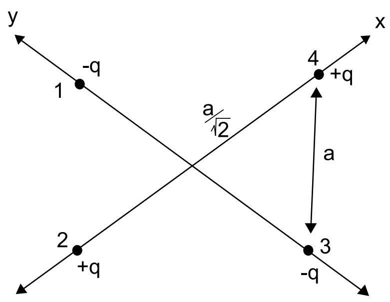
8.022 Lecture Notes Class 13 - 09/28/2006

Find work needed to bind together



$$
\begin{aligned}
W &= qV \\
&= \frac{1}{2}\Sigma_{i=1}^{4} Q_i \cdot V(\vec{r_i}) \\
&= \frac{1}{2}(q \cdot \frac{1}{4\pi\epsilon_0}(\frac{q}{a\sqrt{2}} + \frac{-q}{a} + \frac{-q}{a}) \cdot 2 + 2 \cdot -q \cdot \frac{1}{4\pi\epsilon_0}(\frac{q}{a} + \frac{-q}{a\sqrt{2}} + \frac{q}{a})) \\
&= \frac{1}{a}(\frac{q^2}{4\pi\epsilon_0\sqrt{2}} - 2 \cdot \frac{q^2}{4\pi\epsilon_0} + \frac{q^2}{4\pi\epsilon_0\sqrt{2}} - 2 \cdot \frac{q^2}{4\pi\epsilon_0}) \\
&= \frac{q^2}{4\pi\epsilon_0 a}(\frac{2}{\sqrt{2}} - 4) = \frac{q^2}{4\pi\epsilon_0 a}(\sqrt{2} - 4)
\end{aligned}
$$

Do sides , then diagonals

$$
\begin{aligned}
U &= \frac{\epsilon_0}{2(4\pi\epsilon_0)^2} \int (\frac{q^2}{r^2})^2 r^2 \sin\theta d\theta d\phi dr \\
&= \frac{q^2}{8\pi\epsilon_0} \int_0^\infty \frac{1}{r^2} dr \\
&= \frac{q^2}{8\pi\epsilon_0} \frac{-1}{r}\Big|_0^\infty \\
&= \frac{q^2}{8\pi\epsilon_0}(-\frac{1}{\infty} - (-\frac{1}{0})) \\
&= \infty
\end{aligned}
$$

Charge Up a Capacitor

$$
\begin{aligned}
dW &= \left(\frac{q}{C}\right)dq \qquad \text{work gets harder as there is more charge already on it} \\
W &= \int_0^Q \frac{q}{C}dq \\
&= \frac{q^2}{2C}\Big\|_0^Q \\
&= \frac{Q^2}{2C} \quad C = \frac{Q}{V} \\
&= \frac{1}{2}CV^2
\end{aligned}
$$

$$
\begin{cases}
\nabla^2 V = -\frac{\rho}{\epsilon_0} & \text{Poisson's} \\
\nabla^2 V = 0 & \text{Laplace}
\end{cases}
$$

In one dimension: $\frac{d^2V}{dx^2} = 0$, so $V = mx + b$ (line).

First Uniqueness Theorem

Given V on boundary S of volume $\mathcal{V}$, Laplace's equation gives a unique solution for V in $\mathcal{V}$. ($\mathcal{V}$ does not need to be finite)

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$$
\begin{aligned}
\nabla^2 V_1 &= 0 = \nabla^2 V_2 \\
V_3 &= V_1 - V_2 \\
\text{So } \nabla^2 V_3 &= 0, \text{ and } V_3 = 0 \text{ on boundary}
\end{aligned}
$$

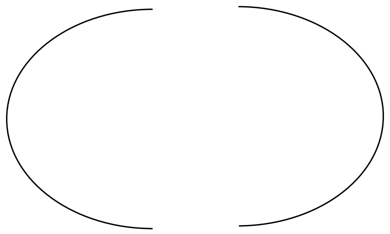
Also no local min/max , so $V_3 = 0$ in V.

Thus $V_1 = V_2$

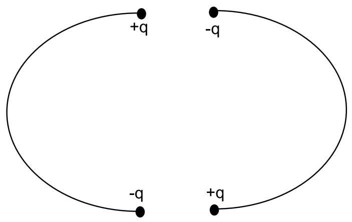
$\square$

Second Uniqueness Theorem

In a volume surrounded by conductors, the total charge on each conductor determines the E-field uniquely.
Will charges spread out ?
Yes!
Thus case below



no charges - (total on each) stable solution



Still no charge on each conductor, so by Second Uniqueness Theorem, same solution as above