

# Experiment ES Estimating a Second

## Introduction: About Estimating

Before measuring and calculating comes estimation; a chance to exercise ingenuity. Often it's a matter of organizing your rough knowledge and experience in quantitative form, or making a simplified model so as to answer questions about numbers, such as: how many hairs are on your head? how much energy does an AA cell store? how many gallons of gasoline are burnt annually in the US? how many piano tuners are there in Chicago? etc. You might say: who wants such a rough answer, perhaps only good to an order of magnitude? Well if you aren't near a reliable reference source or able to measure, it's far better to have some estimate than nothing. A good estimate enables you to consider whether a new idea for an experiment or even a business is possible.

The question about piano tuners was an example from the physicist Enrico Fermi, and we often call these estimations "Fermi problems".

Consider the estimating of amounts, sizes and duration. These correspond to the physical quantities mass, length and time, the fundamental quantities of classical mechanics. One can develop a feel for some of these by using standards based on common objects (for instance a penny, dated 1982 and later has a mass about 2.5 g; earlier pennies have a mass about 3 g. A penny's diameter is about 2 cm. One's pulse, with a period of about 1 second, is a rather variable time standard that you always have around. But there is another way of estimating time which we'll explore here.

## Estimating Time

Some people have a good idea of what time of day it is without looking at a watch, or can feel when an hour has passed; but how do they do it? It often turns out that the estimator has practiced this skill for many years. However, when it's just a matter of estimating a few seconds, most people mark time by counting a word that takes one second to pronounce like: one one thousand, two one thousand, three one thousand, etc., or: one Mississippi, two Mississippi, three Mississippi, etc., To see how well this works we have developed a simple electrical timer which you and your partner can assemble and use to test how accurately you count seconds.

## Parts

**DPDT knife switch:** This "double-pole double-throw" (DPDT) knife switch was chosen because it's obvious how it works and it's easy to connect to. Knife switches, so-called because the motion of the swinging arm is like that of a knife blade, are rarely used now but acronyms like

SPST, SPDT, DPST, DPDT (S for "single") still apply. The words "throw" and "pole" are old usages: "throw the switch", meaning to turn on or off, close or open the circuit, whose conducting wires are connected to the switch terminals, or "poles". See Fig.1 for the schematic of these switches.

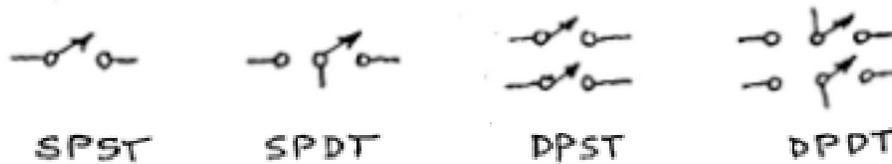


Figure 1: Various switches; SPST, SPDT, DPST, DPDT

Notice that in attaching leads to the screw terminals of the switch you should bend the lead clockwise around the screw so that it is drawn in as you tighten the conventional right-hand screw.

**Two 10 megohm resistors, one 20 ohm resistor:** Resistors are color coded by colored bands read from left to right when you hold them so that the gold band is on the right. Two of the three resistors in your kit have at the left a brown band meaning 1, for the first digit; a black band meaning 0, for the second digit; a blue band meaning 6 zeros-hence

$$1-0-000000 \text{ ohms } (\Omega) = 10^7 \Omega = 10 \text{ megohm } (M\Omega).$$

The gold band on the right means that the resistor is within  $\pm 5\%$  of the rated value. The third resistor is a 20  $\Omega$  resistor having at the left a red band meaning 2, for the first digit; a black band meaning 0, for the second digit; a black band meaning 0 zeros-hence 2-0- = 20  $\Omega$ .

**1.0  $\mu\text{F}$  capacitor:** The capacitor has a brown molded plastic case and the relevant part of the label is 105K. This is a code that has the following meaning. The "5" means place five zeros after the 10 and read the capacitance in picofarads (pF). A picofarad is  $10^{-12}$  F. Thus

$$105 = 1 - 0 - 00000 \text{ picofarads} = 10^6 \text{ pF} = 10^{-6} \text{ F} = 1\mu\text{F} = 1 \text{ microfarad}$$

The "K" means  $\pm 10\%$ . Some capacitors have leads labelled plus or minus, but this one doesn't.

**AA cell and AA cell holder:** The kit contains an AA cell and a holder with a red lead (plus,+) and a black lead (minus,-). These leads are insulated but you will need to remove more of the insulation so that about 1/2 inch , 12 mm, of bare wire are exposed at the end of each wire. Do this with the wire strippers in your toolkit, set so that they just grab a paper clip; too large and they won't cut the insulation or too small and they will cut the wire.

**Three Wire Connectors:** These hollow plastic cones containing conical springs are often called "wiring nuts" and are widely used in 120/240 volt wiring in buildings. They come in various sizes and what you have is about the smallest. Even so the leads are so small, (#22 and #25 AWG: American Wire Gauge) that you'll have to twist them together, bend them over and squeeze them with pliers before screwing on the wiring nut (see Fig. 2). In any case the leads should seem quite firmly connected when pulled or wiggled.

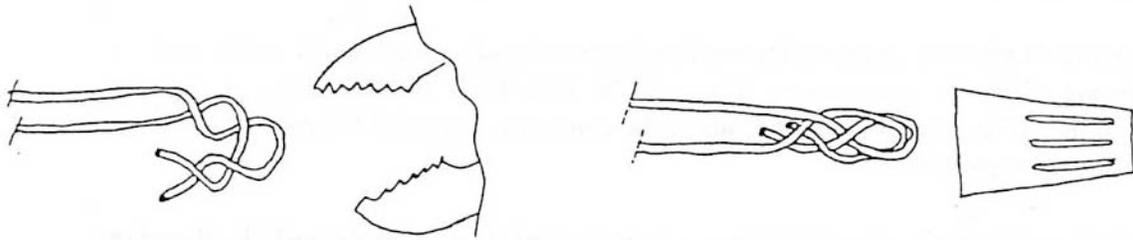


Figure 2: Wiring Nuts

Connections are made by attaching leads to screw terminals on the switch and using the wiring nuts to connect other leads. Alternatively you can use your soldering iron after tinning it. Assemble the circuit according to the sketch of the circuit, Fig. 3, and the circuit diagram, Fig. 4. For this experiment, a single-pole double-throw (SPDT) would do so, but the DPDT provides more mechanical stability when switching.

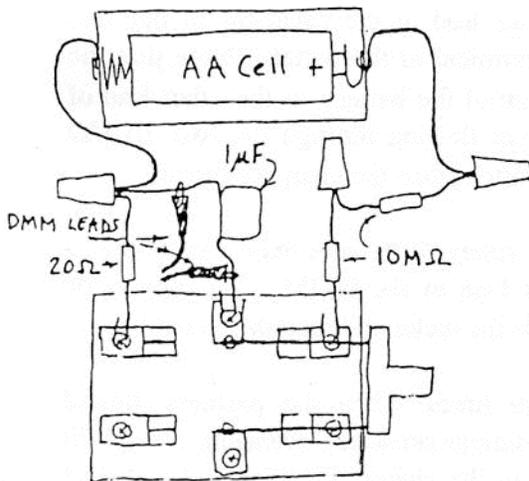


Figure 3: Sketch of circuit

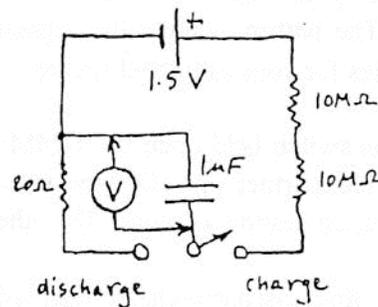


Figure 4: Circuit diagram

## Experiment

In this experiment you will estimate a second by closing a switch for about a second in a circuit that is charging a capacitor. A charged capacitor has a voltage difference between its two leads. Once you open the switch the charging stops and the voltage difference across the

capacitor has a fixed value. The voltage difference is related to the charging time and for our device is nearly linearly proportional. After each trial the capacitor must be discharged until there is zero voltage difference across the leads so that a new trial can begin.

Operate the switch by grasping the small cylindrical handle; don't touch metal parts to avoid introducing electrical disturbances. Connect the two leads of the digital multimeter (DMM), set on the DCV range to the two leads of the capacitor. The DMM displays the voltage difference across the capacitor.

You first need to make sure the capacitor is discharged. Start with the switch, closed on the side that connects the  $1\ \mu\text{F}$  capacitor to the  $20\ \Omega$  resistor. This is the discharge side. Notice that the battery is not connected when the switch is in this position. The  $1\ \mu\text{F}$  capacitor is discharged in a short time by current flowing through the  $20\ \Omega$  resistor until the digital multimeter (DMM), reads zero. Once the capacitor is discharged you can put the switch in the open position.

We can now begin the charging. You will charge up the capacitor put throwing the switch to the other position for you consider to be one second and then opening the switch and reading the voltage difference across the capacitor.

When the switch is closed, you have connecting one lead of the capacitor to the two  $10\ \text{M}\Omega$  resistors, which are in turn connected to the positive terminal of the battery. Note that the circuit is closed by the connection from the negative terminal of the battery to the other lead of the capacitor. The battery charges the capacitor by a current flowing through the two  $10\ \text{M}\Omega$  resistors in series for your estimated second. Opening the switch stops the charging process.

With the switch held open the DMM displays the voltage difference across the leads of the capacitor. The partner operating the switch should not look at the DMM so as not to be influenced when estimating a second. The other partner reads the meter and records the reading.

Repeat this discharge-charge-read-voltage cycle 36 times. Then the partners should exchange tasks. These 72 readings (keep each partner's 36 readings separate), averaging around 70 millivolts ( $1\ \text{mV} = 10^{-3}\ \text{V}$ ) are measures of the time the switch was closed in the charge position, and hence of your estimates of a second.

## **Data**

Use your calculator or your favorite software program to compute the mean and standard deviation of the DMM readings corresponding to your estimate of one second.

Make a histogram of your data as follows. On a sheet of graph paper, draw a line along the long side about 1 inch from the bottom. Label every fifth space, starting at the left so as to

accommodate the range of DMM readings you have made, say, from 40 to 100. Make an X in each space corresponding to each of your 36 values of DMM reading rounded off to the nearest integer. Count the number of Xs in each interval of 5 (eg. 45 through 49) and represent that number by a horizontal line above that interval. Connect the ends of these lines with vertical lines. You now have a bar graph (histogram) of your data.

## Determining the Time Constant $RC$

To interpret the DMM readings in millivolts in terms of time it's necessary to determine the time constant,  $\tau$ , of the resistance-capacitance combination.

Consider the charging circuit in Experiment ES that is shown in figure 4. The DMM is measuring the voltage across the capacitor,  $V_{cap}$ . The rate of change of the voltage,  $dV_{cap}/dt$ , across the capacitor is proportional to two terms. The first is the voltage of the cell,  $V_{cell}$ . This is a constant factor. The second is the negative of the voltage,  $V_{cap}$ , that is already present across the capacitor.

$$\frac{dV_{cap}}{dt} = \frac{1}{\tau}(V_{cell} - V_{cap}).$$

The constant of proportionality  $1/\tau$  has the dimensions of inverse time. Hence  $\tau$  is called the time constant. The rate of change of the voltage is inversely proportional to the resistance,  $R$  measured in ohms, in the circuit. The greater a resistance to the flow of current and hence charge, the slower the rate of change of the voltage across the capacitor. The rate of change is also inversely proportional to the amount of charge the capacitor is capable of storing. This property of the capacitor is called the capacitance and is denoted by  $C$ , measured in farads. So the rate of change of voltage across the capacitor is

$$\frac{dV_{cap}}{dt} = \frac{1}{RC}(V_{cell} - V_{cap}).$$

Thus we can conclude that the time constant  $\tau = RC$ . For the charging circuit,  $R = 20 \text{ M}\Omega$  and  $C = 10^{-6} \text{ F}$  so if the parts were ideal we would have for the charging process

$$\tau = RC = (20 \text{ M}\Omega)(10^{-6} \text{ F}) = 20 \text{ s}.$$

To convert DMM readings to time, measure the AA cell voltage,  $V_{cell}$  and a particular DMM reading is  $V_{cap}(t)$  corresponding to a time  $t$ . The voltage across the capacitor for a charging circuit is given by the relation

$$V_{cap}(t) = V_{cell}(1 - e^{-t/RC}).$$

The power series expansion for the exponential function is given by

$$e^x = \sum_{n=0}^{n=\infty} \frac{x^n}{n!} = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \dots$$

In particular we have that

$$e^{-t/RC} = \sum_{n=0}^{n=\infty} \frac{(-t/RC)^n}{n!} = 1 - \frac{t}{RC} + \frac{1}{2}\left(\frac{t}{RC}\right)^2 - \dots$$

Notice that when  $t \ll RC$ , we can ignore second order terms like  $(t/RC)^2$  so the voltage across the capacitor grows linearly with time for  $t \ll RC$ .

$$V_{cap}(t) = V_{cell}(1 - e^{-t/RC}) \cong V_{cell}(1 - (1 - \frac{t}{RC})) = V_{cell} \frac{t}{RC} = V_{cell} \frac{t}{\tau}.$$

Therefore the time can be related to the voltage across the capacitor according to

$$t = \frac{V_{cap}(t)}{V_{cell}} \tau.$$

**Example:** Suppose  $t \ll \tau = 20$  s,  $V_{cell} = 1.645$  V, and  $V_{cap}(t) = 80$  mV. So

$$t = \frac{V_{cap}(t)}{V_{cell}} \tau = \frac{(.08 \text{ V})}{(1.645 \text{ V})} (20 \text{ s}) = (1.0 \text{ s}).$$

### Experiment ES: Parts

- 1 DPDT knife switch
- 1 AA cell
- 1 AA cell holder
- 3 wiring nuts
- 2 10 M $\Omega$  resistors
- 1 20  $\Omega$  resistor
- 1 1  $\mu$ F capacitor