8.022 Lecture Notes Class 38 - 11/27/2006

$$V = \frac{LI^2}{2}$$

$$\Phi_B = \int_S \vec{B} \cdot d\vec{a} = \oint_l \vec{A} \cdot d\vec{l} = LI$$

$$\begin{aligned} U = & \quad \tfrac{I}{2} \oint_S \vec{A} \cdot d\vec{l} = \tfrac{1}{2} \oint \vec{A}(I \cdot d\vec{l}) \\ = & \quad \tfrac{1}{2} \int \vec{A} \cdot \vec{J} d^3x| \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \\ U = & \quad \tfrac{1}{2\mu_0} \int \vec{A} \cdot (\vec{\nabla} \times \vec{B}) d^3x \\ = & \quad \tfrac{1}{2\mu_0} [\int \vec{B} \cdot \vec{B} d^3x - \int \vec{\nabla} \cdot (\vec{A} \times \vec{B}) d^3x] \end{aligned}$$

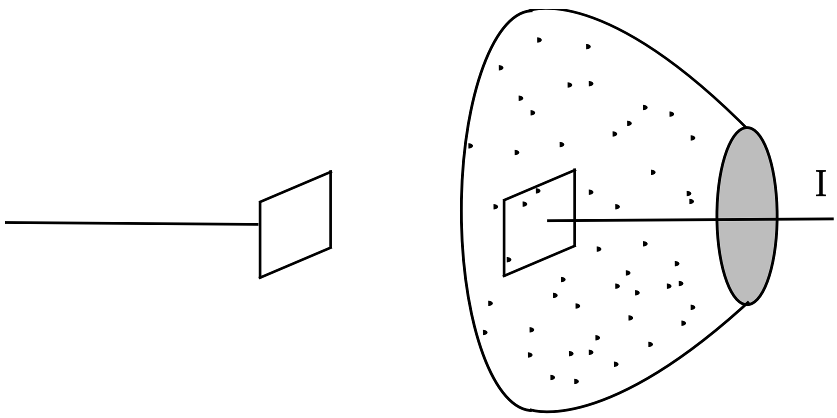
Expand to all over space

$$\begin{aligned} = & \quad [\int_{\text{allspace}} B^2 d^3x - \int_{\text{allspace}} \vec{\nabla}(\vec{A} \times \vec{B}) d^3x] \\ = & \quad (\int_{\text{allspace}} B^2 d^3x - \oint_{\text{surface}} \vec{A} \times \vec{B} d^2x \end{aligned}$$

Choose surface to be WAAAAAY out so it goes to nothing

$$\boxed{U = \frac{1}{2\mu_0} \int B^2 d^3x}$$

$$
\begin{aligned}
\vec{\nabla} \cdot \vec{B} &= \quad 0 \\
\vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \quad \text{Gauss} \\
\vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \quad \text{Faraday} \\
\vec{\nabla} \times \vec{B} &= \mu_0 \vec{J} \quad \text{Ampere}
\end{aligned}
$$



For rightmost side of surface, theres is a current, so $\vec{\nabla} \times \vec{B} \neq 0$. For leftmost side, there is no current, so $\vec{\nabla} \times \vec{B} = 0$.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Take Divergence of both sides.

$$
\begin{aligned}
\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) &= \mu_0 \cdot \vec{\nabla} \cdot \vec{J} \\
0 &= \mu_0 \cdot \vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}
\end{aligned}
$$

Divergence of current is 0? Constant current!

Okay, so something's going from one plate to the other.

$$\begin{aligned}\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) &= -\tfrac{\partial}{\partial t}\epsilon_0\vec{\nabla}\cdot\vec{E}\mu_0\\ &= -\vec{\nabla}\cdot\epsilon_0\mu_0\tfrac{\partial\vec{E}}{\partial t}\end{aligned}$$

So ,

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$$

Call this thing going through th plates Displacement Current.

$$\vec{J}_D = \epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\oint\vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enclosed}} + \mu_0 I_{\text{D,enclosed}}$$

$$\mu_0 I_{\text{D,enc}} = \epsilon_0\frac{\partial\vec{E}}{\partial t}d\vec{a}$$

Source-free Maxwell's equations!

$$\boxed{\begin{array}{ll}\vec{\nabla}\cdot\vec{E}=0 & \vec{\nabla}\times\vec{E}=\frac{\partial B}{\partial t}\\ \vec{\nabla}\cdot\vec{B}=0 & \vec{\nabla}\times\vec{B}=\mu_0\epsilon_0\frac{\partial E}{\partial t}\end{array}}$$

$$\begin{aligned}\frac{dW}{dt} &= \int_V\vec{E}\cdot\vec{J}d^3x\\ &= \int_V\vec{E}\cdot(\tfrac{1}{\mu_0}\vec{\nabla}\times\vec{B}-\epsilon_0\tfrac{\partial E}{\partial t})d^3x\end{aligned}$$

$$\begin{aligned} dW = & \quad \vec{F} \cdot \vec{v} dt \\ = & \quad \vec{E} q \vec{v} dt \\ = & \; \vec{E} \cdot \vec{J} d^3 x dt \end{aligned}$$

$$\vec{v} \cdot (\vec{v} \times \vec{B}) = 0$$

So, magnetic fields do no work

$$q\vec{v} = \vec{J} d^3 x$$

Charge velocity is current

$$\begin{aligned} \vec{\nabla} \cdot (\vec{E} \times \vec{B}) = & \; \vec{B} \cdot (\nabla \times \vec{E}) - \vec{E}(\vec{\nabla} \times \vec{B}) \\ \vec{E} \cdot (\vec{\nabla} \times \vec{B}) = & \quad -\nabla(\vec{E} \times \vec{B}) - \vec{B}\tfrac{\partial \vec{B}}{\partial t} \end{aligned}$$

$$\begin{aligned} \vec{B}\frac{\partial \vec{B}}{\partial t} = & \; \tfrac{1}{2}\tfrac{\partial}{\partial t} \vec{B} \cdot \vec{B} \\ = & \quad \tfrac{1}{2}\tfrac{\partial}{\partial t} B^2 \end{aligned}$$

$$\vec{E}\frac{\partial \vec{E}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t} E^2$$

$$
\begin{aligned}
\frac{dW}{dt} &= \int(-\tfrac{1}{\mu_0}\vec{\nabla}\cdot(\vec{E}\times\vec{B}) - \tfrac{1}{2\mu_0}\tfrac{\partial}{\partial t}B^2 - \tfrac{\epsilon_0}{2}\tfrac{\partial}{\partial t}E^2)d^3x \\
&= -\tfrac{\partial}{\partial t}\int[\tfrac{1}{2\mu_0}B^2 + \tfrac{\epsilon_0}{2}E^2]d^3x + \int_V \tfrac{1}{\epsilon_0}\vec{\nabla}(\vec{E}\times\vec{B})d^3x \\
&= -\tfrac{\partial}{\partial t}\int[\tfrac{1}{2\mu_0}B^2 + \tfrac{\epsilon_0}{2}E^2]d^3x + \oint -\tfrac{\vec{E}\times\vec{B}}{\mu_0}d\vec{a} \\
&= -\tfrac{\partial}{\partial t}U_{E,M} - \oint \vec{S}d\vec{a}
\end{aligned}
$$