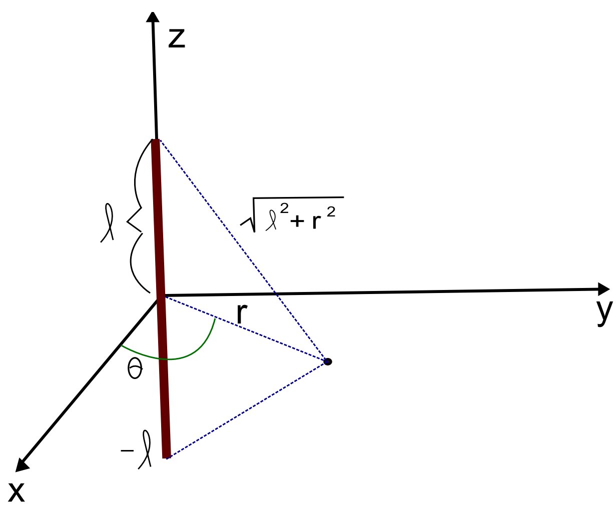
8.022 Lecture Notes Class 10 - 09/21/2006

Electric Potential

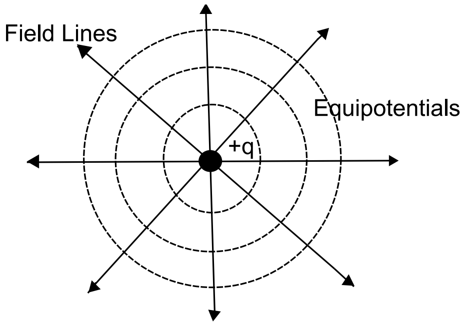
$$\vec{E}(\vec{x}) = -\vec{\nabla} V$$
$$\phi = V$$
$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{x}) d^3x}{|\vec{x}-\vec{x}'|}$$

$V_{x_0}(\vec{x})$ is work done to move charge from x_0 to x.



Find V on xy-plane.
(Hint: cylindrical coordinates)

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int_{-l}^{l} \frac{\lambda dz'}{\sqrt{r^2+z^2}}$$
$$V(r,\theta,z) = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{z+l+\sqrt{r^2+(z+l)^2}}{z-l+\sqrt{r^2+(z-l)^2}}\right)$$
$$V(r,\theta,0) = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{l+\sqrt{r^2+l^2}}{-l+\sqrt{r^2+l^2}}\right)$$



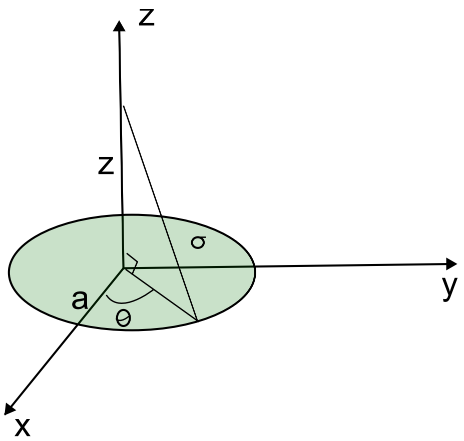
$$V = V_1 + V_2 + ... \qquad \text{(scalar)}$$

Units: $\frac{\text{Joule}}{\text{Coulomb}} = \text{Volt}$.

$$\vec{E} = -\vec{\nabla} V \qquad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla^2 V = \frac{-\rho}{\epsilon_0} \quad \text{Poisson's Equation}$$

$$\text{if } \rho = 0, \quad \nabla^2 V = 0 \quad \text{Laplace's Equation}$$



$$V(z) = \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^a \frac{\sigma r dr d\theta}{\sqrt{r^2+z^2}} \qquad \begin{cases} u = & r^2+z^2 \\ du = & 2rdr \end{cases}$$

$$= \frac{1}{4\pi\epsilon_0}\cdot 2\pi \cdot \frac{\sigma}{2}\int_{z^2}^{a^2+z^2} \frac{1}{\sqrt{u}} du$$

$$= \frac{\sigma}{4\epsilon_0}\cdot 2\sqrt{u}\Big|_{z^2}^{a^2+z^2}$$

$$= \frac{\sigma}{2\epsilon_0}(\sqrt{a^2+z^2} - \sqrt{z^2})$$