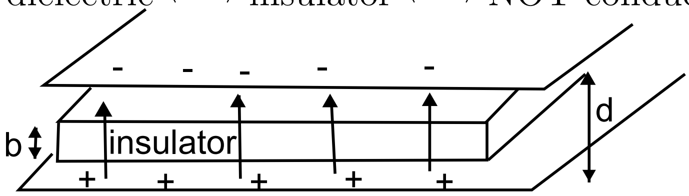
8.022 Lecture Notes Class 17 - 10/16/2006

Today: Dielectrics

A new perspective on p

dielectric $\longleftrightarrow$ insulator $\longleftrightarrow$ NOT conductor



• capacitance increases (Faraday's experiment)

$$C \to k \cdot C$$

• $C = \frac{Q}{V}$

$$V = E \cdot d = \frac{\sigma}{\epsilon_0} d = \frac{Qd}{A\epsilon_0}$$

$$C = \frac{Q}{V} = \frac{A\epsilon_0}{d}$$

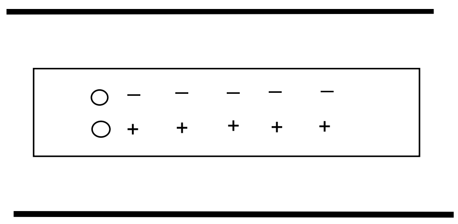
$Q = CV$ where Q constant, C increasing, therefore V decreasing.

$V = Ed$ decreases

$$V_{\text{conductor}} = \frac{\sigma}{\epsilon_0}(d - b)$$

$$C_{\text{conductor}} = \frac{\epsilon_0 A}{d[1 - \frac{b}{d}]}$$

(when $b = d$, $C = \infty$, which is like connecting the two halves of the capacitor)



$$\sigma_{\text{pol}} = \frac{N \cdot A}{A} \cdot q \cdot \delta = |\vec{p}|$$
$$= \quad N \cdot q \cdot \delta$$

(σ_{pol} surface charge density due to polarization , δ distinct charges are pulled apart affects strongly)

$$\Sigma_{\text{allq}} q \cdot \vec{\delta} = N \cdot q \cdot \vec{\delta}$$

$$\vec{p}\alpha\vec{E} \quad \text{(major assumption: } \vec{p} \text{ constant, linear dielecetric)}$$

<u>Original</u>

$$\vec{E_0} = \frac{\vec{\sigma_0}}{\epsilon_0} \quad \text{not surface charge on plates}$$

<u>Dielectric</u>

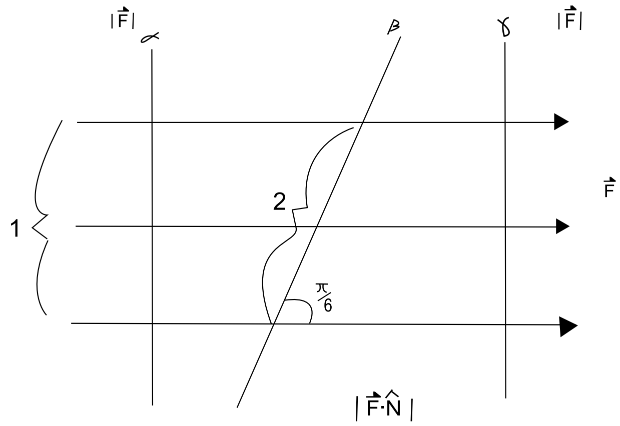
$$\vec{E_{\text{pol}}} = \frac{\vec{\sigma_0} - \vec{\sigma_{pol}}}{\epsilon_0} = \frac{\vec{\sigma_0} - \vec{p}}{\epsilon_0}$$

Define:

$$\vec{p} \equiv \chi \cdot \epsilon_0 \cdot \vec{E},$$
$$\vec{E}_{\text{pol}} = \frac{\sigma_0}{\epsilon_0}\frac{1}{1+\chi}$$
$$V = E_{\text{pol}} - d = \frac{\sigma_0 d}{\epsilon_0(1+\chi)}$$
$$\text{Since , } C = \kappa \cdot C_0$$
$$= \kappa \cdot \frac{\epsilon_0 A}{d} = C = \frac{\epsilon_0 A(1+\chi)}{d}$$
$$\kappa = 1 + \chi \qquad C_0 = \frac{\epsilon_0 A}{d}$$

Consider $\vec{p}$ not uniform

$$|\vec{\sigma_{\text{pol}}}| = \vec{p} = |(\vec{p} \cdot \hat{n})\hat{n}|$$



$$\Delta Q_{\text{pol}} = -\int_S \vec{p} \cdot \hat{n} = \int_V \rho_{\text{vol}} dV$$
$$= -\int_V \nabla \cdot \vec{p} dV$$
$$\text{So , } \rho_{\text{vol}} = -\nabla \cdot \vec{p}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$= \frac{\rho + \rho_{\text{vol}}}{\epsilon_0} \quad \text{two kinds of charge } (\rho_0 = \rho_{\text{other}})$$

$$= \frac{\rho_0 - \vec{\nabla} \cdot \vec{p}}{\epsilon_0}$$

$$\vec{\nabla} \cdot (\vec{E} + \frac{\vec{p}}{\epsilon_0}) = \frac{\rho_0}{\epsilon_0}, \quad \vec{p} = \chi(\vec{x}) \cdot \epsilon_0 \cdot \vec{E}$$

$$\nabla((1+\chi)\vec{E}) = \nabla(\kappa\vec{E}) = \frac{\rho_0}{\epsilon_0}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{p} = \epsilon_0 (1+\chi)\vec{E} = \epsilon_0 \kappa \vec{E}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_0$$

$\vec{D}$ is the divergence field lines end on <u>free</u> charges - lines not ending on the charges caused by polarization

$$\epsilon = \kappa\epsilon_0 \rightarrow \vec{D} = \epsilon\vec{E}$$

$$\epsilon = \epsilon(\vec{x})$$

$$\begin{cases} \text{free - not from polarization} \\ \text{bound - from polarization} \end{cases}$$