ESG 8.022 Fall 2006 Exam 3

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1 Useful Formulae

You may find some of the following formulae useful. Then again, you may not.

Maxwell's Equations: $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}; \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}; \vec{\nabla} \cdot \vec{B} = 0; \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Lorentz Force Law: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Conservation Laws: $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}; \frac{\partial u}{\partial t} + \vec{\nabla} \cdot \vec{S} = -\mathbf{J} \cdot \vec{E}$

Impedance: $V = ZI, Z_R = R, Z_C = \frac{1}{i\omega C}, Z_L = i\omega L$

Admittance: $Y = 1/Z$

Potentials: $\vec{E} = -\vec{\nabla} V; \vec{B} = \vec{\nabla} \times \vec{A}$

Energy Density: $u_{em} = \frac{1}{2\mu_0} B^2 + \frac{\epsilon_0}{2} E^2$

Poynting Vector: $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

Maxwell Stress Tensor: $T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right)$

2 Short (and Sweet) Answer Questions

Do four of the following five problems.

a. Charges are flowing in the $\hat{y}$ direction through a flat plate of a conductor in the x-y plane, in a magnetic field in the $\hat{z}$ direction. The $+\hat{x}$ side of the conductor is measured to have a higher potential than the $-\hat{x}$ side. What is the sign of the charge carriers?

i: positive

ii: negative

iii: both

iv: cannot be determined

b. A cylindrical wire made of imperfect conductor is connected to the two terminals of a battery so that current flows through the wire. Which of the following is true?

i: There is a Poynting flux in the wire and its direction is parallel to the current.

ii: There is a Poynting flux in the wire and its direction is radially outward (away from the central axis of the wire)

iii: There is a Poynting flux in the wire and its direction is radially inward.

iv: There is no Poynting flux in the wire

c. Which of the following is most responsible for paramagnetism?

i: Lenz's law

ii: Alignment of permanent dipoles

iii: The Pauli exclusion principle

iv: Superconductivity

d. The Maxwell stress tensor of a uniform electric field corresponds to which case? (Hint: Think of field lines–don't stress about the tensor)

i: Isotropic pressure (same in every direction)

ii: Isotropic tension (negative pressure)

iii: Pressure along the field lines and tension perpendicular to them

iv: Tension along the field lines and pressure perpendicular to them

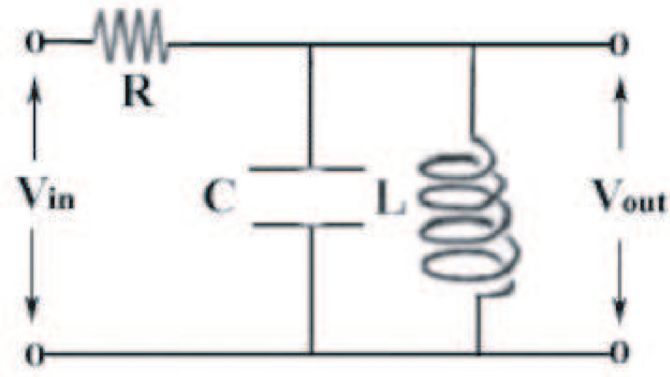
e. A circular disk of radius R has uniform surface charge density σ and rotates like a wheel about its central axis with angular velocity $\vec{\omega}$. The magnetic field for $r >> R$ is given by which of the following expressions? (HINT: Do not solve by brute force. There is a shortcut.) (HINT 2: There are no magnetic monopoles in the universe.)

$$\text{i: } \frac{\mu_0 \sigma \omega R^2}{4\pi r^2}(cos\theta\hat{r} - sin\theta\hat{\theta})$$

$$\text{ii: } \frac{\mu_0 \sigma \omega R^4}{4\pi r^2}(cos\theta\hat{r} - sin\theta\hat{\theta})$$

$$\text{iii: } \frac{\mu_0 \sigma \omega R^2}{16 r^3}(2cos\theta\hat{r} + sin\theta\hat{\theta})$$

$$\text{iv: } \frac{\mu_0 \sigma \omega R^4}{16 r^3}(2cos\theta\hat{r} + sin\theta\hat{\theta})$$



3 Problem with Circuits

Consider the situation where V_{out} is an open circuit (no current goes through there).

a. Given an input voltage $V_{in} = V_0 cos(\omega t)$, what current passes through the resistor?

b. What is the ratio of the output voltage amplitude to the input voltage amplitude?

c. What frequency, ω, should you drive the circuit (as input voltage), to obtain the maximum output voltage amplitude?

d. Electrical engineers call such a circuit a band pass filter. Why does this name make sense given your answers to the previous questions?

4 Displacing the Problem

A capacitor C with circular plates of radius b is charged to a voltage V_0. The space between the two plates is small compared to b so that we can safely ignore any fringing effects. At $t = 0$ the switch is closed and the capacitor discharges through the resistor R. In all the questions below give your answers in terms of C, b, V_0, R, t and any universal constants.

a. Give an expression for the charge $Q(t)$ as a function of time of the positively charged plate (upper one in the figure) of the capacitor.

b. Find the electric field, $\vec{E}(t)$, between the capacitor plates.

c. Find the Maxwell displacement current density, $\vec{J}_d(t)$ between the two capacitor plates.

d. Find the magnetic field, $\vec{B}(t)$, between the capacitor plates. (Hint: Do not assume it is uniform)

e. Find the Poynting vector, $\vec{S}(t)$, between the capacitor plates.

f. Extra Credit: Calculate the time rate of change of the energy stored in the fields between the plates

g. : Extra Credit: Find a relation between your answers from the two previous parts. Comment on why this relation exists.