8.022 Lecture Notes Class 37 - 11/22/2006

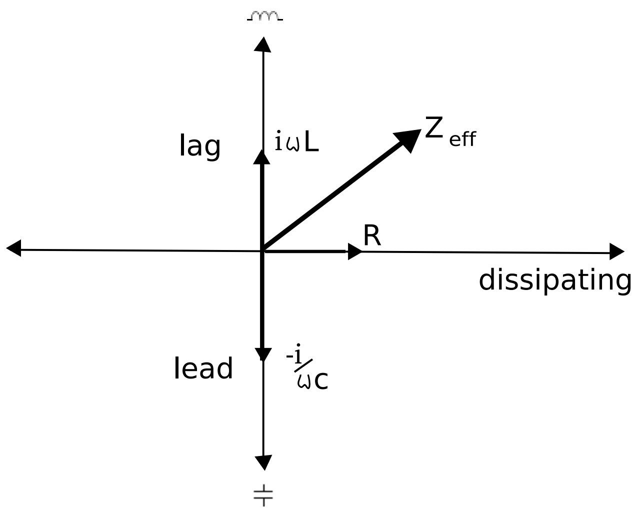
Complex impedence instead of diff eq!
Use fact that everything in RLC circuit has same frequency as driving frequency(?).

$$V(t) = \hat{V}e^{i\omega t} \qquad I(t) = \hat{I}e^{i\omega t}$$

<u>Inductor</u>

$$V = L\frac{dI}{dt}$$
$$\frac{dI}{dt} = i\omega I$$
$$\frac{V}{I} = i\omega L = \chi_L \quad \text{(complex impedence of inductor)}$$

<u>Capacitor</u>



$$\frac{dV}{dt} = \frac{d}{dt}\frac{Q}{C} = \frac{I}{C}$$
$$i\omega V = \frac{I}{C}$$

complex impedence of capacitor

$$\frac{V}{I} = \frac{1}{i\omega C} = \chi_c$$

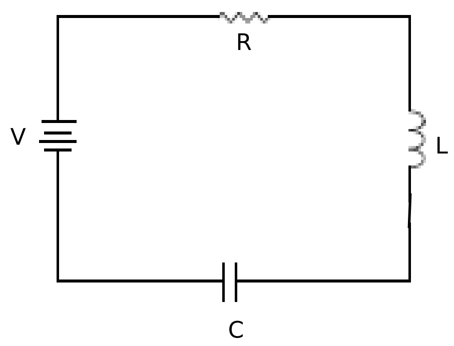
<u>Resistor</u>

$$\frac{V}{I} = R$$

$$V = I \cdot z$$

$$\begin{aligned} \chi_L &= i\omega L \\ \chi_C &= \tfrac{1}{i\omega C} \\ \chi_R &= R \end{aligned}$$

RLC Circuit



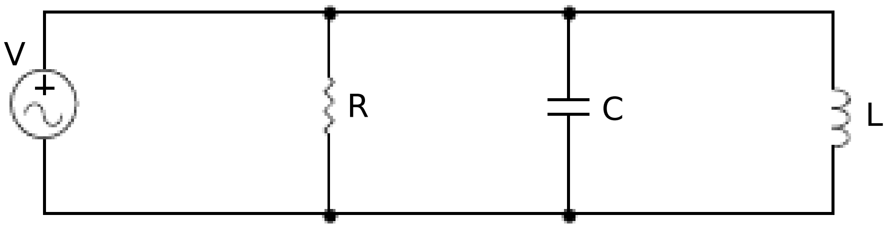
No derivatives any more! Can sum just like resistors in series.

$$\chi_{total} = \chi_R + \chi_C + \chi_L = R + i\omega L + \frac{1}{i\omega C}$$

$$\begin{aligned} I &= \tfrac{V}{\chi_{total}} \\ &= \tfrac{V}{R+i(\omega L - \frac{1}{\omega C})} \cdot \tfrac{R-i(\omega L-\frac{1}{\omega C})}{R-i(\omega L-\frac{1}{\omega C})} \\ &= \tfrac{V(R-i[\omega L-\frac{1}{\omega C}])}{R^2+(\omega L-\frac{1}{\omega C})^2} \end{aligned}$$

$$\hat{I} = \frac{V}{[R^2+(\omega L-\frac{1}{\omega C})^2]^{1/2}}$$
$$\tan\phi = \frac{\omega L-\frac{1}{\omega C}}{R}$$

Parallel RLC Circuit



Let $Y = \frac{1}{\chi}$, $I = V \cdot Y$

admittance current

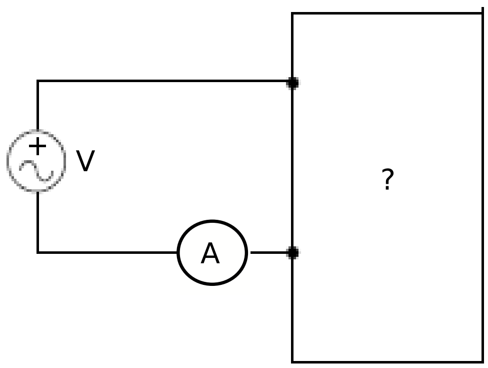
$$Y_L = \frac{1}{i\omega L}$$
$$Y_C = i\omega C$$
$$Y_R = \frac{1}{R}$$

$$I = V(\frac{1}{R} + i(\omega C - \frac{1}{\omega L}))$$

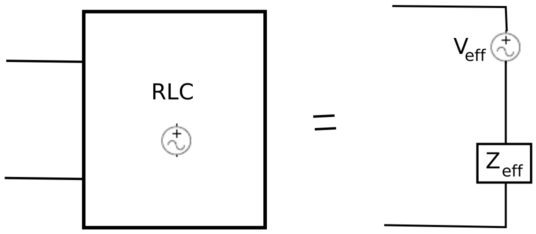
$$\hat{I} = V(\frac{1}{R^2} + (\omega C + \frac{1}{\omega L})^2)^{1/2}$$
$$\tan\phi = R\omega C - \frac{R}{\omega L}$$

Large ω : $\frac{1}{L}$, $V\omega C$ is important.
Small ω : no C , $\frac{V}{\omega L}$ important.

Can we do equivalent of Thevenin's?



$$\frac{V}{I} = \chi_{effective}$$



$$z_{eff} = R_{eff} + i\chi eff$$

First term decays, second term oscillates.

Power Dissipation

R does this! (LC circuit just oscillates, even w/o driver no loss of power).

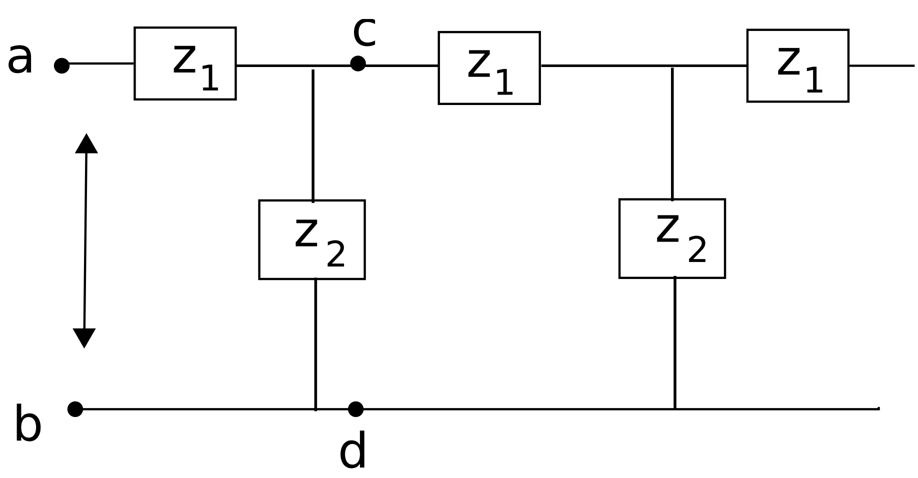
$$\frac{dV}{dt} = RI^2 \qquad (= VI)$$

$$z = R = i\chi$$

$$\begin{aligned} z =& \quad i\chi \\ V =& \quad i\chi I \\ \hat{V}e^{i\omega t} =& \; \chi \hat{I} e^{i\omega t + \frac{\pi}{2}} \end{aligned}$$

$$\begin{aligned} < P >_{avg} =& \quad \tfrac{1}{T}\int_0^T V \cdot I dt \\ =& \int_0^T \hat{I}^2 R\cos^2(\omega t)dt - \tfrac{1}{T}\int_0^T \chi \hat{I}^2 \cdot \cos\omega t \sin\omega t dt \\ =& \quad \tfrac{\hat{I}^2 R}{2} \end{aligned}$$

Ladder Impedence



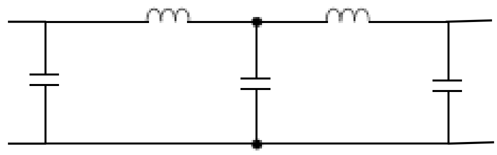
$$z = z_1 + \frac{z_2 z}{z_2 + z}$$

Solve:

$$z = \frac{z_1}{z} + \sqrt{\frac{z_1^2}{4} + z_1 z_2}$$

Let:

$$z_1 = i\omega L$$
$$z_2 = \tfrac{1}{i\omega C}$$



- $$z = \frac{i\omega L}{2} + \sqrt{\frac{-\omega^2 L^2}{4} + \frac{L}{C}}$$

- $$v < 0 \quad \text{for } \frac{\omega^2 L^2}{4} > \frac{L}{C}$$

$$\omega^2 > \frac{4}{LC}$$

- for $\omega^2 < \frac{4}{LC}$, there's a real part = resistance! But from only $L = C$? It's because its infinite! Energy keeps traveling out for certain ω !

Critical Frequency - if you are under, energy will just keep going oout. Otherwise, will go out and come back.