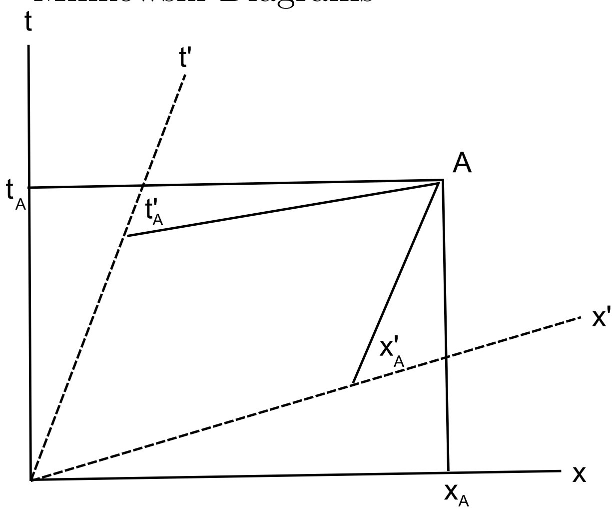
8.022 Lecture Notes Class 20 - 10/19/2006

Reverse Lorentz Transformation

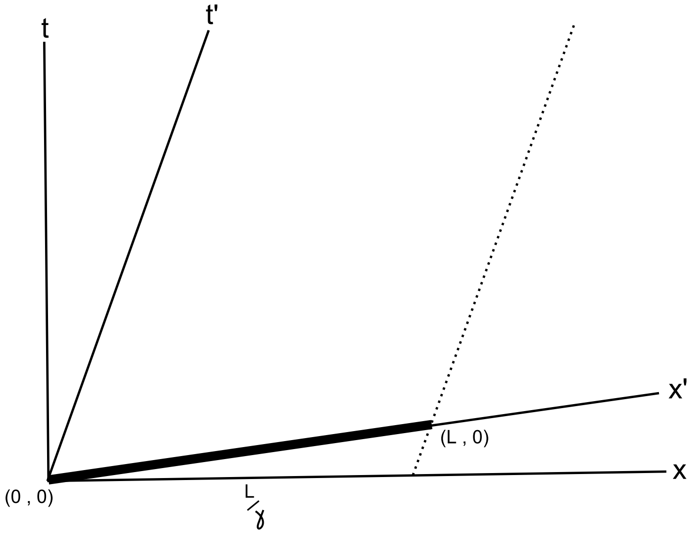
$$\begin{cases} x = & \gamma(x + ut') \\ y = & y' \\ z = & z' \\ t = & \gamma(t' + \frac{ux'}{c^2}) \end{cases}$$

Minkowski Diagrams -



(x' slanting up shows that there is a positive velocity)

Relativistic Plank



- $$t = 0 : L \to R : (0,0) \to (\gamma L, ..)$$

$$t = \frac{1}{v}x - \frac{L}{\gamma v} \to x = \frac{L}{\gamma} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > 1$$

- $$(0, t_a)_S$$

$$t_a = \gamma(t'_a + \frac{vx'}{c^2})$$

$$t_a = \gamma t'_a > t'_a$$

Measuring Intervals

$$c^2(\Delta t')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2$$
$$= c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = (\Delta s)^2$$

Relativistic Velocity

$$x = \gamma(x' + vt') \qquad t = \gamma(t' + \frac{v}{c^2}x')$$

$$dx = \gamma(dx' + vdt')dt = \gamma(dt' + \frac{v}{c^2}dx')$$

$$v = \frac{dx}{dt} = \frac{\gamma(dx'+vdt')}{\gamma(dt'+\frac{v}{c^2}dx')} \cdot \frac{dt'}{dt'}$$

$$v_x = \frac{v'+u}{1+\frac{u}{c^2}v'}$$

$$v_y = \frac{v'_y}{\gamma(1+\frac{uv'_x}{c^2})}$$

$$v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}} \longleftrightarrow v_x = \frac{v'u}{1 + \frac{u}{c^2}v'}$$

$$\frac{dy}{dt} = \frac{dy'}{\gamma(dt'+\frac{udx'}{c^2})} \cdot \frac{dt'}{dt'}$$

$$= \frac{v_y}{\gamma(1+\frac{u \cdot v_x}{c^2})}$$