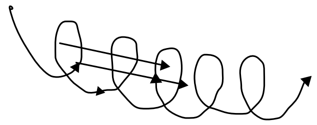
8.022 Lecture Notes Class 26 - 10/31/2006

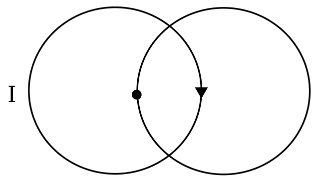


N Turns, Length L

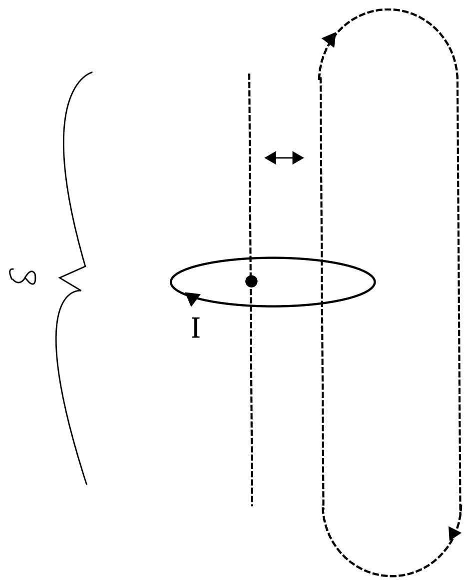
B field lines

$$|\vec{B}| = N\frac{\mu_0 I}{2\pi R}$$

- Single Loop ? $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$



Can't do this b/c of "edge" effect



If we have enough loops, no edge effects!

each gives $\mu_0 IN$

- Use infinite loop for Ampere's - location inside does not matter, so max $\vec{B}$ in solenoid

$$|\vec{B}| = \mu_0\frac{IN}{L} = \mu_0 In \quad (n = \frac{N}{L}\text{ , "turn density" })$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = 0$$
$$\vec{\nabla} \cdot E = \frac{\rho}{\epsilon_0}$$

$$\Phi = V = -\oint E \cdot dl$$
$$\vec{E} = - \qquad \vec{\nabla} V$$

So:

$$\vec{\nabla} \times E = \vec{\nabla} \times (\vec{\nabla} V) = 0 \text{ , b/c gradient of scalar is 0}$$

So we could write similarly,

$$\vec{\nabla} \times \vec{B} = \vec{\nabla} \times (\vec{\nabla}_{--}) = 0$$

Use :

$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$$
$$\vec{B} = (\vec{\nabla} \times \vec{A})$$

Multiple A's have curls equal to B, so can choose easiest $\vec{\nabla} \cdot \vec{A}$,
Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$
$\vec{B} = B_0 \hat{z}$ Find $\vec{A}$

$$\vec{A} = B_0 \times \hat{y} \text{ or } - B_0 y \hat{x}$$

$$\vec{A}_c = \frac{B_0}{2}(x\hat{y} - y\hat{x})$$

$$\begin{aligned}
\vec{\nabla} \times \vec{B} &= \mu_0 J \\
\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) &= \mu_0 \vec{J} \\
&= \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 A
\end{aligned}$$

$$\begin{aligned}
\nabla^2 \vec{A} &= -\mu_0 \vec{J} \quad \text{(Poisson)} \\
\nabla^2 \vec{A} &= 0 \quad \text{(Laplace)}
\end{aligned}$$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{x'}) d^3x'}{|\vec{x} - \vec{x'}|}$$

A is the magnetic vector potential