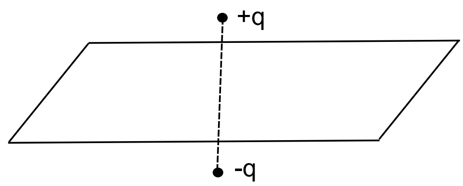
8.022 Lecture Notes Class 19 - 10/18/2006

HW!

$$\frac{d^2x}{dt^2} = \frac{A}{x^2}$$
$$-\frac{dx}{dt} = \sqrt{2A}\sqrt{\frac{1}{x} - \frac{1}{d}} \quad \text{blows up}$$



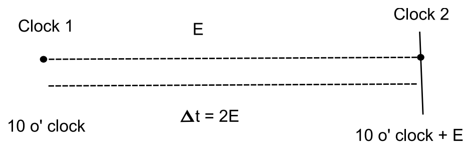
$A = \frac{2}{3} = \pi$

Well , it does blow up because mirror charges annihilate each other. Proceed by separation of variables.

Special Relativity

- c is constant
- all inertial reference frames ($a = 0 = F$) are equivalent

Einstein worked as a clerk in a patent office!

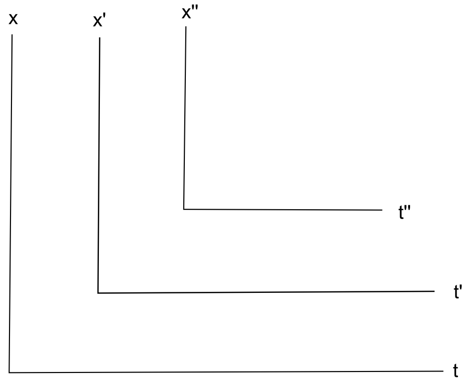


Galilean Transformation (Switch frames of Reference)

Suppose there is a space ship traveling with velocity in $\hat{x}$.

$$\begin{cases} x' = & x - vt \\ y' = & y \\ z' = & z \\ t' = & t \end{cases}$$

$$(x', y', z', t') = (x - vt, y, z, t)$$



Lorentz Transformation

Now, move space shuttle relativistically.

$$\begin{cases} x' = & Ax + Bt =^1 A(x - ut) \\ y' = & y \\ z' = & z \\ t' = & Dx + Et =^2 Dx + At =^3 -\frac{Au}{c^2}x + At \\ t' = & A(\frac{-u}{c^2}x + t) \\ \gamma = & \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \end{cases}$$

- $$\begin{aligned} x' &= 0 \\ Ax + Bt &= 0 \\ Aut + Bt &= 0 \\ Au &= -B \\ B &= -Au \end{aligned}$$

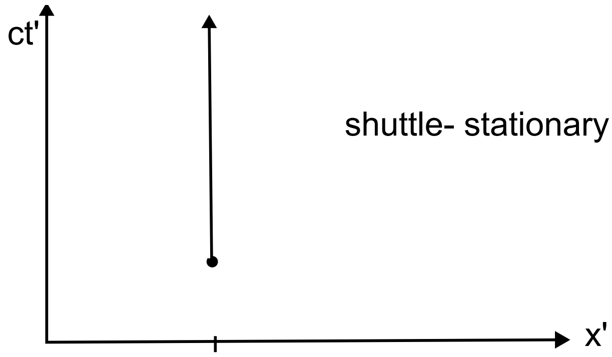
- $$\begin{aligned} x' &= -ut' \quad \text{at shuttle looking at ESG} \\ -u(Dx + Et) &= A(x - ut) \quad x = 0 \\ -uEt &= -uAt \\ E &= A \end{aligned}$$

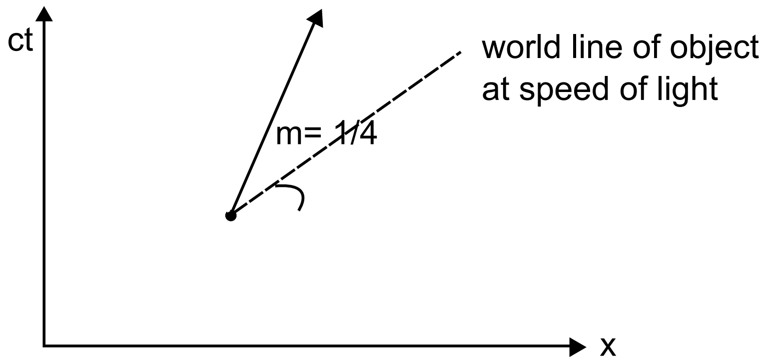
- $$\begin{aligned} x' &= ct' \\ A(x - ut) &= c(Dx + At) \\ A(ct - ut) &= c(Dct + At) \\ A(c - u) &= c(Dc + A) \\ Ac - Au &= c^2 D + Ac \\ D &= -\frac{Au}{c^2} \end{aligned}$$

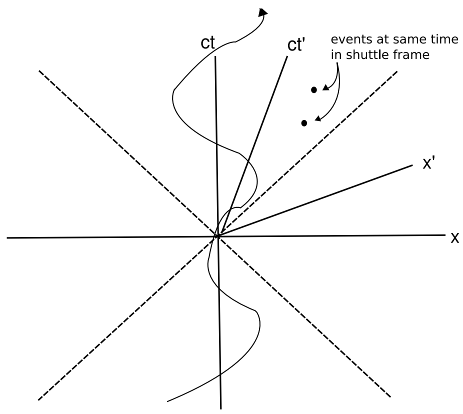
- $$x^2 + y^2 = (ct)^2 \quad x = 0$$
$$\Rightarrow y = ct$$
$$x^{2'} + y^{2'} = (ct')^2$$
$$A^2(x - ut)^2 + c^2t^2 = c^2A^2(\frac{-u}{c^2} + t)^2$$
$$A^2u^2t^2 + c^2t^2 = c^2A^2t^2$$
$$A^2u^2 + c^2 = c^2A^2$$
$$A^2 = \frac{c^2}{u^2 - c^2}$$
$$A = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma$$
$$\text{Also use } \beta = \frac{u}{c}$$

- How fast do you need to go for special relativity to apply? Look at γ : v , Need to be at least 30% of speed of light .
(Also, in $t' = \gamma(\frac{-u}{c^2}x + t)$, $\frac{-u}{c^2}$ is usually very small).

World Lines







This cone of permitted casuality tells us that information can travel no faster than the speed of light

In our frame, 0 and 0 are at different times

In space shuttle, 0 and 0 are at the same time