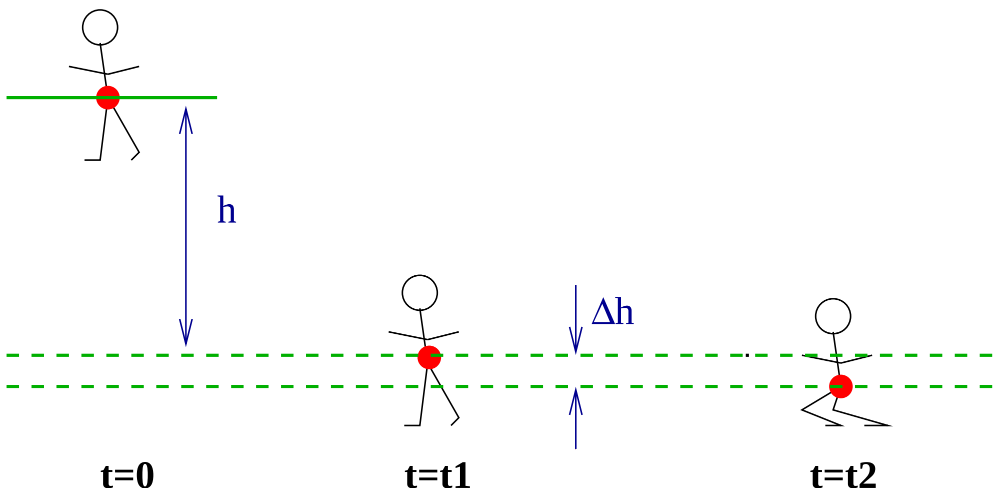
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01X Fall Term 2002

JUMPING FROM A HEIGHT



In this example we will consider what happens if you bend your knees when you hit the ground if you are jumping from a height.

Imagine you have mass m and jump from height h at time $t = 0$. You hit the ground at time $t = t_1$. Then between $t = t_1$ and $t = t_2$ over interval $\Delta t = t_2 - t_1$ you bend your knees and lower your center of mass by a distance Δh. In the figure the red spot indicates your center of mass.

We will ask the questions:

a. What is the average force of the ground on your legs during the impact, in terms of m, g, h and Δh?

b. What is Δt over which the impact happens, in terms of g, h and Δh?

c. If your mass is $m = 60$ kg, what is the maximum $h/\Delta h$ that you can sustain without breaking your tibia? We will assume that the compressive force per area necessary to break the tibia in the lower leg is about 1.6×10^3 bars (1 bar $= 10^5$ Pa $= 10^5 \mathrm{N/m^2}$). The smallest cross-sectional area of the tibia, about 3.2 cm^2, is slightly above the ankle.

For this example, we'll consider you to be a point particle located at your center of mass (we'll see a bit later in the class why this is reasonable).

To solve part (a), first let's consider the first time interval from $t = 0$ to $t = t_1$, during which you are falling. You can get your final velocity at the end of this interval from either 1D kinematics or the work-energy theorem. Let's do the latter. We'll assume no friction, so there's no non-conservative work, so

$$\Delta KE + \Delta PE = 0$$

$$\frac{1}{2}mv_1^2 - mgh = 0 \tag{1}$$

so

$$v_1 = \sqrt{2gh}$$

.

Now let's consider the time interval from $t = t_1$ to $t = t_2$, during the impact. We can apply the work-energy theorem again,

$$\Delta KE + \Delta PE = W_{NC}$$

But this time, there **is** non-conservative work being done on you! There is a contact force from the floor on you, and its direction is antiparallel to your displacement. So the contact force does *negative* work on you, and this non-conservative work is $W_{NC} = -F_{floor}\Delta h$. (We'll assume this force is constant over the interval).

The work-energy theorem for this interval gives:

$$-\frac{1}{2}mv_1^2 - mg\Delta h = -F_{floor}\Delta h$$

(where the "Δ" refers to before and after the impact. Before, you have velocity v_1. After, you have come to a stop and you have velocity zero. Your change in P.E. over the interval is $-mg\Delta h$.)

Now we can plug in $\frac{1}{2}mv_1^2 = mgh$ from equation1, and rearrange terms a little to get

$$\boxed{F_{floor} = \frac{mgh+mg\Delta h}{\Delta h}}$$

So that's part (a).

Now for part (b), let's find the impulse in order to find Δt, since impulse is average total force times time. Impulse is also change in momentum, Δp. We know that your change in momentum over the interval is $\Delta p = p_2 - p_1 = 0 - mv_1 = -mv_1$.

$$\Delta p = F_{tot}\Delta t$$

(which is another way of stating Newton's 2nd Law), where F_{tot} is the *total* average force on you. There are two forces acting on you: gravity and the force of the floor. So $F_{tot} = F_{floor} - mg$, where F_{floor} is the average force of the floor that we just calculated in part (a).

So, the magnitude of Δt is then $\frac{|\Delta p|}{|F_{tot}|}$

Plugging in:

$$\Delta t = \frac{|\Delta p|}{|F_{floor} - mg|} = \frac{mv_1}{\frac{mgh+mg\Delta h}{\Delta h} - mg}$$

The m's cancel, and plugging in for v_1 and massaging a little more we get:

$$\boxed{\Delta t = \frac{\sqrt{2gh}}{g(h/\Delta h)}}$$

For part (c), we use our expression for F_{floor} to get

$$F_{floor} = mg\left(\frac{h}{\Delta h} + 1\right)$$

The maximum force that the smallest area of the tibia can take is 1.6×10^3 bars times 10^5 N/m^2 per bar times 3.2×10^{-4} m^2 times 2 (for two legs) is $F_{max} = 1.0 \times 10^5$ N. If we take $F_{max} = F_{floor}$ when the tibia just breaks, and solving for $h/\Delta h$, we get $\boxed{(h/\Delta h)_{max} = 173}$.

So, if you don't bend your knees (take $\Delta h = 1$ cm), you will break your legs jumping from only 1.7 m. If you bend your knees 0.5 m, your leg bones may survive a leap from 87 m! (Please don't try this yourself though!! This problem considers only damage to bones– in fact other tissues in your body could get damaged in a fall from a height of more than a few meters).

And if you are falling into something soft and cushiony, or into water, Δh (and Δt) are relatively larger. Parachutists are trained to maximize time and displacement of impact when landing by crouching and rolling. And compare a dive to a belly-flop: small Δh and Δt during the collision $\Rightarrow$ hurts more!