8.022 Lecture Notes Class 24 - 10/26/2006

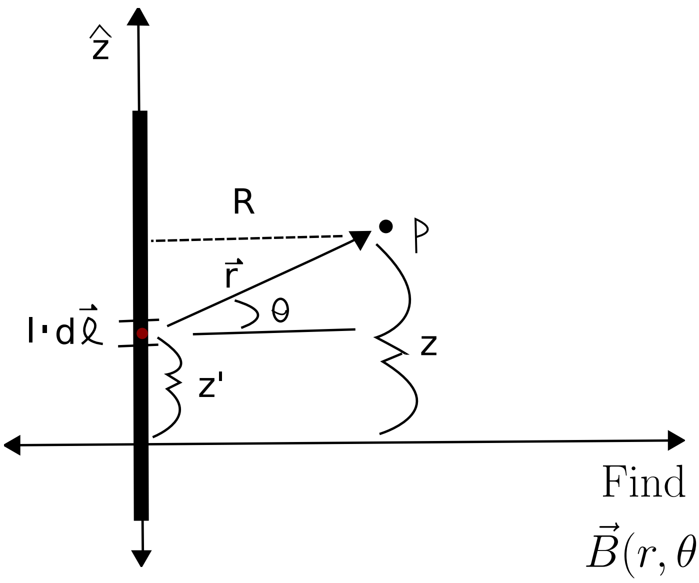
Biot-Savart Law

- works for *steady-state current*

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{Id\vec{l}\times\hat{r}}{r^2}$$
$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{K\cdot d\vec{a}\times\hat{r}}{r^2}$$
$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}d\tau\times\hat{r}}{r^2}$$

$\vec{J}$ is current density

- Not steady-state, since one charge flies by and then no current (I = $\delta(t)$)



Find $\vec{B}(p) = \vec{B}(r,\theta,z)$

$$\vec{B}(r,\theta,z) = \frac{\mu_0}{4\pi}\int_{\text{wire}} \frac{Id\vec{l}\times\vec{r}}{r^3}$$

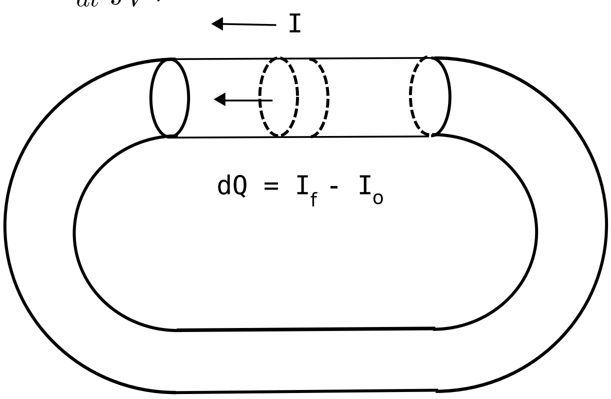
$$\vec{B}(r,\theta,z) = \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty} \frac{I\hat{z}\times(\vec{R}+\vec{z})}{(R^2+(z-z')^2)^{3/2}}dz'$$
$$= \frac{\mu_0}{4\pi}I\cdot\hat{\theta}R\int_{-\infty}^{\infty} \frac{dz'}{(R^2+(z-z')^2)^{3/2}}$$

$\frac{\mu_0 IR}{4\pi} \cdot \frac{2\hat{\theta}}{R^2} = \frac{\mu_0 I\hat{\theta}}{2\pi R}$

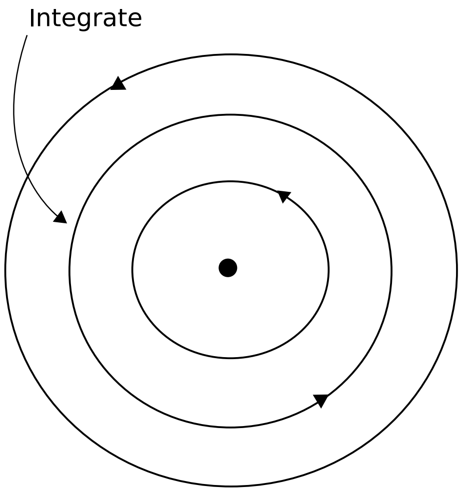
$$\begin{aligned} I &= \tfrac{dQ}{dt} = qn_L \cdot v \\ \vec{J} &= q \cdot n_L \cdot \vec{v} \\ dI &= \vec{J} \cdot dA \\ \vec{\nabla} \cdot J &= -\tfrac{d}{dt}\rho \end{aligned}$$

- Tells you about conservation of charge

$$\oint dI = \oint \vec{J} \cdot d\vec{A} = \int_V \vec{\nabla} \cdot \vec{J} d^3x$$
$$= -\tfrac{d}{dt} \int_V \rho d^5x$$



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$ Ampere's Law (for wire-nice symmetries)



$$\oint \vec{B} \cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi r} dl \cdot \hat{\theta}\hat{\theta}$$
$$= \frac{\mu_0 I}{2\pi r} \int_0^{2\pi r} dl$$
$$= \frac{\mu_0 I}{2\pi r} 2\pi r$$