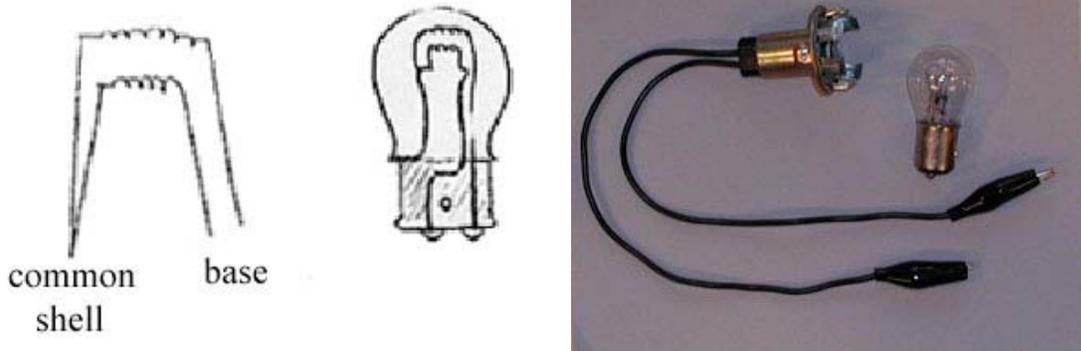
## Trying out Your LVPS

**Do not plug in the LM317T regulator.** You should have already soldered alligator clips to your transformer leads, multimeter leads, and made clip leads. Clip one of the wall transformer leads to one of the LVPS leads of step 17.

Set the MMM to the 25DCV range. Connect the voltmeter across the 1000 $\mu$ F capacitor, red to the plus side and black to the minus side. Plug in the wall transformer. Now touch the second transformer lead to the other ac lead of the LVPS. There should be little or no spark and the meter should read about 17V–18V. Transfer the voltmeter leads to the output loops. Place the regulator in the heat sink with its metal back covered by the heat sink. **Now plug in the**

regulator into the socket with the number LM317T facing the large capacitor. Turn the pot and the output should vary from about 1.2 V to 15V or more.

To make sure that your LVPS is working as it should, use the 1157 lamp (used as a rear brake light in a car) from the plastic bag labeled LVPST (*LVPS Test Kit*) as a load on the LVPS. This lamp has two filaments (tail and stop light) with nominal ratings of 8 watts and 27 watts respectively at an applied voltage of 12.6 V.



**Figure 20: Lamp and socket**

One lead to each filament is connected to the brass shell and the other lead is connected to one of the two terminals (soldered bumps) on the base of the lamp.

Plug the lamp into the socket provided in your Red Box. There are two black leads from the socket. In order to connect one of the filaments to the LVPS, use your clip lead to connect one of the black wires from the socket to one output of the LVPS. Use a second clip lead to connect the other output of the LVPS to anywhere on the socket. (This connects the LVPS to the brass shell of the lamp.) Identify the 8W filament (cold resistance about 2Ω), either with your MMM on the RX1 range or by lighting it with the LVPS---it's the upper filament in the lamp.

## Parts List for LVPS

### LVPS

- 1 perfboard
- 4 rubber feet
- 1 full wave bridge rectifier
- 1 electrolytic capacitor, 1000μF
- 1 socket for LM317T regulator
- 1 potentiometer, 5kΩ
- 1 electrolytic capacitor, 1μF
- 1 resistor, 390Ω 1/2W
- 1 ft wire, #22 bare solid
- 1 voltage regulator LM317T
- 1 heat sink for LM317 regulator

**LVPST**

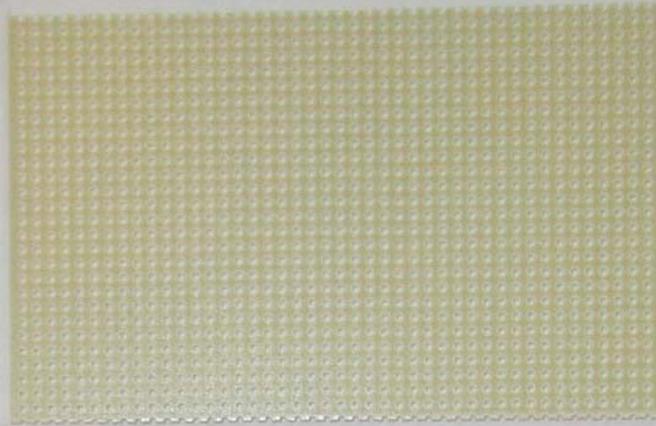
- 1 resistor,  $2.4\Omega$   $2W$
- 1 lamp #1157 automotive

**RED BOX**

- 1 socket for 1157 lamp



RUBBER FEET



PERFBOARD



FULL WAVE  
BRIDGE RECTIFIER



CAPACITOR  
1000  $\mu$ F



SOCKET



LM317  
VOLTAGE  
REGULATOR



HEAT  
SINK



POTENTIOMETER  
5K



#22 COPPER WIRE



RESISTOR  
390  $\Omega$  1/2 W



CAPACITOR  
1  $\mu$ F

## ***Testing the LVPS***

Each of you has built a power supply that converts ac (alternating current) power at  $120V$  (volt), at a frequency of  $60Hz$  (hertz = cycles/sec) from the wall outlet into dc (direct current) power with a voltage range from  $1.2V$  to about  $17V$ . When the output voltage without load (lamp) is set between  $1.2V$  and about  $12V$ , the output voltage will not change appreciably if a load is then placed across it. You will find that range when you place the  $8W$  (watt) filament of an 1157 lamp across the output of the LVPS. See **Building the LVPS: Trying out your LVPS**.

## **Measurements and Data**

You can set your pot at ten different settings from lowest to highest output by turning the top of the pot either clockwise or counterclockwise (depending on how you wired the legs). Set the pot so the no load output voltage is minimum,  $2V$ ,  $4V$ ,  $6V$ ,  $8V$ ,  $10V$ ,  $12V$ ,  $14V$ ,  $16V$ , and maximum. Use the accompanying table to record the results of your measurements. For each setting you will:

1. Measure the output voltage (no-load voltage) of the LVPS when the lamp is not connected;
2. Measure the output voltage (load voltage) of the LVPS when the lamp is connected across the output of the LVPS.
3. Then connect the lamp and measure the load voltage across the terminals of the LVPS.

## **Questions**

1. What range of no-load output voltages remains unchanged after the lamp is connected across the LVPS output terminals?
2. Briefly describe how you distinguished between the  $8W$  filament and the  $27W$  filament?
3. What happens when you connect the outputs of the LVPS to the two black wires in the socket? Can you figure out the wiring diagram for the lamp?

## **Graph**

Plot the output voltage of the LVPS without the lamp connected along the horizontal axis and the output voltage of the LVPS with the lamp connected along the vertical axis.

### Data Table for LVPS

Pot Setting	$V_{LVPS}$ (no load) [volts]	$V_{LVPS}$ (load) [volts]
minimum		
	2	
	4	
	6	
	8	
	10	
	12	
	14	
	16	
maximum		

# Experiment FO Falling Object

## Introduction

We spend most of our time in the earth's gravitational field, so feeling the weight of objects and falling with constant acceleration are among our basic experiences.

The International Committee on Weights and Measures has adopted as a standard value for the acceleration of a body freely falling in a vacuum  $g = 9.80665 \text{ m} \cdot \text{s}^{-2}$ . The actual value of  $g$  varies as a function of elevation and latitude. If  $\phi$  is the latitude and  $h$  the elevation in meters then the acceleration of gravity in SI units is

$$g = 9.80616 - 0.025928 \cos(2\phi) + 0.000069 \cos^2(\phi) - 3.086 \times 10^{-4} h \text{ m} \cdot \text{s}^{-2} \quad (4.1.1)$$

This is known as Helmert's equation. The strength of the gravitational force on the standard kilogram at  $42^\circ$  latitude is  $9.80 \text{ N} \cdot \text{kg}^{-1}$ , and the acceleration due to gravity at sea level is therefore  $g = 9.80349 \text{ m} \cdot \text{s}^{-2}$  for all objects. At the equator,  $g = 9.78 \text{ m} \cdot \text{s}^{-2}$ , and at the poles  $g = 9.83 \text{ m} \cdot \text{s}^{-2}$ . (This is because the radius of the Earth is larger at the equator than it is at the poles by about 26.5 km, and because the Earth rotates at  $2\pi$  radians per day introducing an apparent repulsive force that flattens the spherical shape). Both the magnitude and the direction of the gravitational force also show variations that depend on local features to an extent that's useful in prospecting for oil and navigating submerged nuclear submarines. Such variations in  $g$  can be measured with a sensitive spring balance. Local variations have been much studied over the past two decades in attempts to discover a proposed "fifth force" which would fall off faster than the gravitational force that falls off as the inverse square of the distance between the masses.

One can measure  $g$  by timing either a freely falling object or an object suspended from a support, oscillating as a pendulum.

## Principle of the Method

In this experiment you'll time the free fall of a plastic wire nut by measuring the voltage across a capacitor in an RC charging circuit. The goal is to measure the time of fall of a wire nut as a function of the distance of fall. The distance is measured directly with a ruler. To measure the rather short time of fall, you will measure the voltage developed across a capacitor charged by an essentially constant current that flows only during the time the wire nut is falling.

The diagrams of Fig. 1 show the sequence of events:

- Initially both switches are closed, the upper switch shorts the capacitor and the voltage across it is zero.
- When the wire nut is released and during its fall the upper switch is open and the capacitor charges.
- When the wire nut hits the cup and opens the lower switch the charging current stops.

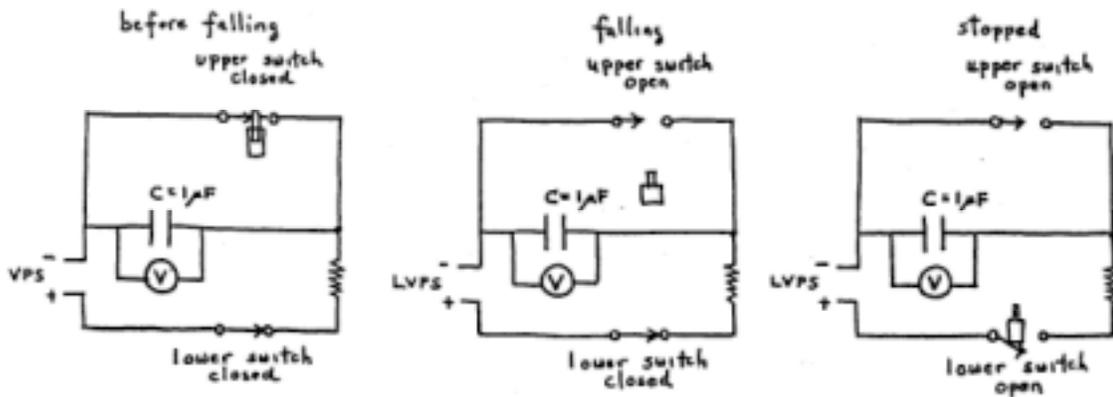


Figure 1: Circuit diagrams for falling object

## Theory

The relation between height and time for free fall is given by

$$h = \frac{1}{2}gt^2 \quad (4.1.2)$$

The time of fall can be determined by measuring the voltage across a capacitor that is charging during the fall.

Let the voltage of the LVPS supply be  $V_0$ . The capacitor  $C$  is charged through the resistance  $R$ . When the capacitor is charging, the voltage across the capacitor varies in time according to

$$V(t) = V_0(1 - e^{-\frac{t}{RC}}) \quad (4.1.3)$$

The voltage as a function of time is graphed in Figure 2.

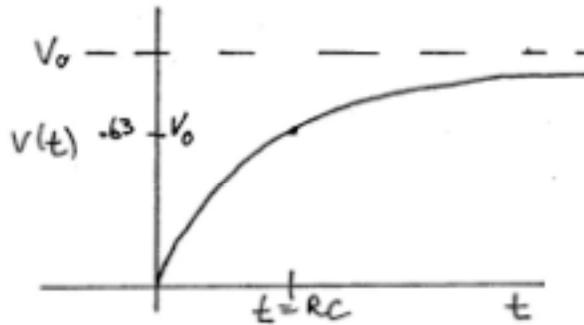


Figure 2: Graph of voltage as a function of time for charging capacitor

You will study capacitors in detail in 8.02X so for the moment we will use this charging voltage as a device to time how long the wire nut takes to fall a height  $h$ . wire nut hits the cup and opens the lower switch the charging current stops. You will then make a voltage measurement. So you will measure the quantities  $V$ ,  $V_0$ ,  $R$  and  $C$ .

The time of flight is experimentally much less than the product  $RC$ . This means that the exponential term in the expression for voltage, eq. (2), is approximately

$$e^{-\frac{t}{RC}} \approx 1 - \frac{t}{RC}.$$

Therefore the voltage is approximately

$$V(t) = V_0(1 - e^{-\frac{t}{RC}}) \cong V_0(1 - (1 - \frac{t}{RC})) = V_0 \frac{t}{RC}.$$

We can solve this equation for the time,

$$t \cong RC \frac{V(t)}{V_0} \tag{4.1.4}$$

This equation tells us that our charging capacitor acts like a clock: the time elapsed is proportional to the voltage across the capacitor.

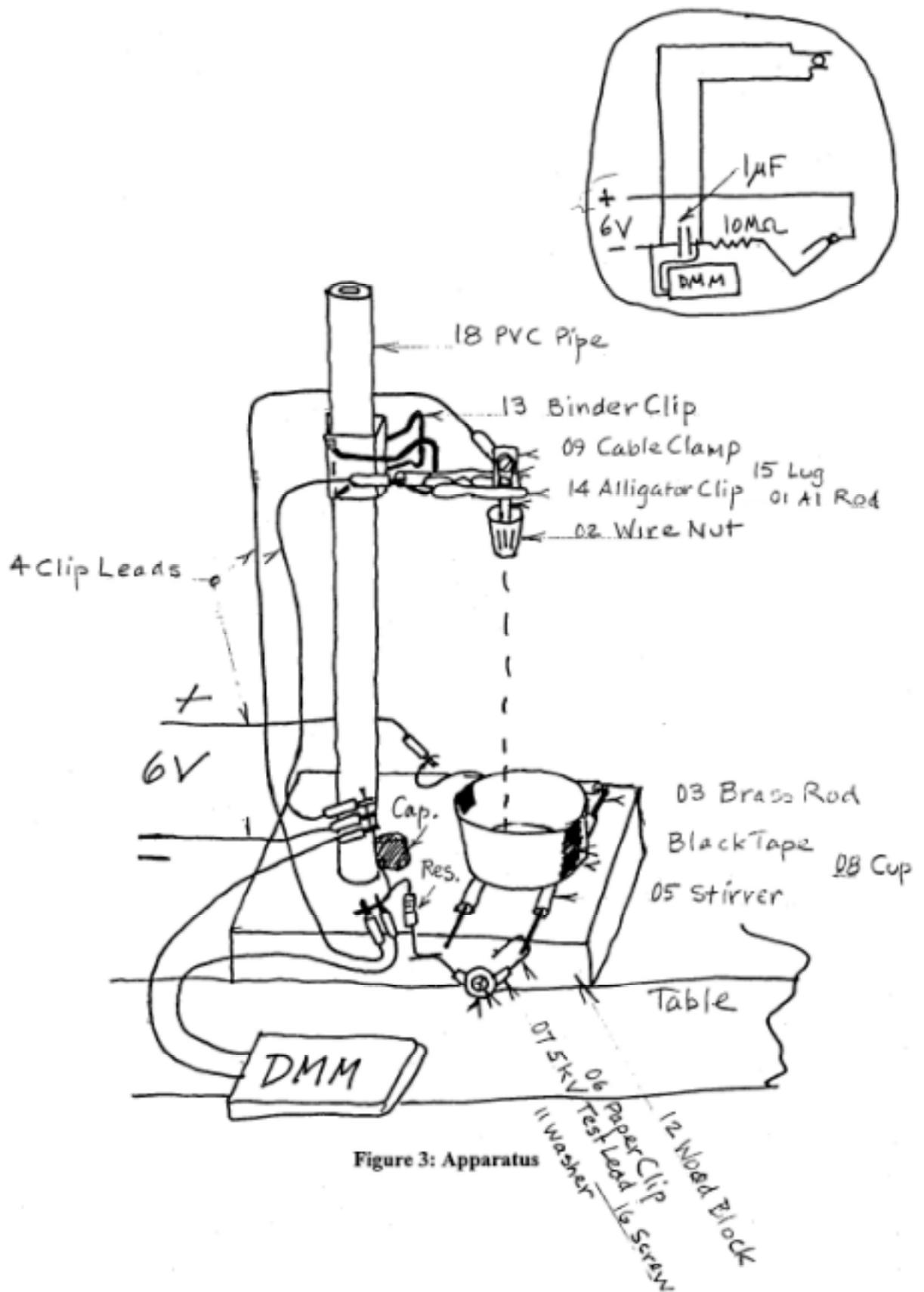
Now that we have measured the time of falling in terms of  $V$ , we can solve eq. (1) for the acceleration of gravity,

$$g = \frac{2h}{t^2} = \frac{2hV_0^2}{(RCV)^2} \quad (4.1.5)$$

The time constant,  $\tau = RC$ , of the 1.0 mF capacitor and the 10 MW resistor is given by the nominal value,

$$\tau = RC = (1.0 \times 10^7 \Omega)(1.0 \times 10^{-6} F) = 10s,$$

that could be off by 10 % or more.



## Apparatus

The apparatus is shown in figure 3. The wire nut has an aluminum rod screwed into it so that it can be held in the jaws of an alligator clip and released by squeezing the clip open. There is an insulated contact on one jaw of the clip; releasing the wire nut opens a circuit and allows the charging of the capacitor to begin. The binder clip that holds the alligator clip can be slid to various positions on the vertical piece of PVC pipe pressed into a wooden block clamped to your desk with a C-clamp.

On the wooden block is a switch made of a pivoted piece of bent brass rod, a paper clip and a cup to catch the wire nut. When the wire nut hits the cup, the switch opens and the charging current to the capacitor stops.

The apparatus is made of every-day stuff to illustrate what can be done without complicated, costly, specialized material. Just as describing a simple action like tying shoelaces in words can be lengthy, so are the instructions for assembling the 20 items into an apparatus that works properly and yields reasonably good data. **Assembly instructions and explanatory drawings are at the end of this hand-out.**

## Experiment

After wiring up the circuit as shown in the diagrams, clamp the wire nut in the release switch, leaving the lower switch open. Set the LVPS to near 6 V and record the reading (and recheck it from time to time). Higher voltages up to 10 V may give better accuracy, but the automatic range-switching feature of the DMM between millivolt ranges and volt ranges, can give unreliable and unsteady readings.

Use your plastic ruler to measure the height (to the nearest millimeter) from the bottom of the suspended wire nut to the bottom of the cup in its lower position (lower switch open). Also measure the distance you have to raise the cup (just a few millimeters) to just close the lower switch from below—the position it has when it just opens. This distance has to be subtracted from the height

Close the lower switch. The DMM should read zero. Then release the wire nut by opening the alligator clip decisively but without shaking the whole works — squeeze it firmly between thumb and index finger — and **hold it open** until the wire nut has dropped into the cup and the DMM reading has reached a steady maximum value that you should record.

Put the wire nut back in the release switch; **then** close the lower switch. (This sequence avoids unnecessary charging of the capacitor. Then drop the wire nut again. Make 5 voltage measurements at each of 4 heights, eg. 0.2 m, 0.15 m, 0.1 m and 0.05 m.

Unsteady readings while holding the upper switch open arise from electrical leakage. Be sure that you are not touching anything besides the alligator clip. Leakage may be caused by high humidity, and a warm electric lamp nearby may help dry the apparatus out.

## **Analysis**

For each height  $h$ , average the voltage measurements and using the formula in equation (3) for the time of a charging capacitor, calculate the time of fall.

Plot the square of the time (vertically) against  $h$  in m. There's also a point at the origin since it takes 0 time to fall 0 distance.

Fit the best straight line (by eye), and determine the slope. Since the relation between height and time for free fall is given by

$$h = \frac{1}{2}gt^2,$$

the slope of your graph can be used to calculate your average value for  $g$ ,

$$g = \frac{2}{\text{slope}}.$$

## Step-by-Step Assembly of Experiment FO Apparatus

### Parts List

The parts are numbered as they appear in figures 4 and 5.

01	1"	1/8" Al rod
02	1	Wire Nut
03	8"	1/16" brass rod *
04	1'	#22 stranded wire
05	2	plastic stirrers
06	2	paper clips
07	2"	5kV test lead
08	1	paper cup *
09	2	cable clamps
10	2	#6 sheet metal screws
11	3	#6 steel washers
12	1	pre-drilled wood block * (ALSO USED IN EXPT. FM)
13	1	5/8" binder clip
14	1	alligator clip
15	1	solder lug
16	1	6-32 x 1/2" steel screw
17	1	6-32 steel nut
18	12"	1/2" PVC pipe *
19	1	10 MW resistor, 5%
20	1	1.0 mF 100-V capacitor, 10%,

The four items marked with an asterisk \* are in the Redbox. The remainder are in a plastic bag.

### Preliminary

1) Fig. 5a Hold the 1 inch long piece of 1/8 inch aluminum rod (01) with your slip-joint pliers and screw the wire nut (02) onto it so that they are firmly joined and the wire lies along the central axis of the nut. This is the "falling object", and is shown in Fig. 5a.

2) Fig. 4a. Hold the 8 inch long piece of 1/16 inch brass rod (03) with your slip-joint pliers so that 4-3/4 inches sticks out sideways from the jaws of the pliers. Bend the rod to a right angle as in Fig. 4a.

- 3) Strip 1-1/2 inches of insulation off one end of the piece of #22 stranded wire (04). Don't twist up the strands. Cut the wire to 8 inches length and strip 1/4 inch off the other end and twist up the strands. Use the wire stripper, properly set for stripping the #22 wire. If in doubt, practice on the extra 4 inch piece.
- 4) Cut, with scissors, one of the plastic stirrers (05) into 3 pieces, one piece 1 inch long and two pieces 2 inches long.
- 5) Fig. 4i: Use long-nose pliers to straighten the smaller radius end of one of the paper clips (06) so that it becomes a hook. Hold the clip so that 1-1/2 inches sticks out sideways from the jaws of the pliers and bend it 90° into a V-shape as in Fig. 4i.
- 6) Cut a 1 inch long piece of 5 kV (kilovolt) test lead (07) with wire cutters. Use your long nose pliers to pull out the wire strands and wrapping thread from the rubber insulation, leaving in effect a piece of small-inside- diameter, thick-wall tubing.
- 7) Use scissors to cut around the paper cup (08) parallel to its rim 1-1/2 inches from the bottom so that the cup becomes smaller and less deep.

### **Assembly of the Lower Switch**

- 8) Figs. 4a and 4b: Grasp the shorter bent part of the brass rod (03) and wrap the 1-1/2 inch stripped part of the stranded wire (04) about the rod, starting at the bend as shown in Fig. 4a. Slide the 1 inch length of plastic stirrer (05) over the rod and the wrapped wire, turning it as if to screw it on (Fig. 4b).
- 9) Fig. 4c: Hold the longer part of the brass rod (03) with your slip-joint pliers so that 3-1/4 inches sticks out sideways from the jaws of the pliers. Bend the rod to a right angle as in Fig. 4c so as to form a "U". Adjust the bends so that the arms of the U are parallel and in the same plane.
- 10) Fig. 4d: Bend the wire (04) back 180° so it's next to the stirrer and slip the plastic cable clamp (09) with its **flat side down** over the piece of stirrer and the wire as in Fig. 4d.
- 11) Fig. 4e: Use one of the sheet metal screws (10) and washers (11) to fasten the assembly of Fig. 4d to the wood block (12). Be sure that you use the pilot hole closer to the top of the block. The top side of the block is the one with the largest hole, 5/8 inch. A side view is shown, about 3 times full size, in Fig. 4e. Note: There should be enough friction so that the U-shaped piece barely falls under its own weight—this is to prevent it bouncing back when the falling object hits.

12) Fig. 4f: Slide onto the two arms of the U the two 2 inch pieces of stirrer, making sure that they don't extend beyond the edge of the block. Lift up the two arms and place a 6 inch length of black electrical tape, **adhesive side up**, on the wooden block with one end of the tape at the end nearest the 5/8 inch hole. See Fig. 4f. Press the arms of the U down so that the stirrer pieces stick to the tape.

13) Fig. 4g: Hold the cut-off cup (08) centered over the space between the two 2 inch pieces of stirrer (Fig. 4f) and pull up the tape on each side and down into the cup, as in Fig. 4g. Press the tape up so that it stretches and sticks to the bottom of the cup.

14) Fig. 4h: Screw a sheet metal screw (10) with a washer (11) part way into the wooden block (12) using the pilot hole closer to the bottom of the block.

15) Figs. 4h and 4i: Slip the piece of rubber insulation (07) over the right angle bend in the straightened paper clip (06) and attach it to the wooden block by tightening the screw and thus clamping the rubber insulated paper clip between the washer and the wooden block. See Figs. 4h and 4i. Note that the loop of the paper clip should be so positioned that the brass rod sits on it and can slip off easily. The straightened left end of the paper clip should be bent down as indicated by the dotted lines in Fig. 4h. To avoid electrical leakage, **no part** of the paper clip **should touch the wood block**.

### **Assembly of the Upper Switch**

16) Fig. 5a: Clip the 5/8" binder clip (14) to the 12 inch long piece of 1/2 inch PVC pipe (18). Notice that by squeezing the clip levers slightly while keeping it pressed against the pipe you can slide it back and forth.

17) Figs. 5a: Take the alligator clip (14), remove the screw and put a #6 washer (11) on it. Replace the washer and screw with a few turns. Hook the screw and washer so the lever of the binder clip (13) is between the washer and the alligator clip as in Fig. 5a, and tighten the screw firmly.

18) Fig. 5b: Put the solder lug (15) on the 1/2 inch 6-32 machine screw (16), put the screw through the hole in a cable clamp from the flat side and screw on the 6-32 nut (17). With the small hole end of the lug lying on the flat side of the clamp tighten the nut moderately. See Fig. 5b. Use pliers to bend the small hole end of the lug so that it conforms to the curve of the clamp.

19) Fig. 5c: Loosen the nut (17) so that you can slip the clamp (09) over the stationary jaw of the alligator clip (14). Tighten the nut firmly. The lug is now sandwiched between the two jaws as shown in Fig. 5c.

## **Final Assembly**

20) Fig. 6: Fit the PVC pipe into the 5/8 inch hole in the wood block with a careful rocking and/or twisting motion. A few taps with your slip-joint pliers may help seat the pipe firmly.

21) Fig. 6: With a well tinned soldering iron, tin the left end of the paper clip and solder one end of the 10 megohm resistor—no need to twist wires; simply hold them parallel and let the solder join them. Likewise, solder the 1 microfarad capacitor to the resistor, as shown in Fig. 6.

22) Clamp the apparatus to a desk or table. Use a C-clamp on the corner of the wood block furthest from the paper clip of the lower switch.

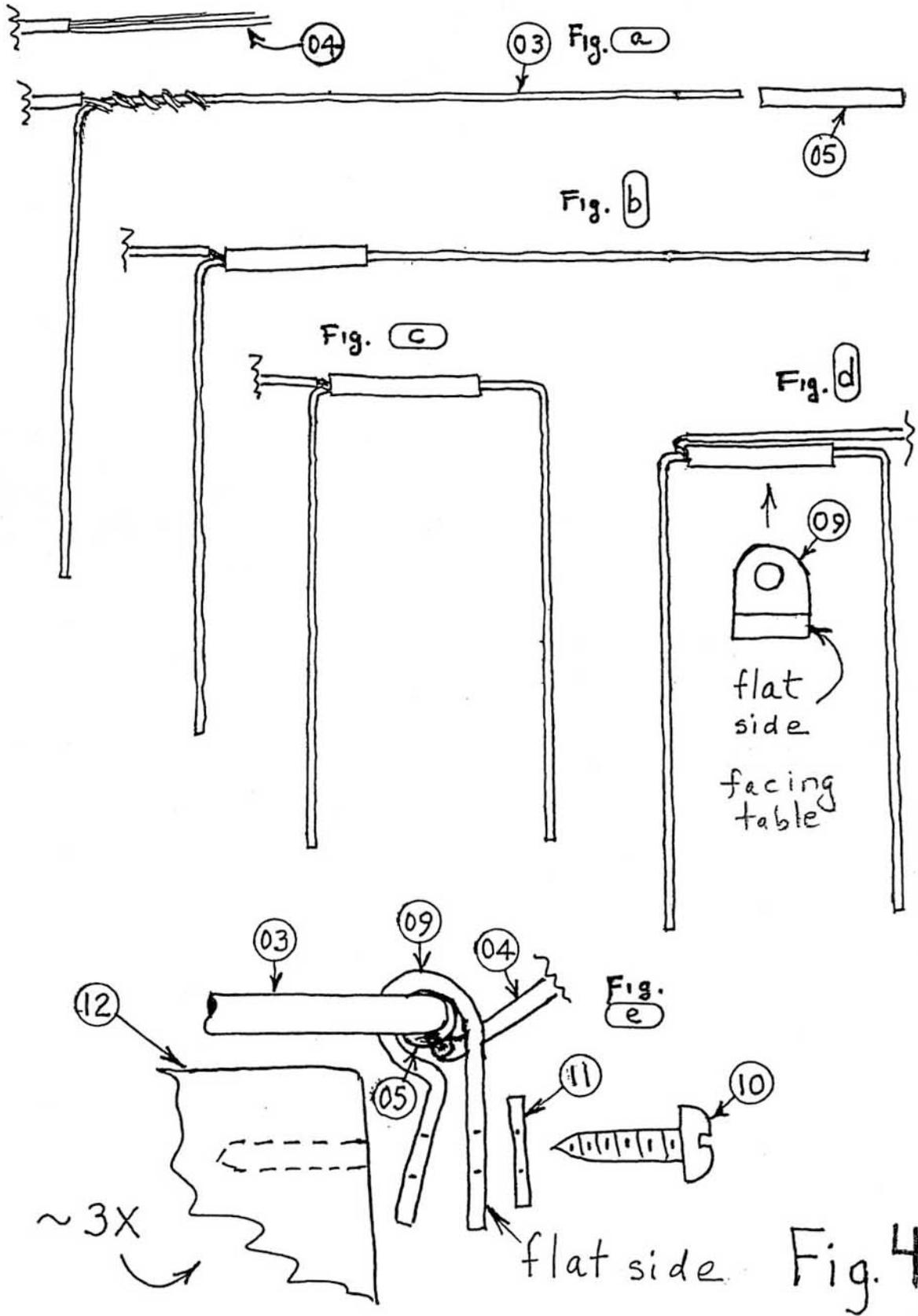
23) Connect the two switches, an LVPS, and a DMM to the resistor and capacitor with clip leads, as shown in Fig. 6.

## **Adjustment and Operation**

24) Figs. 4h and Fig. 6: Slide the brass U sideways in its cable clamp hinge so that the right arm of the U rests on the U-shaped part of the paper clip as shown in Figs. 4h and Fig. 6. Hold the wire nut in your fingers and drop it into the cup from a height of ~2 inches—this should operate the switch and open the circuit.

25) Fig. 6: Slide the binder clip to the top of the PVC pipe with the jaws of the alligator clip approximately centered over the cup. Grab the aluminum rod of the falling object in the alligator clip, with the rod pointing straight down and with its end flush with the upper edges of the jaws. Again, see Fig. 6.

26) Try dropping the wiring nut and observe the action of the lower switch. It shouldn't bounce back up when the wire nut hits. Reclosing the circuit on bouncing will cause incorrect high readings. This was the reason for having some friction in the hinge. That friction can be increased by tightening the clamp screw or even by adding a straightened paper clip between the clamp and the piece of stirrer. If the switch doesn't open there's too much friction which can be reduced by loosening the clamp screw. Another way of preventing bouncing is to put a piece of electrical tape, adhesive side up, under the U of the lower switch so that the pieces of stirrer stick to it. Use three more pieces of tape to hold the first one to the wooden block.



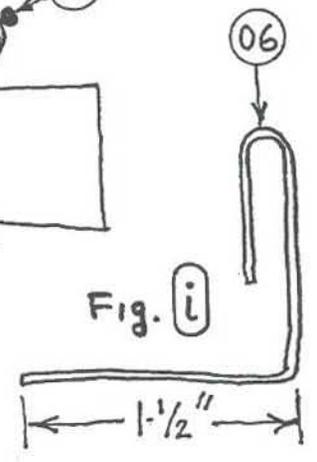
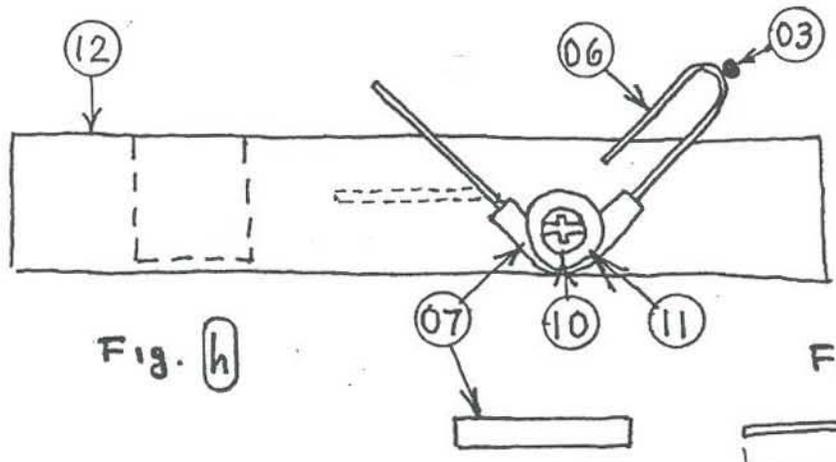
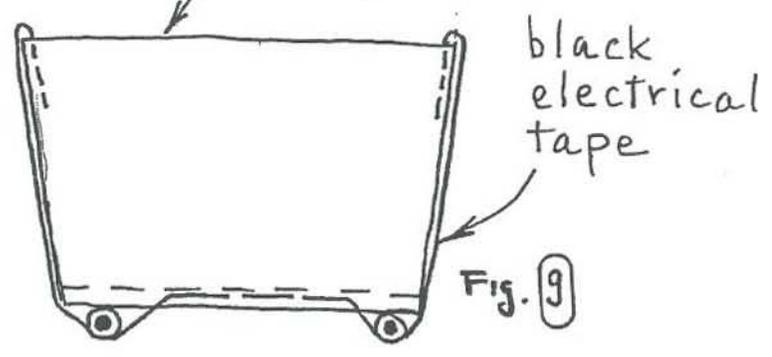
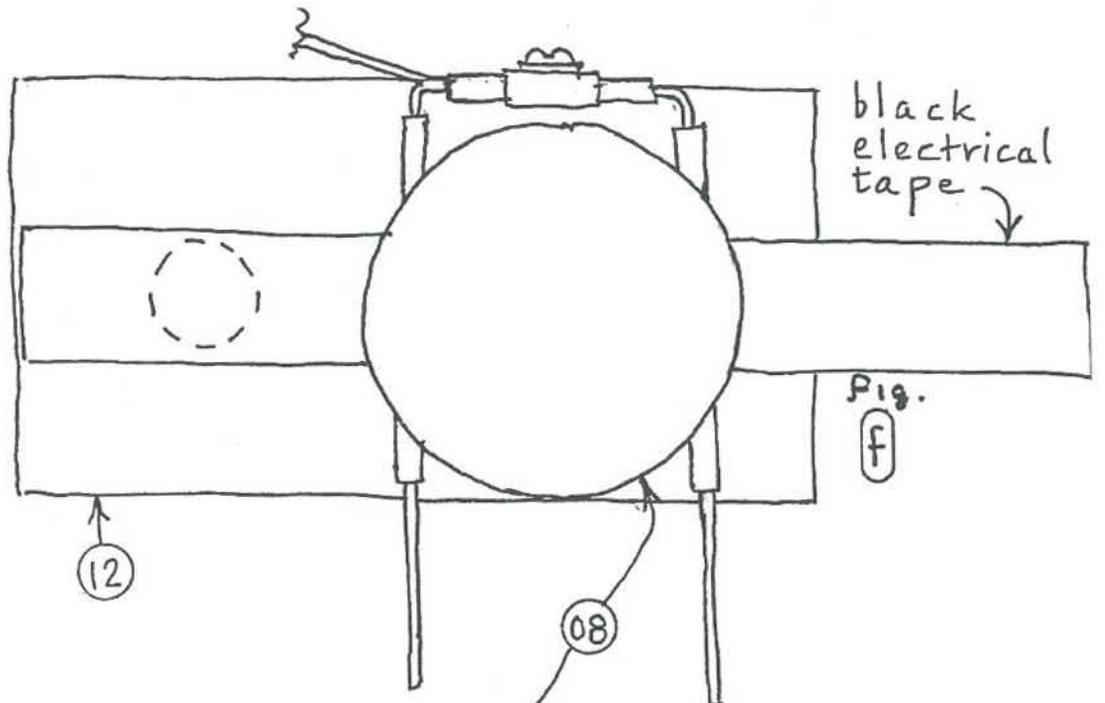


Fig. 4 (continued)

Fig. 5a

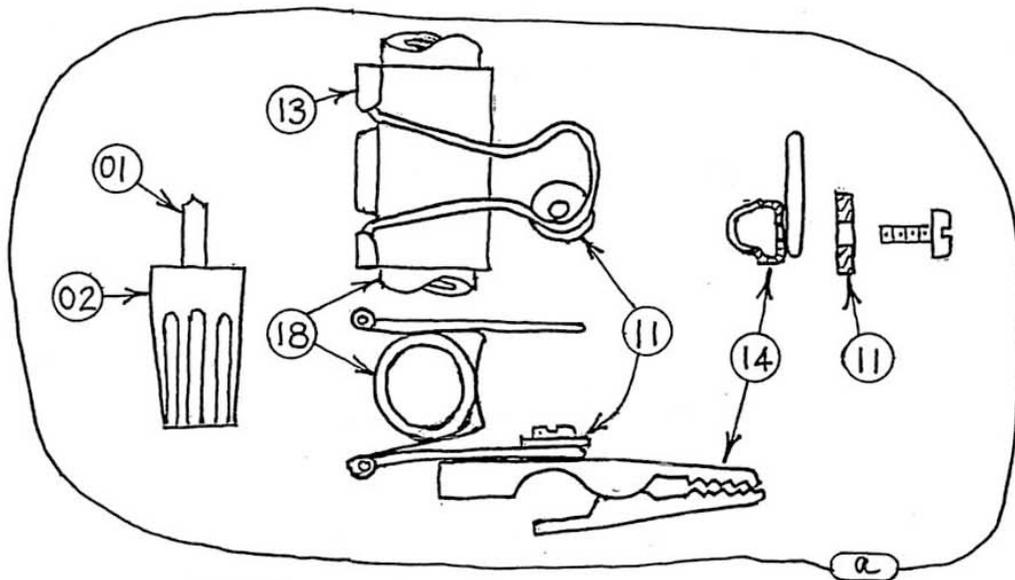


Fig. 5b

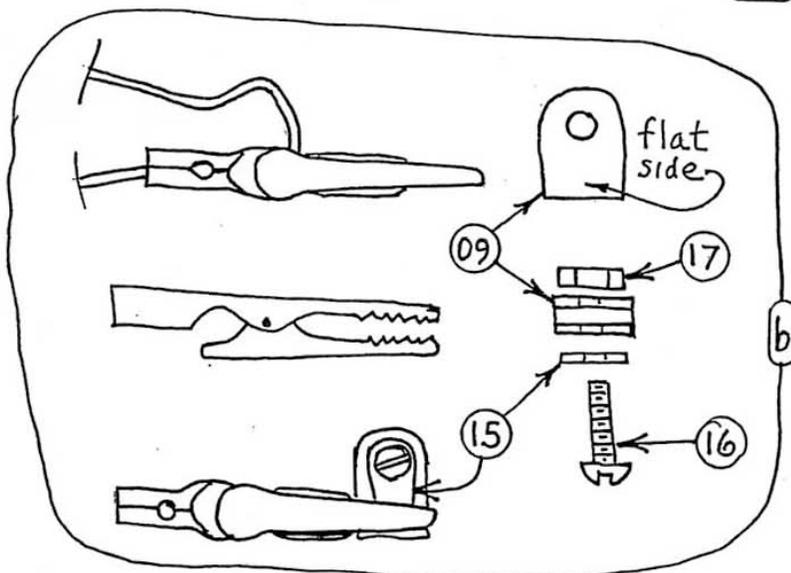


Fig. 5c

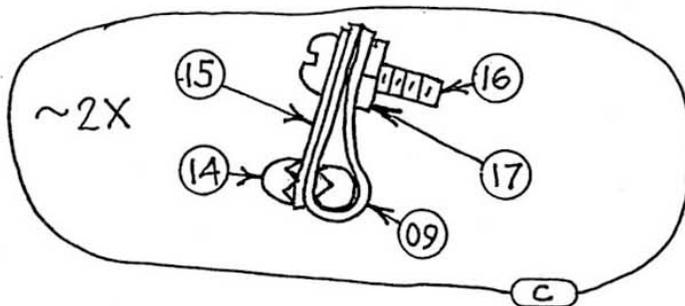


Fig. 5

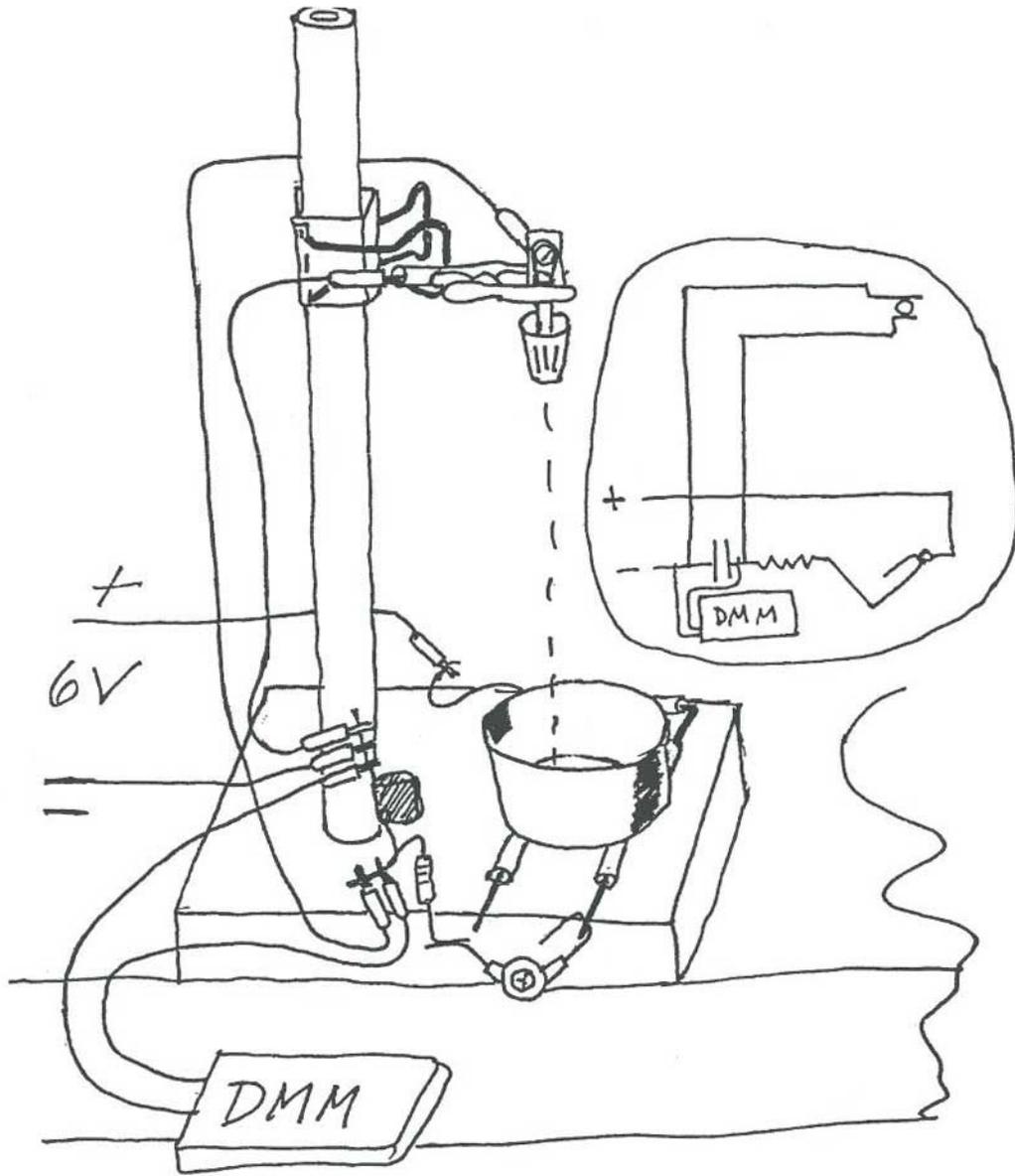


Figure 6

# Experiment FM Force between Magnets

## About Forces and Fields

There are forces that don't change appreciably from one instant to another, said to be constant in time, and forces that don't change appreciably from one point to another, said to be constant in space. The weight of a mass on the earth is an example,  $\vec{F}_{grav} = m\vec{g}$ .

There are forces that increase as you move away. When a mass is attached to one end of a spring and the spring is stretched a distance  $|x|$ , the spring force increases in strength proportional to the stretch,  $|\vec{F}| = k|x|$

There are forces that stay constant in magnitude but always point towards the center of a circle, for example when a ball is attached to a rope and spun in a circle, the tension force acting on the ball is directed towards the center of the circle. This type of attractive central force is called a centripetal force

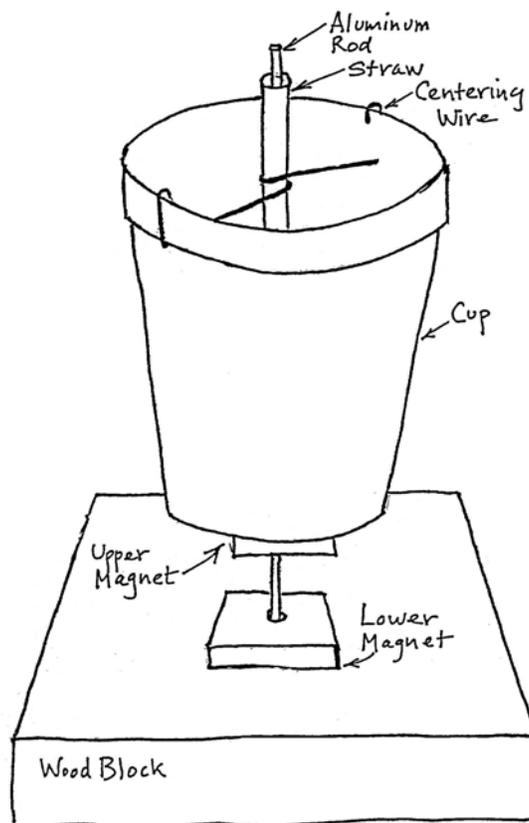
There are forces that spread out in space such that their influence becomes less with distance. Common examples are the gravitation and electric forces. The gravitational force between two masses falls off as the inverse square of the distance separating the masses provided the masses are of a small dimension compared to the distance between them. More complicated arrangements of attracting and repelling things give rise to forces that fall off with other powers of  $r$ : constant,  $1/r$ ,  $1/r^2$ ,  $1/r^3$ , etc.,. Thus you might expect the force between 2 rectangular magnets to vary with distance in some not very obvious way—but calculable with time and patience.

In this experiment you'll measure the force  $F$  newtons (N) required to press two rectangular magnets together as a function of the distance between the centers of the magnets. One magnet will be fixed to a block and the other magnet will be suspended above. A cup is balanced on top of the upper magnet. Pennies will be added to the cup pressing the magnets together.

## Assembling the Apparatus

- Press the aluminum wire into the hole in the wooden block. Don't bend the wire.
- Cut off a 100 mm length of the 7 mm OD plastic soda straw
- Make a hole (from below) in the center of the bottom of the Styrofoam cup with the Phillips screwdriver in your kit.
- Carefully enlarge the hole with the long-nose pliers so that the plastic soda straw fits tightly in the hole with one end flush with the bottom of the cup. NOTE: You can practice this first in the side of the cup if you like.

- Take the 140 mm length of number 22 wire and form a loop in the middle that will fit around the straw—just wrap it around. This is the centering wire.
- Make two holes diametrically opposite in the side of the cup about 13 mm from the top edge, just below the reinforcing thicker part of the rim of the cup.
- Put the wire loop over the straw. Bend the straw first to one side and then to the other in order to put the ends of the wire through the two holes.
- Squeeze the cup a few millimeters out of round, until the straw is centered in the opening and bend up the wire on one side.
- Again squeeze the cup out of round, until the straw is centered in the opening and bend up the wire on the other side.
- Clamp the wooden block to your desk, put the two magnets over the aluminum wire in such a way that one repels the other and slide the cup with its central straw over the wire.

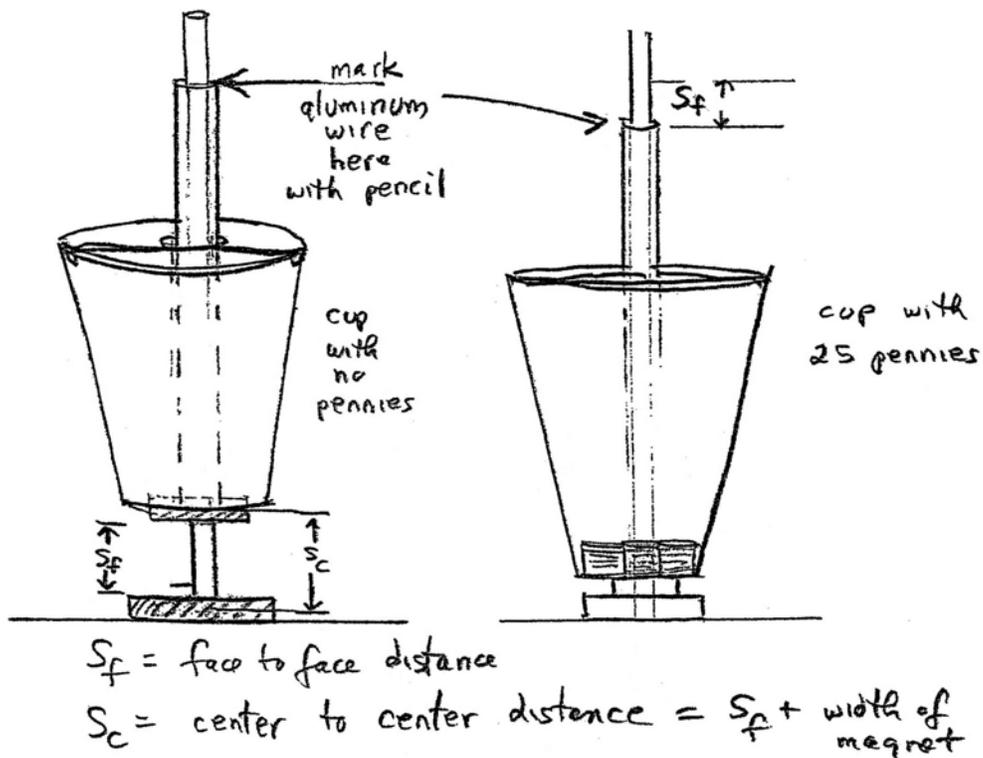


To make sure the apparatus is OK, press down on the cup with two fingers on a diameter until the magnets touch and release it—the cup should bounce up and down about 3 or 4

times. Give the cup a twist of say  $20^\circ$  —it should oscillate a few times, 3 or 4. Now remove the cup.

## Experiment

- The first measurement you will make are the width of the magnets. You should find this width to be about 4 mm.
- The second measurement you will make is the distance between the magnets without the weight of the cup and pennies acting on the suspended magnet. To do this, place your ruler on the block and measure the distance to the center of the upper magnet. You may notice that the upper magnet is tilted at an angle so try to estimate the location of the center of the upper magnet. Subtract half the width of a magnet (about 2 mm) in order to obtain the center distance  $S_c$ , between the faces. This distance corresponds to a force equal to the weight of 1 magnet,  $0.074 \pm 0.002$  N. Make this the first entry in your table of separation between magnet faces,  $S_c$  (in mm) and  $F$  (in N).



$$S_c = S_f + 4 \text{ mm}$$

- Place the upper magnet and cup on the wire. The magnets are now closer because of the weight of the cup, straw and centering wire,  $0.025 \pm 0.003$  N. Make a pencil mark on the aluminum wire next to the top of the straw.

- Put 5 pennies in the cup placed symmetrically around the straw at the bottom. Make another pencil mark on the aluminum wire next to the top of the straw.
- Continue adding pennies in groups of 5, always placing them symmetrically around the straw, and making pencil marks on the aluminum wire.
- As the magnets get close you may need to bend the aluminum wire slightly so that the magnet faces are parallel. With about 25 pennies the magnets are essentially touching and the center to center distance,  $S_c$ , corresponds to the width of a magnet (about 4 mm).

## Data

Remove the cup and upper magnet. Measure the distance between your highest pencil mark and the lowest pencil mark. This corresponds to the face-to-face distance between the magnets when there are no pennies in the cup. Add the width of one magnet to get the center-to-center distance,  $S_c$ . Continue this procedure after putting 5 pennies in the cup, measuring the distance between your highest pencil mark and your lowest pencil mark. This corresponds to the face-to-face distance between the magnets when there are 5 pennies in the cup. Add the width of one magnet to get the center-to-center distance,  $S_c$ . Estimate your measurements to the nearest 0.5 mm. Continue for 10, 15, 20, and 25 pennies.

Post 1982 pennies have a mass of 2.50 g and therefore a weight of 0.0245 N. Compute the cup and penny weights in newtons from their masses. Thus your cup, straw and centering wire mass, 2.50 g, should be added to the magnet mass,  $7.6 \pm 0.2$  g. For example, 5 pennies, with a mass 12.5 g yield a total mass of 22.6 g, corresponds to a force of 0.22 N.

Tabulate the data, with two columns labeled Force (N), and Center-to-Center Distance,  $S_c$ , (mm). Make two plots of the data; one on linear paper and the other on log-log paper with center-to-center distance  $S_c$  (in mm), along the horizontal axis and Force (in N) along the vertical axis.

## Analysis

On the log-log paper try to fit a straight line between the data points to match your best-fit curve. If you cannot match one straight line, you may be able to find two different regions where there are straight line fits. This means that the force between these magnets can be described by different inverse powers at different distances. In particular when they are very close and the separation is small compared to their width and length we often say they are infinite planes. Since any finite gap is in effect zero compared to infinity, you would not expect the force to vary—another way of thinking about this is

that with  $\infty$  featureless planes there is no scale of length and no way of telling how far away you are. This is why there is a slower variation of  $F$  with  $S_c$  when  $S_c$  is small. On the other hand, for very large distances one can expect  $F$  to go with some inverse power law.

Calculate the slope of the  $\log F$  vs.  $\log S_c$  best-fit straight lines. This gives the approximate power law for the force between the magnets for different ranges of center-to-center separation distance. You may use a program to find the best fit straight line. If you want to calculate the slope note that if the force is a power law

$$F = a(S_c)^b,$$

where  $a$  is a constant and  $b$  is the power. Then

$$\log F = \log(a(S_c)^b) = \log(a) + \log((S_c)^b) = \log(a) + b \log(S_c).$$

The slope of the  $\log F$  vs.  $\log S_c$  graph is the power exponent  $b$  and the intercept is the constant  $a$ . On the log-log graph paper choose two points that lie on your best-fit straight line. For example, suppose you choose the points

$$(x_1, y_1) = (9.0 \text{ mm}, 0.9 \text{ N})$$

$$(x_2, y_2) = (4.0 \text{ mm}, 3.0 \text{ N}).$$

Then the slope is

$$\text{slope} = \frac{\log y_2 - \log y_1}{\log x_2 - \log x_1} = \frac{\log(3.0 \text{ N}) - \log(0.9 \text{ N})}{\log(4.0 \text{ mm}) - \log(9.0 \text{ mm})} = \frac{\log(3.0 \text{ N} / 0.9 \text{ N})}{\log(4.0 \text{ mm} / 9.0 \text{ mm})} = -1.5$$

## Parts

- 1 wooden block
- 1 length #8 (1/8 in) aluminum wire, 140 mm
- 1 styrofoam cup
- 1 plastic soda straw
- 1 length #22 copper wire, 140 mm
- 1 paper clip #1
- 2 rectangular magnets with ~4 mm hole
- 50 US pennies

## Experiment CF Centripetal Force

Note: You will need both of your LVPS for this experiment. So build your second LVPS if you have not done so already.

### Introduction

This experiment is about centripetal force, the force that keeps objects moving in a circular path at a constant velocity. This force is always at right angles to the motion and directed towards the center of the circular orbit. By Newton's Second Law when a centripetal force acts on a mass, the force continually changes the direction of the velocity without changing its magnitude. This change in the direction of the velocity is always towards the center and accounts for the centripetal acceleration of the mass.

Examples of centripetal forces are tension in a string connected to a rotating mass; the gravitation force between a planet and a rotating satellite; and magnetic forces on moving charged particles in a magnetic field.

### Experiment

In this experiment, the centripetal force is the restoring force of a stretched rubber band with one end attached to the shaft of a dc permanent-magnet motor powered by your LVPS and the other end attached to a # 6-32 nut. The nut has a mass  $0.92 \pm 0.01\text{g}$  and the rubber band has a mass of  $0.22 \pm 0.02\text{g}$ . The motor whirls the nut in a circular path at a rotational frequency  $f$  that you will set using a stroboscopic method. You will measure the radius of the circular path,  $r$ , for various rotation speeds. This will allow you to calculate the centripetal acceleration

$$|a_r| = r 4\pi^2 f^2 .$$

You will set the rotational frequency by using a light-emitting diode (LED) that flashes 60 times per second when connected to the wall transformer. (The wall transformer supplies a nominal 12 Volts AC at 60 Hz). You can adjust the motor speed by adjusting the output voltage of the LVPS. You can then set your motor speed by a stroboscopic method. You will place a pattern of lines on the shaft of the motor. While the LED is directed downward onto the motor shaft, you adjust the LVPS until you see a stationary pattern of lines. This will correspond to a frequency that is some fraction of 60 Hz.

The rubber band stretches until the restoring force of the rubber band exerts the necessary centripetal force on the nut to keep the nut spinning in a circular orbit. You calibrate the rubber band by hanging pennies in a cup that stretch the rubber band and measure the stretched length of the rubber band as a function of the hanging weight. Thus by measuring the stretched length of the rotating rubber band (the radius of the circular orbit) you can use your calibration to

determine the centripetal force acting on the spinning nut. You can then verify that Newton's Second Law is indeed correct, namely that

$$|\vec{F}_{radial}| = m|\vec{a}_{radial}| = m r 4\pi^2 f^2$$

## Theory

When an object is moving in circular motion the direction of velocity is always tangent to the circle. Therefore the direction of the velocity is always changing towards the center of the circle. This means that there is a non-zero component of the acceleration directed radially inward. This acceleration is called the *centripetal acceleration*. If our object is increasing its speed or slowing down there is also a non-zero *tangential acceleration*. But when the object is moving at a constant speed in a circle then only the centripetal acceleration is non-zero. Therefore there must be a '*centripetal force*', a radial force pointing inward, that produces this acceleration. Since Newton's Second Law is a vector equality,  $\vec{F} = m\vec{a}$ , can be applied to the radial direction yielding (in magnitude)

$$\vec{F}_{radial} = m\vec{a}_{radial}$$

## Kinematics of Circular Motion

We choose the origin of our coordinate system as the fixed central point. We shall choose coordinates for a point P in the plane as follows (Figure 1). One coordinate,  $r$ , measures the radial distance from the origin to the point P. The coordinate  $r$  ranges in value from  $0 \leq r < \infty$ . Our second coordinate  $\theta$  measures an angular distance along the circle. We need to choose some reference point to define the angle coordinate.

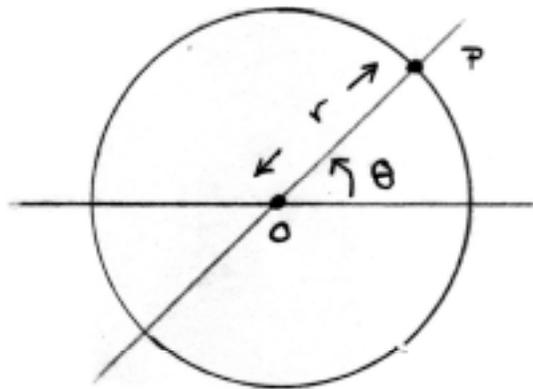


Figure 1: Polar Coordinate System

We choose a 'reference ray', a horizontal ray starting from the origin and extending to  $+\infty$  along the horizontal direction to the right. (In a typical Cartesian coordinate system, our reference ray is the positive x-direction). We define the angle coordinate for the point P as follows. We draw a ray from the origin to the point P. We define the angle  $\theta$  as the angle in the counterclockwise direction between our horizontal reference ray and the ray from the origin to the point P. All the other points that lie on a ray from the origin to infinity passing through P have the same value as  $\theta$ . For any arbitrary point, our angle coordinate  $\theta$  can take on values from  $0 \leq \theta < 2\pi$ .

### Velocity and Angular Velocity

The rate of change of angle with respect to time is called the angular velocity and is denoted by the Greek letter  $\omega$ ,

$$\omega \equiv \frac{d\theta}{dt}.$$

Angular velocity has units [rad/sec]. The component of the velocity  $v$  for circular motion is given by

$$v = r \frac{d\theta}{dt} = r\omega.$$

### Uniform Circular Motion

When the magnitude of the velocity (speed) remains constant, the object moves in *uniform circular motion*. Since the speed  $v = r\omega$  is constant, the amount of time that the mass takes to complete one circular orbit of radius  $r$  is a constant. This time interval is called the period and is denoted by  $T$ . In one period the object travels a distance  $s$  equal to the circumference,  $s = 2\pi r$ . Since the distance traveled in one period is the product of speed and period, we have

$$s = 2\pi r = vT.$$

Thus the period  $T$  is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{r\omega} = \frac{2\pi}{\omega}.$$

The frequency  $f$  is defined to be the inverse of the period,

$$f = \frac{1}{T} = \frac{\omega}{2\pi},$$

The units of frequency are  $[\text{sec}^{-1}] = [\text{hertz}] = [\text{Hz}]$ .

### **Radial Acceleration**

The magnitude of the component of radial acceleration (*centripetal acceleration*) is given by

$$|a_r| = |v\omega|.$$

The direction of the radial acceleration is towards the center of the circle.

The centripetal acceleration can be expressed in several equivalent forms since the velocity and the angular velocity are related by  $v = r\omega$ . Thus we have for the magnitude of the centripetal acceleration two alternative forms:

$$|a_r| = r\omega^2 \tag{6.1.1}$$

and

$$a_r = \frac{v^2}{r} \tag{6.1.2}$$

Recall that the angular velocity is related to the frequency by  $\omega = 2\pi f$ . So we have a third expression for the magnitude of the centripetal acceleration

$$a_r = r4\pi^2 f^2 \tag{6.1.3}$$

A fourth form commonly encountered uses the fact that the frequency is defined to be the inverse of the period  $f = 1/T$ . Thus we have the fourth expression for the centripetal acceleration

$$a_r = r4\pi^2 f^2 \tag{6.1.4}$$

### **Experiment: Building the Apparatus**

Note: Do not plug in the wall transformer until you have completed the apparatus.

#### **Clamping the motor**

Take the 2-inch corner brace in the experiment kit and use the hose clamp to attach the motor to it as shown. Handle the motor carefully to avoid damage to the terminals of the motor. The clamp

should be tight enough to keep the motor from wiggling around, but not so tight as to keep it from turning freely. Use the 'C-clamp' to attach the assembly rigidly to your desk.

### Attaching the nut

Attach the 6-32 nut to the rubber band and the rubber band to the motor shaft by looping the rubber band over itself as shown. Loop it in the direction such that the rotation of the shaft tends to tighten the loop. Try to keep the two strands the same length and not twisted up.

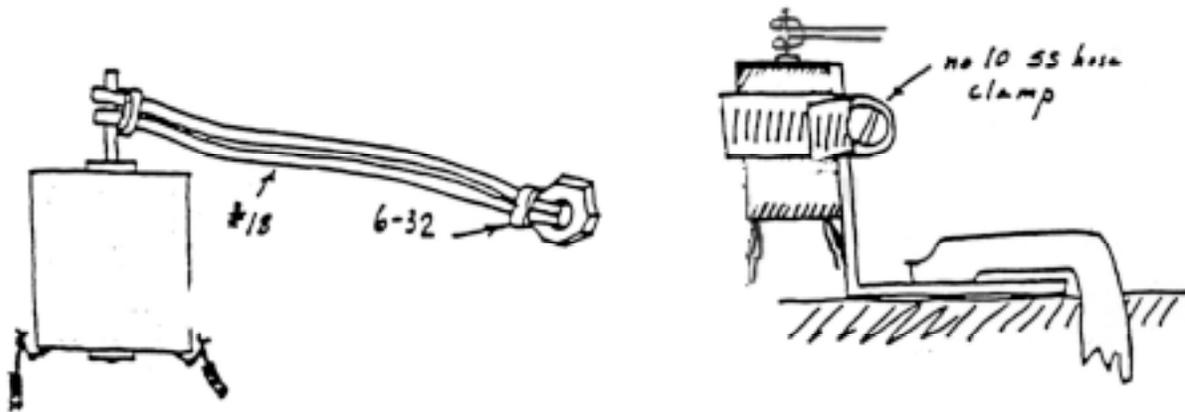


Figure 2: Clamping the motor and attaching the nut

### Attaching the stroboscopic pattern

Cut out the four-line pattern (you can find it on the last page of this write-up), make a hole in the center with a pushpin and press it onto the motor shaft above the rubber band.

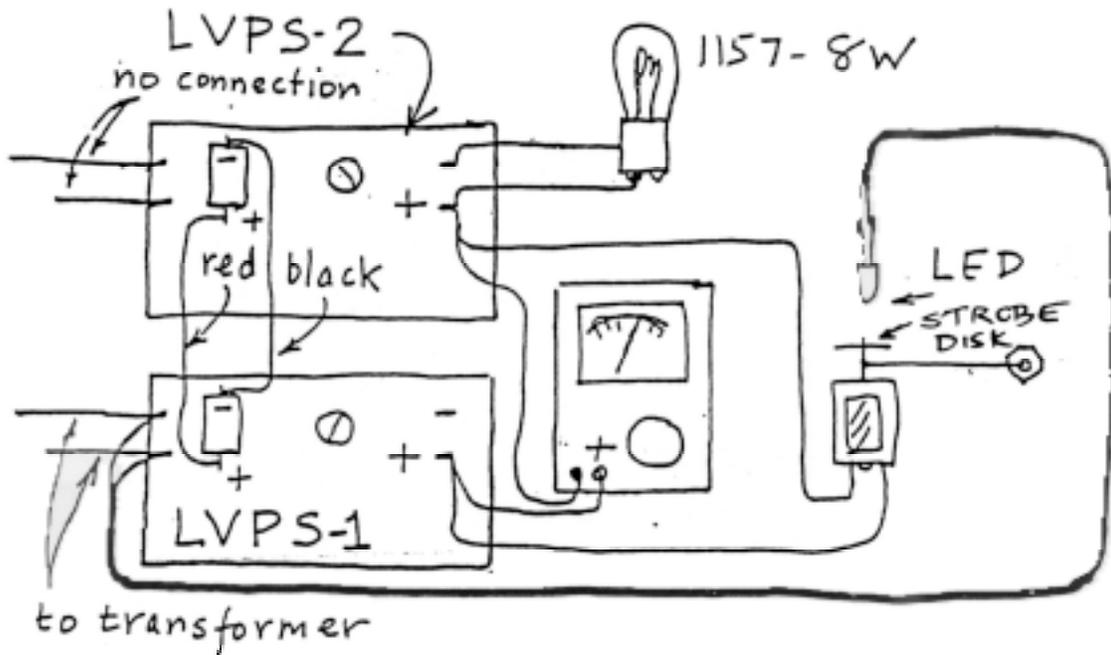
### Connecting the LED

The LED needs an alternating voltage in order to flash so you will connect it to the ac input terminals of the rectifier in one of your partnerships' LVPS (call this one LVPS-1). Split the 'speaker wire' (two conductor stranded wire in transparent plastic insulation) an inch or so at each end of the 4 foot length provided in the experiment kit. Remove about 1/4 inch insulation from each of the ends and tin the exposed wire. Solder the pair at one end to the LED without twisting the wires. Use 1 1/2 inches of black tape folded lengthwise to protect the LED leads from shorting each other. Solder the pair at the other end to the ac input terminals of the rectifier in the LVPS-1. Again don't twist the wires.

### Connecting the two LVPS

Your LVPS only goes down to 1.5 volts, which is still too high to obtain the low rotational speeds you will need for this experiment. You will use your other LVPS (call this one LVPS-2) with the voltage reversed to adjust the voltage down to zero voltage.

You only need one wall transformer to drive both LVPS. You will do this by connecting the output of LVPS-1 rectifier to the  $1000\mu F$  capacitor of LVPS-2. Use a red clip lead to connect the positive sides of the  $1000\mu F$  capacitors of LVPS-1 and LVPS-2 together. Use a black clip lead to connect the negative sides of the  $1000\mu F$  capacitors of LVPS-1 and LVPS-2 together.



**Figure 3: Connecting the two LVPS**

### Connecting the 1157 lamp

You will use the 8-watt filament of your 1157 lamp for a current regulating effect that makes settings less critical and more stable. As the current increases in the filament the resistance goes up. This increased resistance tends to limit the increase in the current. Solder the leads of the 8 W filament of the 1157 lamp to the output terminals of the LVPS-2.

### Connecting the motor

Connect the motor leads to the plus terminals of the two LVPS with clip leads. By connecting the motor to the two positive terminals you are effectively subtracting the differences between the two voltage settings to get a net voltage across the motor between 0 and 1.5 volts. Connect

the input leads of the multimeter (MMM) at the 5 volt DC setting to the two positive output terminals of the LVPS so that you can measure the voltage across the motor.

## Doing the Spinning Nut Experiment

You will first measure the radius of the circular orbit of the spinning nut for various rotational frequencies, 10 Hz (six stationary lines), 12 Hz (five stationary lines), 15 Hz (four stationary lines), and 20 Hz (three stationary lines). Then you will calibrate the rubber band by stretching the rubber band with pennies in a hanging cup.

### Procedure:

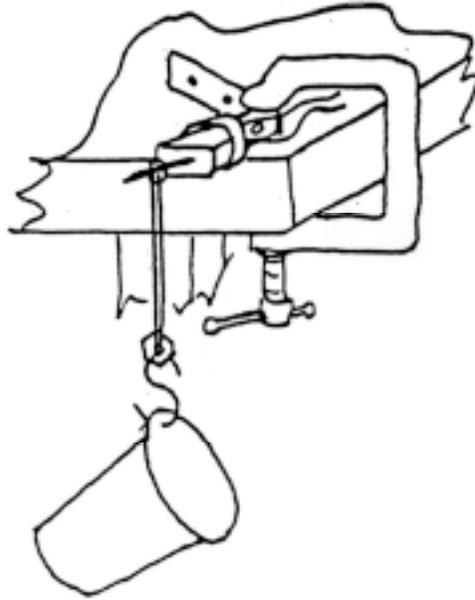
- **Unclip one motor lead and plug in the wall transformer.** Adjust the voltage of LVPS-2 to about 7.5 volts. This is enough to make the 8 W filament of the 1157 lamp glow brightly. Turn down LVPS -1 all the way and then reconnect the motor.
- Hold the nut so the rubber band is slightly stretched; the band should not rub on the motor housing. Turn up LVPS-1 so that there is 0.5 to 0.75 volts across the motor. You will see the shaft start to turn; immediately launch the nut into orbit. Start all over if the rubber band gets twisted up around the shaft.
- In order to see the stroboscopic pattern you will need to make the area immediately around the experiment dark. One partner can hold the LED about 1 inch above the motor and continually adjust the LVPS-1 until the 4-line pattern appears to be stationary. This means that (the simplest case) the band is advancing 90 degrees = 1/4 of a revolution each 1/60 of a second, and therefore the shaft is turning at a rotational frequency  $f = 15$  Hz.
- The other partner can measure the radius of the circular path of the spinning nut from the center of the shaft to the center of the nut.
- **Repeat these measurements for the various patterns corresponding to 10 Hz, 12 Hz, 15 Hz, and 20 Hz rotational frequencies.**

## Calibrating the Rubber Band

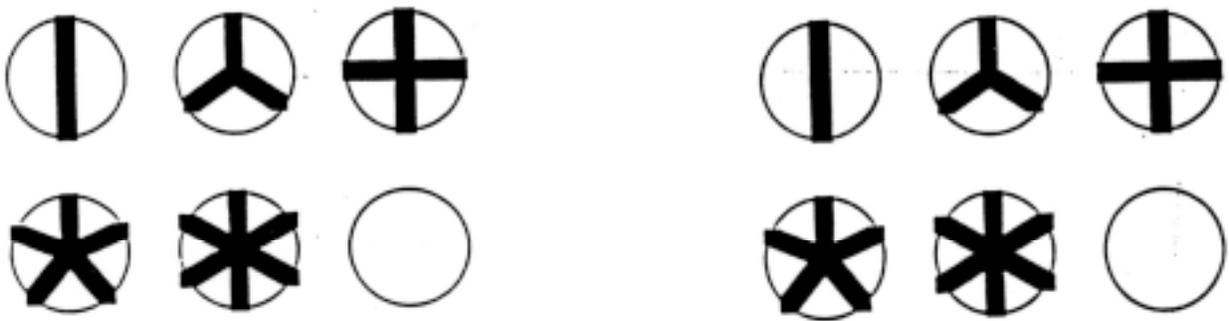
Clamp the corner brace on its edge with the motor shaft extended well out over the edge of your table or desk so that the rubber band and nut hang freely. Bend a paper clip into an S-shaped hook. Press one end through the paper cup just below its rim and hook the other end to the nut.

Now add pennies to the cup and measure the length,  $L$ , of the rubber band. Keep adding pennies to cover the range of lengths you measured when spinning the nut. The mass of the paper cup and the paper clip together is  $5.8. \pm 0.01$ g .

The weight required to stretch the rubber band to a length equal to the radius of the circular path of the nut is approximately equal to the centripetal force needed to accelerate the nut inward in its motion. We say "approximately", because the rubber band has mass and is stretched by the rotation even when there's no nut at the end. This isn't too serious, as you can see by spinning the rubber band by itself; it doesn't stretch much, and if it did, you could take data to make an appropriate correction. You should also note that excessive stretching of the rubber band could weaken it permanently; this could invalidate your results.



**Figure 4: Calibrating the rubber band**



**Figure 5: Stroboscopic line patterns**

## **PARTS LIST**

- 1 permanent magnet DC motor**
- 1 2" corner brace**
- 1 hose clamp #12 SS**
- 1 LED, wt. internal resistor**
- 4' #22 speaker wire**
- 2 nuts, 6-32 (5/16) steel**
- 2 rubber bands, #18**
- 1 paper clip, #1**
- 1 pushpin**
- 1 paper cup**
- 100 pennies**

## Experiment ET Energy Transformation & Specific Heat

We have introduced different types of energy, which help us describe many processes in the world around us: mechanical, electrical, chemical and heat energy. We have distinguished between energy stored in some system, potential energy: and energy associated with the motion of objects, kinetic energy. We have described some of the conversions or transformations in which one kind of energy turns into another kind, and we have pointed out that the final form is invariably heat. Consider Niagara Falls: the water's gravitational potential energy turns into kinetic energy as it falls. The turbines and alternators spin and electrical energy is generated. It's sent to you and your soldering iron turns it into heat, or your refrigerator turns it into mechanical energy to make the refrigerant flow to remove heat from the inside to the outside. There are an immense number of processes like that going on in both the man-made and the natural world.

In the Part I of the experiment you will calibrate an electrical temperature sensor, called a "thermistor" by immersing it in water whose temperature is measured with a glass thermometer. The thermistor (smaller and faster in response than the glass thermometer) is a compressed pellet of semi-conducting metal oxides whose resistance depends on temperature. Calibrating the thermistor means finding a relation between its electrical resistance and the temperature.

In the Part II of the experiment, electrical energy will be transformed into thermal energy resulting in the heating up of a sample of water. You will thus be able to find the specific heat,  $c_w$ , of water. The specific heat is the energy it takes to heat one kilogram one degree Kelvin (or equivalently one degree Celsius).

### Part I Calibrating the Thermistor

#### Apparatus

Start by soldering two wires, each about 1 foot long, to the thermistor.

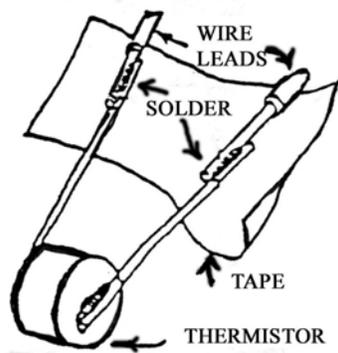


Figure 1: Thermistor

(It is a small cylindrical object with two leads; see Figure 1.) Use tape to prevent electrical shorts. Treat the thermistor gently, as the leads can be pulled or melted off rather easily.

Set the digital multimeter (DMM) to " $\text{K}\Omega$ " (kilo-ohms =  $1000\Omega$ ) and connect it to the thermistor with the clips or with your clip leads. A reading of about  $100\Omega$  should appear. Breathe on the thermistor and note its speed and sensitivity compared to the glass thermometer.

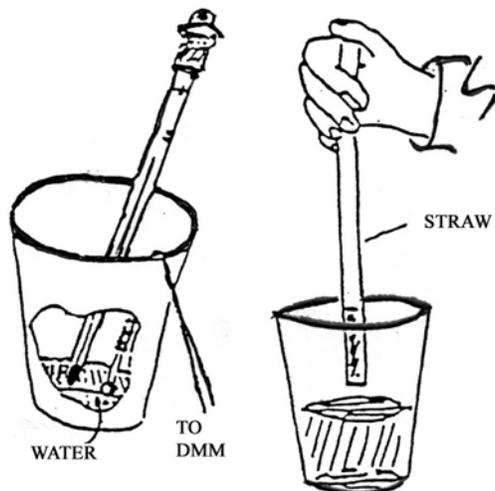
You will make many resistance readings with the DMM or the magnetic multimeter (MMM) and these use battery power. Get in the habit of turning off the DMM when not actually making resistance readings, and don't leave the MMM with its test leads connected for long periods.

Clear a work area so that if water spills there is no problem. Go to a nearby lavatory and fill two of your four styrofoam cups  $2/3$  full, one with hot water and the other with cold water. Do experiments in a third cup, and use the fourth to hold water when you're through with it. In these experiments you should watch out for heat sources like lights or heaters, and for drafts from windows and doors. It generally improves your measurements if you stir the water by gentle shaking to make sure it's well mixed and therefore at one temperature.

You can put the 1157 lamp into the hot water cup as an immersion heater to warm up water as needed. Connect the 8 W filament directly to the wall transformer (no LVPS). The 8W filament will maintain a cup of water near  $45^\circ\text{C}$ . We provide a lid for the experiment cup, which may be helpful in reducing heat loss, especially, that due to evaporation when using hot water.

### **Calibrating the Thermistor**

Put enough hot water into the third cup (the experiment cup, which serves as a calorimeter) to cover the end of the thermometer and the thermistor. Take and record temperature and resistance readings and also the time as the water continues to cool. Stir the water with the thermometer (or gently shake the cup) before each reading.



**Figure 2: calibrating the Thermistor**

When cooling slows, continue the calibration by adding cold water so as to cover the range from hot (45 C to 50 C) to cool (20 C to 25 C), taking data at about 10 intermediate values. You can use a soda straw as a pipette to add small amounts of water. **Plot the thermistor resistance versus temperature, with temperature along the horizontal axis. (See Data Analysis at end of lab write-up.)**

## **Part II Measuring the Specific Heat of Water**

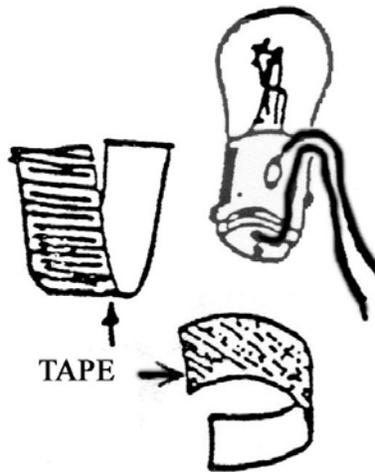
Now that you have calibrated the thermistor you can do the second experiment. You'll put the 1157 light lamp in a cup containing a known amount of water and measure the electric power delivered to the lamp and the rate of temperature rise of the water. This rate of rise, together with knowledge of the rate of energy input, will give you the specific heat,  $c_w$ , of the water in units of  $[J/kg \cdot K]$  if you assume energy conservation during the transformation. The actual value is  $c_w = 4185 J/kg \cdot K$ .

### **Apparatus**

The calorimeter is the experiment cup used previously. Fill it to a depth of 35 mm. You can mark this depth with a pen mark on the inside of the cup. Use a lid for the cup to prevent evaporation. Use your calibrated thermistor and the DMM to measure water temperature.

Before starting the experiment, stir the water with a glass thermometer, and take a temperature reading to check the correctness of your thermistor reading at one point.

Carefully fold back the wires soldered to the 1157 lamp as shown and tape over the end of the lamp base with black tape so as to insulate it electrically from the slightly conducting water in the cup. This will prevent current from flowing, which could affect the thermistor reading.

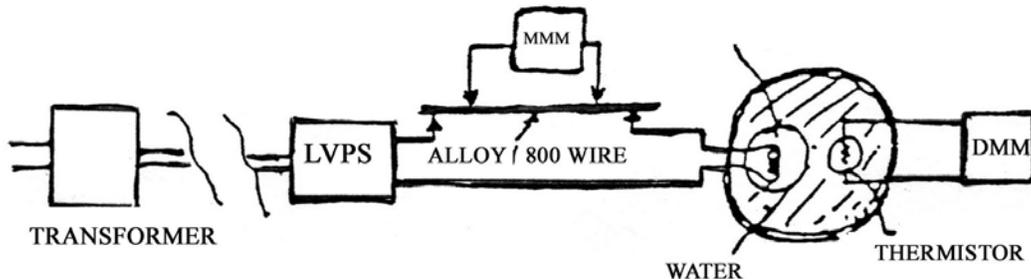


**Figure 3: Heat Source**

Locate the Alloy 800 resistance wire (stiff, thick, uninsulated wire of length  $\sim 0.3$  m). It's resistance is  $1.02\Omega$  per meter of length, so the resistance of your piece is about  $0.3\Omega$ . (Your DMM is quite inaccurate in this regime, but it will indicate  $0.3 - 0.5\Omega$ .)

### Measuring the Electric Power

Connect the 8 W filament of the 1157 lamp in series with the Alloy 800 resistance wire and the LVPS output (see sketch). Then disconnect the minus lead of the LVPS and plug in the wall transformer.

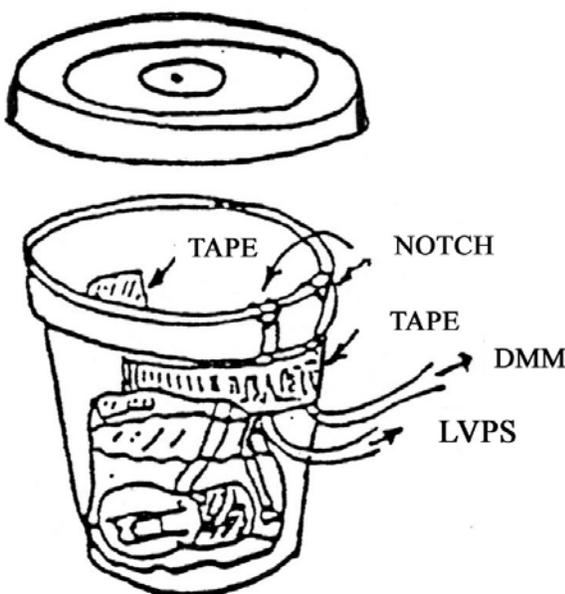


**Figure 4: Circuit Diagram for Experiment ET**

Use the MMM on the 25 VDC range to set the voltage of the LVPS to 10.0 V. Reconnect the minus lead of the LVPS and the lamp should light up but the regulator should keep the voltage at 10.0 V. Once the voltage is set, disconnect the MMM and rotate the MMM rotary knob to the " $50\ \mu\text{A}$  ( $250\ \text{mV}$ )" position on the DCA (DC Amperes) range, *not* the 250 mA position. The meter will then read 250 mV full scale; use the "250" scale on the face. Clip the MMM leads across a *carefully measured* length of the resistance wire (see sketch above), e.g. 0.25 m and **measure the voltage across this portion of the resistance wire**. Then unplug the wall transformer.

## Doing the Experiment

Put water (approx. room temp.) in the cup to a depth of 35 mm. Wedge the 1157 lamp (which tends to float) in the bottom of the cup. Make a small notch in the rim of the cup for the leads and pass them through the notch bending them and taping them in place (see sketch). Put the thermistor approximately in the middle of the water on one side of the lamp, bend its leads over the rim (again with a small notch), and tape. Make sure that the thermistor doesn't touch the cup or the lamp. Allow time for the entire apparatus to come close to room temperature. The thermistor readings give the water temperature. The thermometer can be used to keep track of room temperature. (As usual, watch out for drafts and unwanted sources of light and heat).



**Figure 5: The Calorimeter**

Press the lid onto the cup. You may need to make a few vertical cuts (say 4) around the cup rim to make it fit the lid. Work the lid on gently; a piece of tape will hold it on.

Disconnect the LVPS minus output lead. (You don't want to start heating the water until you're ready to take data.) Plug in the wall transformer, and set the LVPS output again to 10.0 V. Connect the DMM (set on the 250 mV range) across the same carefully measured points on the resistance wire.

## Taking Data

Since the lid on the cup prevents you from stirring with a thermometer, you should agitate the water by moving the cup in a horizontal circle about 1 to 1.5 inches in

diameter about twice a second, reversing direction every few seconds. The electrical connections had better be firm and the wires must have enough slack.

With the transformer plugged in, but the LVPS minus lead still not connected, start agitating and read resistances every 15 seconds. **One partner should agitate the water and keep track of the time. The other partner reads thermistor resistance values and records them with the time. When the readings are steady, or changing only 0.1 or 0.2Ω, reconnect the LVPS minus lead and note the MMM reading. Continue taking data every 15 seconds for 5 minutes. After 5 minutes, disconnect the LVPS, and continue agitating and recording resistances so as to get a value for the initial rate of cooling.**

## Data Analysis

### Part I: Fitting the Calibration Data

The decreasing exponential can fit the plot of your thermistor data,

$$R(T) = R_0 e^{-\alpha T}$$

where  $R(T=0) = R_0$  and  $\alpha$  are constants and  $R(T)$  is the resistance at Celsius temperature  $T$ . Note that  $R$  goes to 0 as  $T$  goes to 0. You can linearize this expression by taking the natural log (ln) of both sides,

$$\ln R = -\alpha T + \ln R_0$$

This becomes a linear equation if the dependent variable is taken to be  $\ln R$ . Your data, if truly exponential, should follow a straight line on a plot of  $\ln R$  versus  $T$ .

Plot  $\ln R$  versus  $T$  and fit a straight line to the data points (by eye and ruler). This yields the constants  $\ln R_0$  (the intercept at  $T = 0$ ) and  $\alpha$  (negative of the slope). The temperature  $T$  can then be obtained from

$$T = (\ln R_0 - \ln R) / \alpha$$

for any measured value of  $R$ . Check it for a couple of the values of  $R$  you measured to see if you get the proper temperatures.

### Part II Determining the Specific Heat of Water

#### Determining the Electrical Power of the Lamp

The measured voltage  $\Delta V$  across the resistance wire allows you to deduce the current flowing in the circuit. Suppose there is 0.25 m of resistance wire between the clips on the

MMM leads, and the measured voltage across the wire is  $\Delta V = 100 \text{ mV}$  (still using the "50  $\mu\text{A}$  (250mV)" setting and the "250" scale on the meter face). Ohm's law then gives

$$I = \frac{\Delta V}{R} = \frac{0.10 \text{ V}}{0.25 \Omega} = 0.40 \text{ A}$$

The power dissipated in the lamp is  $P = I\Delta V_L$  where  $\Delta V_L$  is the voltage across the lamp, i.e.,

$$P = I\Delta V_L = (0.40 \text{ A})(10.0 \text{ V} - 0.10 \text{ V}) = 3.96 \text{ W}$$

### Determining the Mass of the Water

You can estimate the mass of water in the calorimeter from its depth,  $d$ , before inserting the lamp and estimated 'average' radius (measured with one of your rulers). More precisely, use the bottom radius,  $r_1$ , and the top radius,  $r_2$ , of the cup and its height  $h$ . Here's the formula for the volume  $V$  of the water (can you derive it?).

$$V = \pi r_1^2 d + \pi r_1 \left(\frac{r_2 - r_1}{h}\right) d^2 + \frac{1}{3} \pi \left(\frac{r_2 - r_1}{h}\right)^2 d^3$$

For the styrofoam cup we provide, with  $r_1 = 2.0 \text{ cm}$ ,  $r_2 = 3.4 \text{ cm}$ , and  $h = 8.2 \text{ cm}$ , this yields a volume in [ $\text{cm}^3$ ]:

$$V(\text{cm}^3) = 12.6d + 1.07d^2 + 0.03d^3$$

From the measured depth  $d$  in cm and you can find the water volume  $V$  in  $\text{cm}^3$  and the mass in grams (numerically equal, closely enough). Convert your value for the mass of your volume into kilograms. Is your answer consistent with your first estimate?

### Analyzing the Data

Use the thermistor calibration to convert resistances to degrees C. Plot the temperature versus time, using an expanded ordinate scale that includes only the range of temperatures encountered, i.e., do not start the ordinate at  $0^\circ \text{C}$ ; start it at  $\approx 20^\circ \text{C}$ . Fit a straight line by eye which goes through the data points, and determine the slope  $dT/dt$  in units of [deg/s] for both the heating and cooling parts of your graph.

The quantity of heat  $Q$  required to increase the temperature  $\Delta T$  of a mass  $m_w$  of water is equal to

$$Q = m_w c_w \Delta T$$

where  $c_w$  is the specific heat capacity of water.

The rate that heat is flowing in time, the power, is the time derivative

$$P = \frac{dQ}{dt} = m_w c_w \frac{dT}{dt}$$

Treat the water as a system. Electrical power flows into the water via the 8W filament of the 1157 lamp.

$$P_{in} = I \Delta V_L$$

Power also flows out of the water during the heating process via the radiant heat loss, which you can approximate by

$$P_{out} \cong m_w c_w \left| (dT / dt)_{cooling} \right|.$$

This is approximate because the radiant power loss varies as a function of temperature and we are only using the power loss after the water has already been heated.

The power absorbed by the water is

$$P_{absorbed} = m_w c_w (dT / dt)_{heating} .$$

The difference between the power in and the power out is the power absorbed by the water,

$$P_{in} - P_{out} = P_{absorbed}$$

thus

$$I \Delta V_L - m_w c_w (dT / dt)_{cooling} \cong m_w c_w (dT / dt)_{heating} .$$

From your data you can obtain a value for the specific heat,  $c_w$ , of water in [J/kg · K].

## Error Analysis

Estimate the error on the slope you obtain as well as other input quantities to obtain an error for  $c_w$ . You may find that your experiment could have been done with more care regarding heat loss, agitating, etc. If so, try it again, it doesn't take long.

How serious was heat loss during your runs? (You might wish to devise a way to correct for it.) What is the approximate effect of the heat capacity of the light lamp, which has a mass of about 9 g (note that  $c_{glass} \approx 1000 \text{ J/kg} \cdot \text{K}$  and  $c_{brass} \approx 300 \text{ J/kg} \cdot \text{K}$ )? The effect of heat absorbed by the cup mass, 1.6 g is negligible.

## Experiment ET: Parts List

4	Styrofoam cups		1 ft	Resistance wire, alloy 800 (in Red box)
1	lid for same	1		#1157 lamp (in LVPST)
1	drinking straw	1		thermometer (in Red box)
1	thermistor		4	paper towels

# Experiment VS Vibrating Systems

## Introduction

Many vibrating or oscillating systems around us are visible or audible, but most are hidden. The motion of a clock pendulum, or of a child on a swing, can be seen. The vibrations of a bell, or of any musical instrument, are not easily seen but are certainly audible. The oscillations of current in the tuned circuits of a radio, or in atoms emitting light, are not directly perceived but can be measured or inferred.

These systems and others have several things in common: some quantities (position, sound pressure, electric field, etc.) vary sinusoidally in time at one or more natural frequencies depending on the pattern of motion or mode of vibration. In each cycle of vibration there is an exchange of energy from kinetic to potential and back, or from electrical to magnetic and back. In the case of mechanical oscillators the potential energy is associated with the work done against a linear restoring force provided by the deformation of the spring in a mass/spring system. In the case of the pendulum the potential energy is associated with the work done against the component of the gravitational force along the direction of the circular arc of a pendulum's bob. The pendulum is only an approximate oscillator for small angles of arc. The kinetic energy is associated with the moving mass.

In this experiment you'll investigate the properties of four mechanical oscillating systems, simple pendulum with two strings, simple pendulum with one string, mass/spring oscillator, and cantilevered beam. All these systems have nearly linear restoring forces.

## Mass/Spring Oscillator

In its ideal form, the mass/spring oscillator has a massless spring of spring constant  $k$ , attached to a point-like with mass  $m$  and a rigid support. We'll use rubber bands and the cup of pennies as an approximation. Clamp the piece of wood to a desk or table, and suspend from it a chain of rubber bands (use all that you have of one kind) with a paper clip at each end. Join the rubber bands as shown in Figure 1.



Figure 1: Rubber Band linkage

Hang the cup from the lower clip and measure the distance between the nearest points of the two paper clips (i.e., the length of the chain of rubber bands) for total suspended penny masses of about 10, 20, 50, 100, 120, 150, 180, 200, and 250 g.

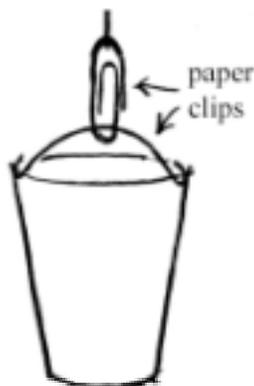
Note that the mass of a penny before and including 1981 is 3.1 g; from 1982 on it is 2.5 g. The paper cup has mass  $6.10 \pm 0.02$  g. Each paper clip has mass  $0.40 \pm 0.02$  g. Each rubber band has mass  $0.02 \pm 0.01$  g. Plot a graph of extension against suspended mass for this system. Is the graph linear?

Now measure the time for ten vertical oscillations and calculate the period of one vertical oscillation with a total suspended mass of about 150 g. Repeat for two different amplitudes of oscillation (one larger and one smaller).

Observe other possible oscillations -- pendulum and twisting motions -- of this oscillator. Look closely for any unusual features of the pendulum motion.

### **SIMPLE PENDULUM:**

We'll approximate the ideal "simple" pendulum -- a point mass on a massless inextensible string attached to a rigid support -- by a cup, containing pennies, attached by a long string (about 2 m) to a piece of wood clamped horizontally to a high support, for example a bookshelf, so that it projects well beyond the edge. Make a handle for the cup by first straightening out a paper clip. Pierce the ends of the paper clip through the top rim of the cup and then bend the ends of the clip with your needle nose pliers forming a jay shaped hook on each end. Attach the handle to the string using a paper clip (Figure 2).

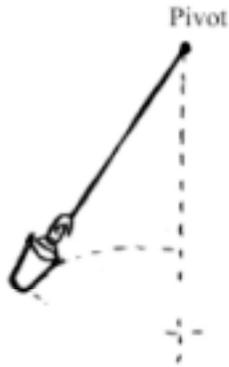


**Figure 2: Paper Cup Pendulum**

Suspend the cup, with 50 pennies, and time the period for swings of small amplitude -- say 10 degrees. Time a group of at least 10 swings. Don't forget to measure all lengths from the

attachment point to the center of gravity of the "bob" -- the pennies in the cup. Time how long it takes for the amplitude to decrease by 50%, and repeat with 5 pennies in the cup. Observe whether the period changes with the number of pennies in the cup. Repeat the period measurements with half the length of string.

Set the pendulum in circular motion forming a conical pendulum (Figure 3) and measure its period and other relevant parameters, for example the radius.



**Figure 3: Conical Pendulum**

### **EXPERIMENT VS -- PARTS LIST**

- 1 Length Twine
  - 1 Paper cup from Expt ET
  - 5 Rubber bands
  - 2 Paper clips
  - 1 Piece of wood  $1/2 \times 1/2 \times 10$
- Also C-clamp, and pennies from previous experiments.

# Experiment AM Angular Momentum

## Introduction

If an object, e.g., a heavy ball, comes straight at you and you catch it, you may stagger back while exerting the impulse -- some force for some time -- needed to bring its linear momentum down to zero. If you were on a frictionless surface you wouldn't be able to exert any forces to stop the ball, and you and it would move together, conserving linear momentum. Were you to catch the same ball as it passes beside you, with your arms outstretched to one side, you would have to exert an *angular* impulse, applying a *torque* for some interval of time, to bring its *angular* momentum down to zero. Suppose while catching the ball you were sitting on a stool firmly fixed to the ground but the seat of which could turn without friction. If you hung on to the ball, the ball and you would spin together indefinitely. If the stool were not anchored but free to slide and you caught the ball, then you, the ball and the stool would move in a combination of translational and rotational motion. Both the linear momentum and the angular momentum would be conserved during this collision since there would be no external forces or torques to provide impulses to change the total of each kind of momentum.

Just as conservation of linear momentum is a hidden part of many everyday happenings at all size scales, so is the conservation of angular momentum. The collisions of particles, the emission and absorption of radiation, the leaps and gyrations of cats, dancers, athletes, the motions of astronomical objects, all involve angular impulses and exchanges of angular momentum. In any isolated system angular momentum is conserved (as is linear momentum), despite mechanical energy losses ranging from zero (elastic collisions) to the maximum consistent with the conservation of linear momentum.

Part of the richness of phenomena and complexity of description in rotational dynamics comes from the fact that the same torque can be obtained with different combinations of lever arm  $\vec{r}$ , force  $\vec{F}$  and angle between them  $\theta$ . The same angular momentum can arise with different combinations of radius  $r$ , mass  $m$ , and velocity  $\vec{v} = \vec{\omega} \times \vec{r}$  or moment of inertia  $I$  and angular velocity  $\vec{\omega}$ . Some or all of these can change in any given situation.

## Experiments

You'll investigate two kinds of rotational phenomenon using a motor and a hub as a fixed axis of rotation.

In the first experiment you will measure the total frictional torque responsible for the slowing down of a rotating washer. You will place a washer on the hub. When the motor is turned on, the angular velocity of the shaft increases. When the motor is turned off, the total frictional torque decreases the angular velocity until the assembly comes to a stop. During the deceleration,

the motor will act as a generator providing a voltage that is a measure of the instantaneous angular velocity.

In the second experiment, you will measure the change in angular momentum due to an inelastic rotational collision in which a stationary washer is dropped on a spinning washer. During the collision there is a rotational frictional torque between the washers, slowing one washer down and speeding the other washer up until the washers are moving at the same angular velocity. The total angular momentum is nearly conserved during this collision. You will measure how closely angular momentum is conserved.

## Theory

When a **torque**,  $\vec{\tau}_S$ , is applied to a body about a point  $S$ , the body will acquire an *angular acceleration*,  $\vec{\alpha}$ . If the body is constrained to rotate about a *fixed axis of rotation* then the component of angular acceleration will be proportional to the component of torque about the axis,

$$\tau_S^{total} \propto \alpha,$$

where  $S$  denotes the point where the fixed axis passes through the center of the orbit. The constant of proportionality is called the *moment of inertia about the axis passing through  $S$* ,  $I_S$ , and it is a measure of how the mass is distributed about the axis of rotation,

$$\tau_S^{total} = I_S \alpha$$

The moment of inertia of a set of  $N$  masses about an axis of rotation is given by

$$I_S = \sum_{i=1}^{i=N} \Delta m_i (r_{\perp,i})^2$$

where  $r_{\perp,i}$  is the distance the  $i$ th mass  $m_i$  lies from the axis of rotation. For a continuous body the sum over all the masses becomes an integral over the body

$$I_S = \int_{body} dm (r_{\perp})^2$$

where  $r_{\perp}$  is the radius of the circular orbit of the mass element  $dm$ .

Let the angle  $\theta$  parameterize some point on the rigid body in a plane perpendicular to the axis of rotation. The *angular velocity*,  $\vec{\omega}$ , about the axis of rotation is the rate of change of the angle  $\theta$ , and the component is given by

$$\omega = \frac{d\theta}{dt}.$$

The component of angular acceleration is defined to be the rate of change in time of the component of angular velocity,

$$\alpha = \frac{d\omega}{dt}.$$

Consequently the applied torque will either increase or decrease the angular velocity.

The *angular momentum*,  $\vec{L}_S$ , of the mass about the axis of rotation passing through  $S$  is proportional to angular velocity,  $\vec{\omega}$ , with the moment of inertia  $I_S$  as the constant of proportionality,

$$\vec{L}_S = I_S \vec{\omega}.$$

Differentiating the above equation shows that the rate of change in time of angular momentum is equal to the applied torque

$$\frac{d\vec{L}_S}{dt} = I_S \frac{d\vec{\omega}}{dt} = \vec{\tau}_S.$$

When the total applied torque on a rigid body about some point  $S$  is zero then the angular momentum,  $\vec{L}_S$ , is conserved. If there is a constant applied torque,  $\vec{\tau}_S$ , over an interval of time  $\Delta t = t_f - t_0$ , then the change in angular momentum,  $\Delta \vec{L}_S = \vec{L}_{S,f} - \vec{L}_{S,0}$ , which is known as the *angular impulse*, and is given by

$$\Delta \vec{L}_S = \vec{L}_{S,f} - \vec{L}_{S,0} = \int_{t_0}^{t_f} \vec{\tau}_S dt.$$

The *rotational work*,  $W$ , which an applied torque,  $\vec{\tau}_S$ , does on a body in rotating that body about a fixed axis through an angle  $\Delta\theta = \theta_f - \theta_0$  is given by

$$W = \int_{\theta_0}^{\theta_f} \tau_S d\theta.$$

The rate of change in time of the work done by the torque is the *instantaneous rotational power* and is given by

$$P = \frac{dW}{dt} = \tau_s \omega.$$

## Apparatus

You have a permanent-magnet dc motor of the kind used in portable tape players which serves three purposes: (i) it provides a pivot, bearing or axis about which things can rotate with relatively low, but significant, friction, (ii) it is, of course, a motor which, when powered by your LVPS, can exert torque, rotate objects and give them angular momentum, and (iii) it can act as a DC generator that produces a dc voltage proportional to the angular velocity of its motor if it is made to rotate mechanically. In the latter mode, it can be used as an angular velocity sensor. The voltage generated by the motor is linearly proportional to the angular velocity of the motor,  $\omega = \beta V$ , where  $\beta$  is the constant of proportionality. You'll need a calibration of your generator to determine  $\beta$ , and we'll describe how to do this later. We'll use the motor in all these ways in this experiment.

You have a plastic bushing used to attach casters to tubular furniture, a wooden dowel in which we've drilled a hole, and a piece of rubber insulation. These can be assembled to make up a hub on which washers can be placed so that they can be rotated and so that their angular velocity can be measured.

The double pole, double throw slide switch (DPDT) will be used to connect the motor to the low voltage power supply (LVPS) when you want to accelerate a rotating object, and to the digital multimeter (DMM) when you want to measure angular velocity as the object coasts down.

To make readings at uniform intervals of, let's say, one second, you'll either have to call out seconds and record what your partner reads, or use the fact that the DMM flashes at a rate of two flashes a second.

## Assembling the Apparatus

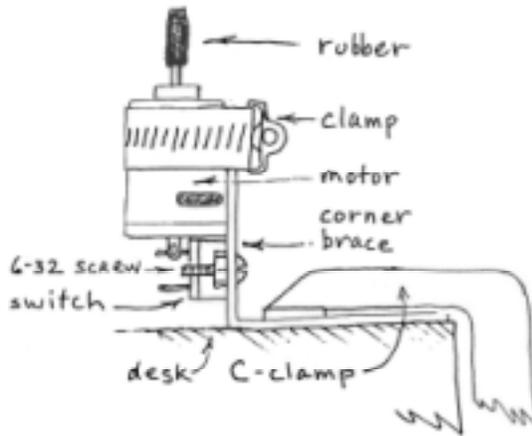


Figure 1: Overall assembly diagram

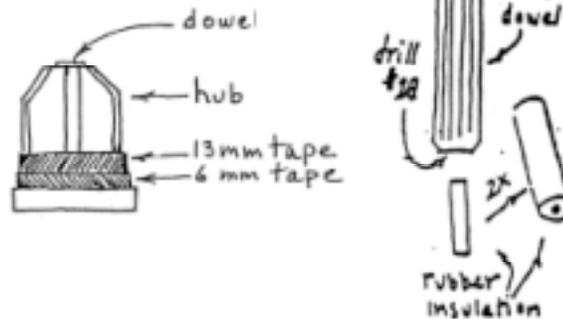


Figure 2: Hub diagrams

The assembly of the apparatus involves the following steps:

- construct the axis of rotation using the dowel, hub, tape, and rubber insulation as connector to shaft of motor (steps 1-4 below);
- connect the double pole double throw switch (DPDT) and low pass filter to motor (steps 5-7 below);
- connect the hub to shaft of motor clamped to table (steps 8-9 below);
- connect the LVPS to DPDT and motor (step 10 below).

1) Look at the white plastic object that will serve as a hub to carry the washers. It has a flat base and a conical top. Press the wooden dowel into the hub. The end of the dowel without a hole should be at the top of the hub and the end with the hole at the base.

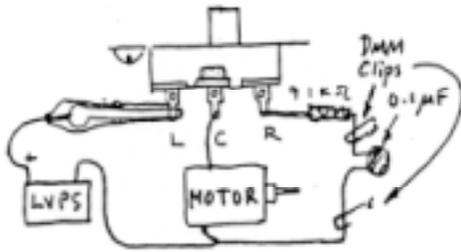
2) Cut off a length of black electrical tape about 300 mm long. Stick one end to your desk so that you can use scissors to cut the tape lengthwise into 2 strips, one narrow, about 6 mm wide, the other wider, 13 mm wide. Carefully wind the wider piece around the lower part of the hub, taking care not to cover the rim at the base. Now wind the narrow piece around the lowest part of the hub, again taking care not to cover the rim at the base. See if one of the washers can be pressed onto the taped hub. The washer should fit neither loosely nor so tightly that it's a struggle to press it on. Add or remove tape as appropriate to get the right fit.

3) Strip off a 12 mm piece of rubber insulation from the length of red 5 kV test lead in your kit. Look at the piece of insulation and remove any thread or strands of wire. What you want is

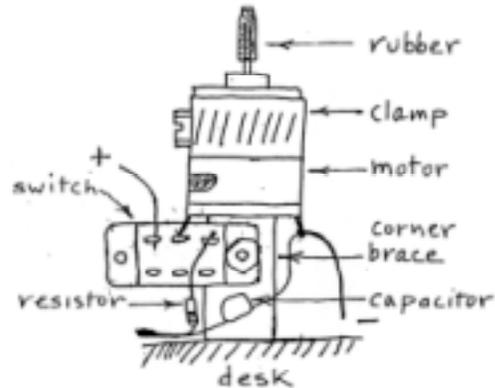
essentially a small piece of rubber tubing. Press it onto the motor shaft so that 9 mm are on the shaft and the rest isn't.

4) Clamp the motor to the corner brace with the stainless steel hose clamp. The clamp should be flush with the end of the corner brace and the top of the shaft end of the motor should be a few mm above that. Tighten the clamp firmly.

The double pole double throw switch (DPDT) will connect the motor alternately to the LVPS or to the DMM. The hookup diagram below shows the circuit.



**Figure 3: Hookup diagram for DPDT**



**Figure 4: DPDT Switch Connections**

- 5) Attach the DPDT switch to the corner brace below the motor using the 6-32 screw and nut.
- 6) One of the motor terminals may be so near the center terminal of the DPDT switch that you can simply solder them together. If not use a short length of bare wire to connect them.
- 7) Solder the 15 kΩ resistor to the end terminal of the DPDT switch that's right under the motor. Solder the 0.1μF capacitor from the free end of the resistor to other motor terminal. This low pass filter reduces commutator ripple that would otherwise cause wrong readings on the DMM. You will clip your digital multimeter (DMM) leads across the 0.1μF capacitor.
- 8) Press the hub and washer assembly onto the motor shaft with its rubber tube. It should be pressed on far enough so that no rubber is visible but still with some clearance (1 to 2 mm) so that nothing rubs on the brass bearing at the top of the motor.
- 9) Clamp the corner brace firmly to your desk or table with the C clamp.
- 10) Use clip leads to connect the negative output of the LVPS to the motor terminal that has one end of the 0.1μF capacitor soldered to it. Connect the positive output of the LVPS to the end terminal of the DPDT switch that has nothing soldered to it.

## Running the Experiment

The DPDT switch allows you to turn the motor on and off. When it's off the motor is connected to your voltmeter and acts as a generator whose output is proportional to its angular velocity.

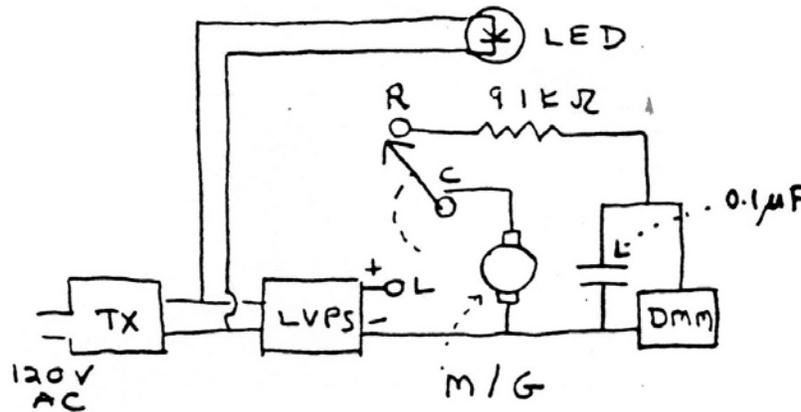


Figure 5: Circuit Diagram for Expt AM

Place one washer on the hub. (**Make sure you note the mass; it is printed in grams on the washer.**) Let the motor come up to speed, as you can tell by listening, operate the switch and observe the readings on your DMM. Since the voltage readings change twice a second on the DMM (sampling rate) they are hard to read. The first thing to do is blank off the end digit on the right with a piece of black tape so you have 2 or 3 digits to read instead of 3 or 4.

Make three runs in which you allow the motor to speed up and then coast down, taking voltmeter readings at regular intervals.

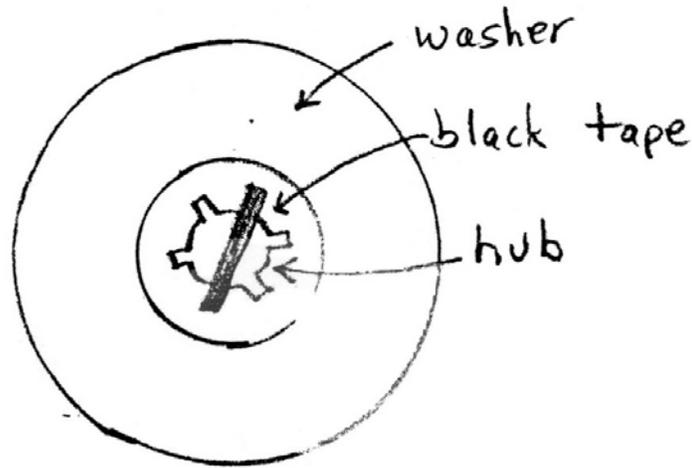
One person should read the meter and calls out numbers while the other writes them down (very rapidly) on a sheet of paper. This will not be easy; so practice and work out some procedure for taking reliable measurements at constant intervals of time.

## Calibration of the Motor/Generator

Connect the LED from Experiment CF to the 12V AC output from your wall transformer with leads sufficiently long to allow you to position the LED over your motor. (Note the transformer output will also be connected to the LVPS.)

Put a single washer on the hub. Place a small narrow piece of electrical tape completely across the top of the hub, as shown in figure 6. Since there are 60 flashes of light per second, the

tape should appear stationary when you reach a rotational speed of 30 Hz. (Wobble may make it sort of blurry.)



**Figure 6: Setting up the strobe pattern**

Accelerate the washer to as high a speed as you can achieve without the washer flying off the hub. Then disconnect the motor with the switch and read your voltmeter, as the motor coasts down. As the hub approaches 30 Hz, the LED will reflect off the tape at the top of the hub and the stroboscopic pattern will appear to slow down and spin the other way. (This will appear blurry but it is noticeable.) Just as the strobe pattern stops, you have reached 30 Hz. Record the value of the voltage. Repeat this three times and average your voltages. The angular velocity  $\omega$  of the motor is linearly proportional to the voltage  $V$  generated by the motor,  $\omega = \beta V$ , where  $\beta$  is the constant of proportionality. Recall that the angular frequency  $\omega = 2\pi f$ , where  $f$  is the frequency. Do not confuse the two quantities or their units:  $\omega$  [rad/sec] and  $f$  [Hz]. Calculate your value for  $\beta$ . From this data you can now calibrate your motor generator output so that you can determine angular velocity for any measured voltage with an assumption of linearity down to zero volts and zero Hz.

## Experiment One

With one washer on the hub, set the LVPS to 3.0 volts. Let the motor come up to speed. Turn the motor off and measure the voltages generated by the motor as it slows down (see above for ways to record the voltages).

Plot the voltage readings versus time for the three runs, arranging to have the first reading always at the zero of time. Because you are not starting each run at the same rotational speed the initial readings may differ, but one would expect the deceleration (represented by the slope) to be the same for each run.

## Experiment Two

Make three runs in which after about 6 s you drop a washer on the spinning hub, producing an inelastic rotational collision. Take data as before. Make sure you have at least three or four data points before and after the collision.

Plot voltage readings versus time (in arbitrary units) for the three runs on three sheets of linear graph paper. The collision takes about 0.5 s; that's the time during which the dropped washer is being accelerated and the one beneath decelerated by the friction torque between them. The meter reading process introduces additional uncertainty.

## Parts for Experiment AM

### From previous experiments:

C-clamp,  
2" corner brace,  
10-32 screws,  
2 LVPS,  
DC permanent magnet motor,  
stainless steel hose clamp,  
LED

### New Items:

1 5/16" hardwood dowel with #18 drilled hole  
1 1" plastic chair caster bushing  
2" 5-kV test lead wire  
2 1" US Standard Washer (2-1/2" OD, approx. 5/32" thick)  
1 paper clip #1  
1 slide type DPDT switch  
1 6-32 X 1/2 RH screw  
1 6-32 nut  
1 resistor 91 k $\Omega$  1/2 W  
1 capacitor, 0.1  $\mu$ F

# Experiment FL Flow

## Introduction

Steady flows are driven by forces that are balanced by resisting forces. For instance, the amount of water coming out of a shower depends on the water pressure as provided by private or municipal water systems, and the resistance of the many small holes in the shower head. Depending on the diameter of the holes and the velocity of the flow, the resistance can be due to viscosity (the friction of water against water when there are differences in velocity within the flow), momentum given to the fluid as it speeds up as it passes through the holes, and additional resistance when the flow becomes turbulent and there is vortex motion in the fluid.

## Experiment

You'll measure the rate at which water flows out of a container through a tube placed near the bottom. You'll do this for different length tubes. For each tube length you will calculate characteristic time constant for the flow rate. You will then compare these time constants as a function of the tube length. This experiment is primarily about taking data but later in the semester you will interpret your data to determine the viscosity of water.

## Apparatus

You have a 0.5 liter clear plastic bottle which you will cut to make a cylindrical container, four cups, one piece sugarless chewing gum, four stirrers (plastic tubes about 130 mm long and 2.8 mm inside diameter), two paper plates, one push pin, one 8 penny nail, one 1 1/2" length poly tube, and paper towels. You also have a photocopy of a ruler on your parts list. You can use thermometer in your Red Box that will be used in Expt ET. You'll need either a stopwatch, or a clock or watch with a sweep second hand for timing the drop in water height.

Use the attached three copies of *Data Sheet for EXPERIMENT FL: FLOW* for experiments I, II, and III, with 3, 2 and 1 stirrers, respectively. There are spaces for recording the length of the tube, the diameter of the tube (see parts list), room and water temperatures at start and finish of each experiment, and a table with six columns: one for water level (the "head"), three columns for three sets of time measurements, one for the calculated average time, and one for the standard deviation of the average time. Record any relevant circumstances, phenomena, troubles, etc. in the space below labeled *notes*.

There are attached to the write-up four pieces of linear graph paper and three pieces of semi-log graph paper although you may plot your data using a computer.

## Procedure

Open up the cap of the bottle and empty the contents. You should have one piece of sugarless chewing gum, four stirrers (plastic tubes about 130 mm long and 2.8 mm inside diameter), one and one half inch length of poly tube, one 8 penny nail, rubber bands, and a push pin.

You will first connect three stirrers together with small pieces of poly tubing. Cut a 15 mm piece of the poly tubing and place the end of one stirrer into the tubing (Figure 1).



**Figure 1: Stirrer inserted into poly tube**

Fit a second stirrer into the other end of the tubing until the stirrers meet together in the middle of the poly tube (Figure 2). This step is delicate, you can easily rip the stirrer.



**Figure 2: Two stirrers connected by a poly tube**

Now cut another 15 mm piece of the plastic tubing and connect a third stirrer to the other two. **Measure and write down the length of this tube. (It should be about 390 mm).**

Cut the plastic bottle with scissors about 15 mm from its bottom so as to make a cylindrical container that tapers down to the screw cap at the end. In order to cut the container begin by making a hole with the push pin and enlarging it with the nail so that you can get the scissors in to cut the bottle.

Make sure the cap is screwed on tight. Turn the container upside down and put it in one of the cups. You will make a hole in the container at a point that is just above the rim of the paper cup. Mark a spot for your hole with a pen. This hole will be about 65 mm from the bottom of the cap.

(You can now take the container out of the cup).

Start making the hole with a push pin. Enlarge this small hole with the point of the nail, turning the nail like a drill. The stirrer should just fit; press it in about 10 mm. (If you wish, you can practice beforehand making holes in the otherwise useless cut off part of the container). Use a small amount of chewing gum to seal around the plastic tube and black tape or gum to seal undesired holes. If necessary, cut away the rim of the cup to make room for the gum seal.

You'll need to measure the depth of water in the container in 10 mm steps to within  $\pm 1$  mm or less. You have a photocopy of a ruler on the side of your Experiment Flow parts list. Attach the ruler to the side of the container with rubber bands so that zero of the metric scale is at the center of the hole and so that it extends upward about 90 mm.



**Figure 3: Container, ruler, and stirrer inserted in hole**

Your apparatus will consist of the upside down container in a cup; an upside down second cup that supports the tube; and a third cup that will collect the outflow. Make sure that the stirrers are straight. Figure 4 shows the apparatus when two stirrers are connected together.



**Figure 4: Apparatus with two stirrers**

Remove items from your desk that might be damaged by water and arrange your apparatus as shown in figure 4 but with three stirrers instead of two. Sand, pebbles or coins might be put in the bottom of the container support cup to make it sit steadier. Use the paper plates under the container support cup and the collector cup to contain spills. Paper, folded or wadded, can be used to adjust heights so that the tube is reasonably level.

### **Taking Data**

Your first data measurements will use a tube made from three stirrers. You will fill the container and then record the time it takes for the water level in the container to pass successive 10 mm (1 cm) scale marks. You will repeat the experiment three times. Then you will detach the third stirrer and repeat the process making three more trial runs. You will then detach the second stirrer and repeat the process making three more trial runs.

Fill a cup with water as close to room temperature as possible, as indicated on your thermometer that you will find in your red box. **Record room and water temperatures at the start and finish of your experiment. Record the length of your stirrers. Record your results on the accompanying Data Sheet for Experiment Flow.**

Fill the container and observe what happens as the water level drops. You and your partner should decide on a method for taking data. For instance, one partner can call out as the water level passes successive 10 mm (1 cm) scale marks and the other writes down the time to the nearest second as the level passes each mark. Refill the container using the collector cup, keeping water from flowing until you're ready by putting a finger lightly over the end of the tube. Have another cup at hand to catch any extra flow. Note that at some level the nature of the flow

changes from a continuous stream to a series of drops. Either stop timing or, if you wish, record the level and time corresponding to this change. **Take data for three trial runs. Record your results on the accompanying Data Sheet for Experiment Flow.**

Repeat the above procedure after shortening the plastic tube from 3 stirrers (about 390 mm) to 2 stirrers (about 260 mm) and then to 1 stirrer (about 130 mm). **Take data for three trial runs for two stirrers and take data for three trial runs for one stirrer. Record your results on the accompanying Data Sheet for Experiment Flow.**

## **Averaging**

For each combination of stirrers, you have three time measurements for each height that the water level passes. Average the three times to give an average time corresponding to each water level. **Record your results on the accompanying Data Sheet for Experiment Flow.**

## **Graphing the Data**

You have four linear and three semi-log graph papers. In addition you have one linear graph paper for the time constants vs. length of tube graph.

On three linear graph papers, **plot the level of the water above the hole (the "head") in mm versus the average time in seconds to reach that level for each of the tube lengths.** (You should have three graphs.)

On the semi-log paper, **plot the head vs. average time for each of the tube lengths.** On semi-log paper, the horizontal axis is a normal linear scale, but the vertical axis is marked off in proportion to the logarithms or natural logarithms of the numbers represented. (Recall logarithms in base 10 are proportional to natural logarithms according to  $\log_{10} u = \ln u / \ln 10$ .) So you can choose the numbers 1,2,3 etc on the vertical axis to represent 10 mm, 20 mm, 30 mm, etc. So a data point like (30 mm, 55 sec) is placed at the 3 on the vertical axis and at the 55 on the horizontal axis. You should have three semi-log graphs. Draw the best straight lines through the points as judged by eye. (When choosing the 'best straight line', consider which points are most reliable. Are the first and last measurements as reliable as the others?)

## **Reporting the Data**

Your linear graph of head vs. average time should be an exponentially decaying function,

$$h(t) = h_0 e^{-\alpha t} \quad (10.1.1)$$

where  $h_0$  is the value of the head at  $t = 0$  and  $\alpha$  is a constant. The time constant  $\tau$  associated with this exponential decay is defined to be the time that it takes for the head to reach a value of  $h(\tau) = h_0 e^{-1} = h_0 / 0.368$ . Since  $h(\tau) = h_0 e^{-\alpha\tau}$ . The time constant  $\tau$  is related to the constant  $\alpha$  according to

$$\alpha\tau = 1 \text{ or } \tau = 1/\alpha .$$

Your semi-log plot should be nearly a straight line. The natural logarithm of Equation 10.1.1 is

$$\ln h(t) = \ln h_0 - \alpha t .$$

So a plot of  $\ln h(t)$  vs. time  $t$  will be a straight line with

$$\text{slope} = -\alpha = -1/\tau .$$

## Finding the Time Constant

### Method 1

Obtain the time constant  $\tau$  for the flows with the three tube lengths by the following procedure that will use results from both your semi-log graph or your linear graph. Use your best straight line in the semi-log paper graph to determine the value of the head,  $h_0$ , at  $t = 0$  for each of the three experiments. (Note: if you just choose your initial value from your data sheet you are ignoring the rest of your data values.) You can obtain the time constant from the linear graph of head vs. average time by directly reading off the time that the head reaches the value  $h_0 / 0.368$ . Determine the time constants for each of your three experiments. **Report your results in the table Time Constants for Experiment Flow that is attached to the write-up.**

### Method 2

From your semi-log plot, calculate the slope of your best-fit straight line. Compute the time constant according to  $\text{slope} = -\alpha = -1/\tau$ .

**Make a plot of tube length vs. time constant. Is there any nice curve that passes through the data points? What does the extrapolation to zero tube length mean?**

## Data Sheet for Experiment Flow

Expt. I: Length of tube = \_\_\_\_\_

Room Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

Water Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

Water level (mm)	T <sub>1</sub> (sec)	T <sub>2</sub> (sec)	T <sub>3</sub> (sec)	T <sub>ave</sub> (sec)

Notes:

## Data Sheet for Experiment Flow

Expt. II: Length of tube =

Room Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

Water Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

Water level (mm)	T <sub>1</sub> (sec)	T <sub>2</sub> (sec)	T <sub>3</sub> (sec)	T <sub>ave</sub> (sec)

Notes:

## Data Sheet for Experiment Flow

Expt. III: Length of tube =

Room Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

Water Temperature: Start \_\_\_\_\_ Finish \_\_\_\_\_

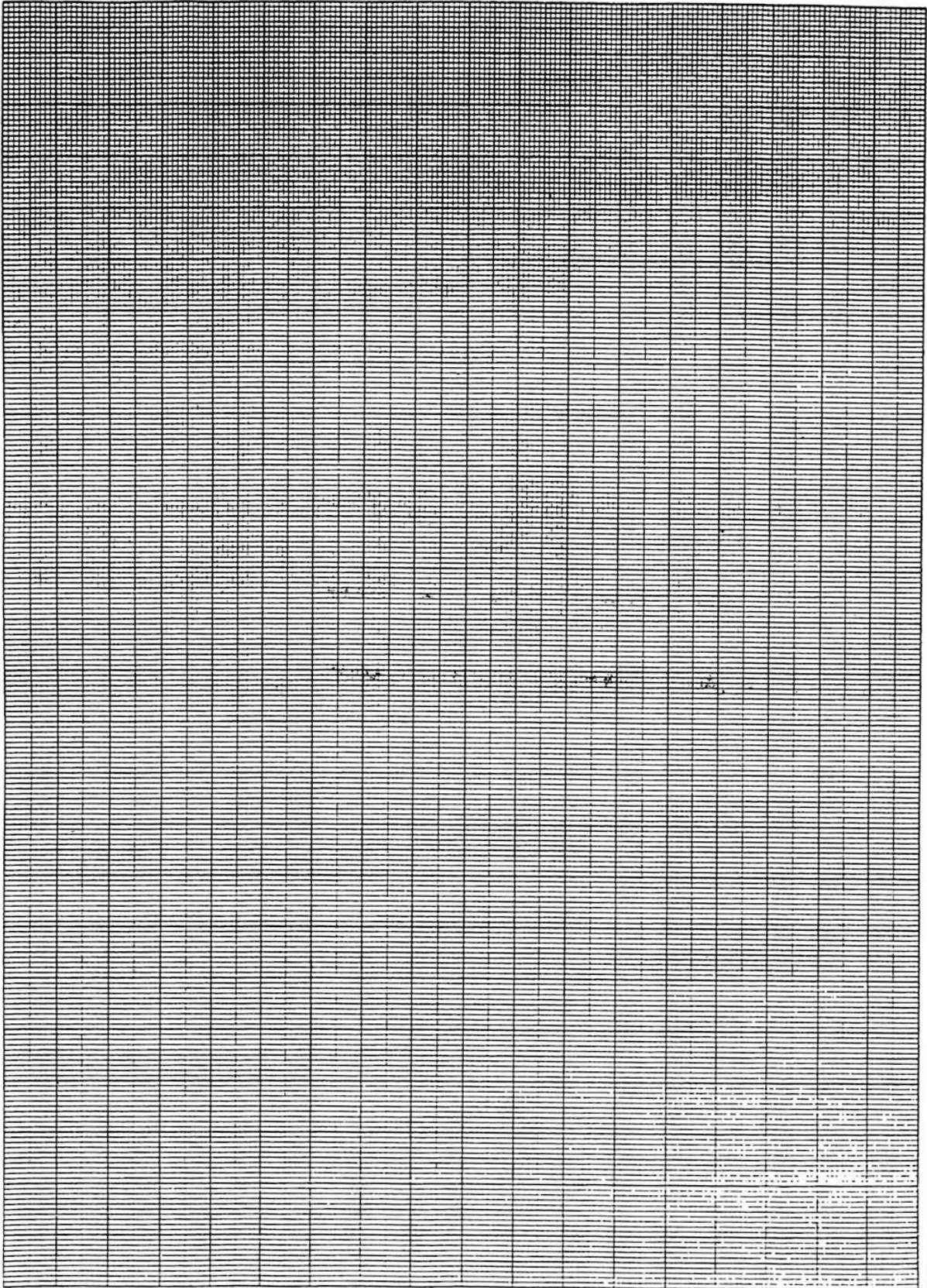
Water level (mm)	T <sub>1</sub> (sec)	T <sub>2</sub> (sec)	T <sub>3</sub> (sec)	T <sub>ave</sub> (sec)

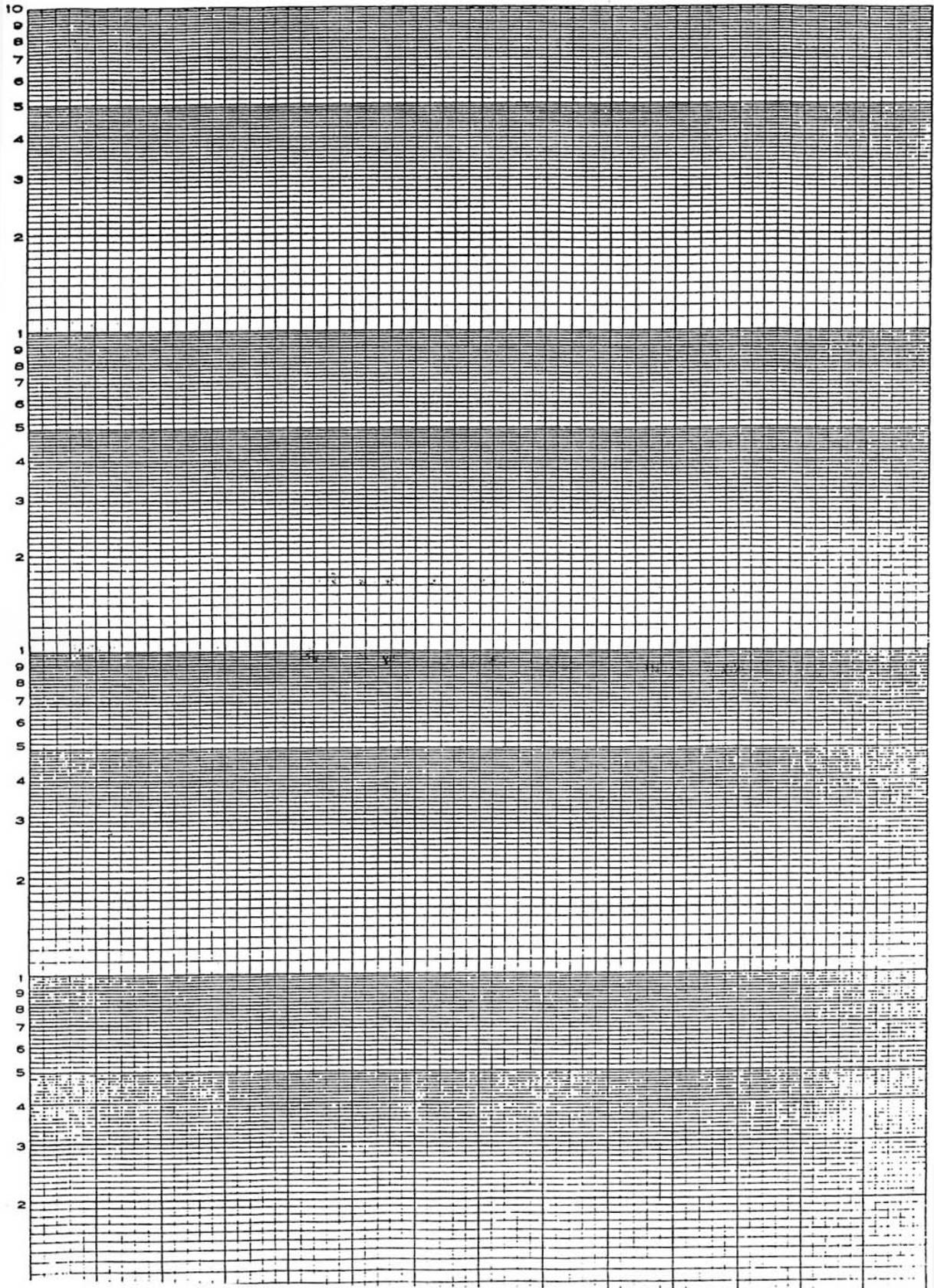
Notes:

**Time Constants for Experiment Flow**

Trial	Tube length (mm)	Time constant (sec)	Container diameter (mm)	Temperature start ( <sup>0</sup> C)	Temperature finish ( <sup>0</sup> C)
Experiment I					
Experiment II					
Experiment III					

Notes:





## **Red Box and Toolkit Contents**

### **Red Box Top Tray**

- 1 magnetic multimeter
- 1 digital multimeter
- 1 transformer
- 1 thermometer
- 1 roll electrical tape
- 1 meter/transformer kit
- 1 C-clamp
- 1 piece of resistance wire
- 1 clip lead kit
- 100 US pennies
- 1 piece of emery cloth
- 1 length, spare hookup wire

### **Bottom of Red Box**

- 2 LVPS

1 each: AM, CF, ES, ET, FM, FO, LVPS, LVPST, VS

wood block, 1/2" pipe, brass rod, paper cups, paper towels

### **Toolkit**

- 1 soldering iron
- 1 tube solder
- 1 pr. slip joint pliers
- 1 pr. long nose pliers
- 1 wire stripper
- 1 pr. scissors
- 1 small slotted screw driver
- 1 large slotted screw driver
- 1 phillips screw driver
- 1 12" plastic ruler
- 1 6" steel ruler
- 1 tape measure

## Kits Parts List (\* in Red Box)

### Clip lead kit

- 6 black insulated alligator clips
- 6 red insulated alligator clips
- 3' #22 black stranded wire
- 3' #22 red stranded wire

### Meter/Transformer kit

- 4 black alligator clips
- 2 red alligator clips
- 1 AA cell
- 3 spare fuses
- 2 spare button cells (old style meters only)

### ES

- 1 knife switch (DPDT)
- 1 AA cell
- 1 battery holder
- 3 wiring nuts
- 2 10 meg  $\Omega$  resistors
- 1 20  $\Omega$  resistor
- 1 1.0  $\mu$ F polyester capacitor

### LVPS

- 1 Perfboard
- 4 rubber feet
- 1 full wave bridge rectifier
- 1 1000  $\mu$ F electrolytic capacitor
- 1 socket for LM317 regulator
- 1 5 k $\Omega$  potentiometer
- 1 1 $\mu$ F, electrolytic capacitor
- 1 390  $\Omega$  1/2W resistor
- 1ft. #22 tinned copper wire
- 1 LM317 voltage regulator
- 1 heatsink for LM317 reg AM
- 2 1" plated steel washers
- 1 1" plastic caster bushing
- 1 hardwood dowel with #19 drilled hole
- 2" 5kV test lead wire
- 1 paper clip
- 1 slide type SPDT switch
- 1 6-32 X 1/2 screw
- 1 6-32 nut
- 1 91 K $\Omega$  1/2 W resistor

1 0.1  $\mu$ F ceramic disc capacitor

### **LVPST**

1 resistor 2.4  $\Omega$ , 2 W

1 #1157 tail lamp

### **FO**

1" 1/8" Al rod

1 wire nut

8" 1/16" brass rod\*

1' #22 stranded wire

2 plastic stirrers

2 paper clips

2" 5k test lead

1 paper cup\*

2 cable clamps

3 #6 washers

2 #6 sheet metal screws

1 pre-drilled wood block\*

1 5/8" binder clip

1 alligator clip

1 solder lug

1 6-32 machine screw

1 6-32 nut

12" 1/2" PVC pipe\*

1 10M $\Omega$  resistor 1/2W, 5%

1 1.0  $\mu$ F polyester cap.10%

### **FM**

2 magnets

1 paper cup\*

1 piece aluminum rod

1 soda straw

1' solid wire

1 pre-drilled wood block\*

### **CF**

1 12V DC motor

1 2" corner brace

1 hose clamp #12 SS

1 LED, wt. internal resistor

1 .1 $\mu$ F ceramic disc capacitor

4' #22 speaker wire

2 nuts, 6-32 (5/16) steel

2 rubber bands, #18

1 paper clip,#1

- 1 push pin
- 1 paper cup (in red box)

### **ET**

- 4 #6 cups
- 1 cup lid
- 1 soda straw
- 1 thermistor
- 4 paper towels (in red box)

### **VS**

- 1 paper cup ( in red box)
- 1 6' piece of string
- 1 10" hacksaw blade
- 1 fender washer
- 1 4-40 X 1/2" screw
- 1 4-40 nut
- 6 #18 rubber bands
- 1 12" piece of wood
- 2 thumbtacks
- 2 paper clips

### **AM**

- 2 1" plated steel washers
- 1 1" plastic caster bushing
- 1 hardwood dowel with #19 drilled hole
- 2" 5kV test lead wire
- 1 paper clip
- 1 slide type SPDT switch
- 1 6-32 X 1/2 screw
- 1 6-32 nut
- 1 91K $\Omega$  1/2W resistor
- 1 0.1 $\mu$ F ceramic disc capacitor

### **FL**

- 1 500ml plastic bottle
- 4 plastic stirrers
- 1 1 1/2" length poly tube
- 4 paper cups
- 2 paper plates
- 4 paper towels
- 1 push pin
- 1 piece of gum (sugarless)
- 1 2 1/2" nail
- 2 rubber bands