8.022 Lecture Notes Class 39 - 11/28/2006

Poynting Vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

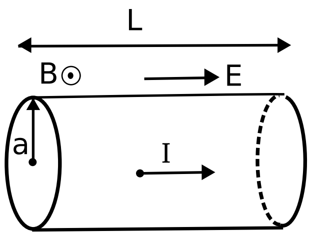
$$\begin{aligned}\frac{dW}{dt} &= \qquad -\tfrac{dU_{\text{E,M}}}{dt} - \oint \vec{S}\cdot d\vec{a} \\ &= \tfrac{d}{dt}\int U_{\text{mechanical}} d^3x \qquad \text{by def}\end{aligned}$$

$$\frac{d}{dt}\int U_{\text{mechanical}} + U_{\text{EM}} d^3x = -\oint \vec{S}\cdot d\vec{a} = -\oint_V \vec{\nabla}\cdot\vec{S} d^3x \quad \text{by Gauss}$$

Poynting Vector $\Longleftrightarrow$ Energy Flow

$$\frac{\partial}{\partial t}[U_{\text{Mechanical}} + U_{\text{EM}} = -\vec{\nabla}\cdot\vec{S}]$$

Like $\frac{\partial}{\partial t}\rho = -\vec{\nabla}\cdot\vec{J}$



Wire with constant current.

$$\begin{aligned}\oint S\cdot da = \frac{2\pi a L\cdot VI}{2\pi a L} &= VI \\ &= \tfrac{\partial}{\partial t}U\end{aligned}$$

$$|\vec{B}| = \tfrac{\mu_0 I}{2\pi a}$$
$$|\vec{E}| = \tfrac{V}{L}$$

If you've got charges and potential field, gives/takes energy to move through potential.

$$\oint \vec{S} \cdot d\vec{a} = 2\pi a L \cdot S$$

$$|\vec{S}| = \frac{1}{\mu_0} E \cdot B = \frac{1}{\mu_0} \cdot \frac{V}{L} \cdot \frac{\mu_0 I}{2\pi a} = \frac{VI}{2\pi a L}$$

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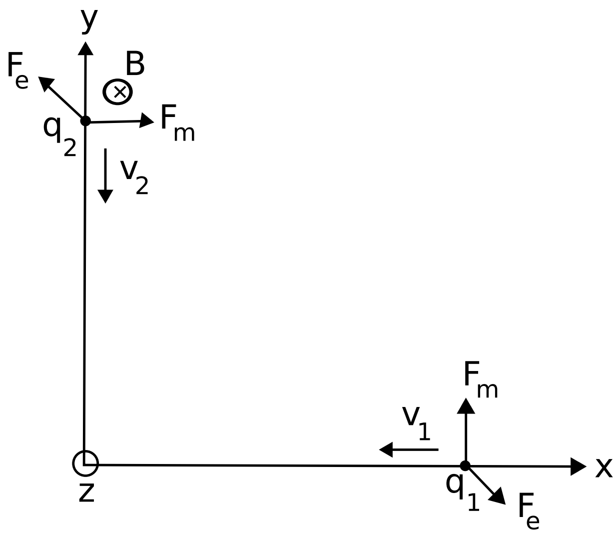
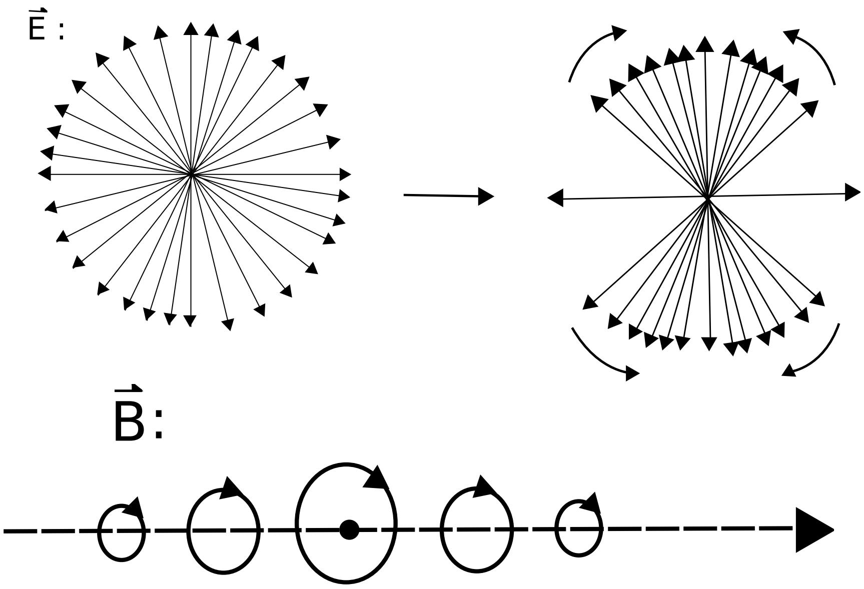


Wait !! F_m's are not anti-parallel violated Newton's third law

<u>Rest frame of charges</u>



What if fields push back on charges too?
Fields have momentum too!

$$\begin{aligned}
\vec{F} &= \int_V (\vec{E} + \vec{v} \times \vec{B}) \rho d^3x \\
&= \int_V (\rho \vec{E} + \vec{J} \times \vec{B}) d^3x \qquad \vec{v} \cdot \rho = \vec{J} \\
f &= \rho \vec{E} + \vec{J} \times \vec{B} \\
&= \epsilon_0 (\vec{\nabla} \cdot \vec{E}) \vec{E} + (\tfrac{1}{\mu_0} \vec{\nabla} \times \vec{B} - \epsilon_0 \tfrac{\partial \vec{E}}{\partial t}) \times \vec{B}
\end{aligned}$$

Same mathematical tricks as last time

$$\begin{aligned}
\frac{\partial}{\partial t}(\vec{E} \times \vec{B}) &= \tfrac{\partial E}{\partial t} \times \vec{B} + E \times \tfrac{\partial B}{\partial t} \; , \; \tfrac{\partial B}{\partial t} = -\nabla \times E \text{ from 4 eqs.} \\
\frac{\partial E}{\partial t} &= \tfrac{\partial}{\partial t}(\vec{E} \times \vec{B}) + E \times (\nabla \times E)
\end{aligned}$$

$$f = \epsilon_0 [(\vec{\nabla} \cdot \vec{E}) \vec{E} - \vec{E} \times (\vec{\nabla} \times \vec{E})] - \frac{1}{\mu_0} [(\vec{\nabla} \cdot \vec{B}) \vec{B} - \vec{B} \times (\vec{\nabla} \times \vec{B}) - \epsilon_0 \frac{\partial}{\partial t} (\vec{E} \times \vec{B})]$$

$$\vec{\nabla}(E^2) = \qquad 2(\vec{E}\cdot\vec{\nabla})\vec{E} + 2\vec{E}\times(\vec{\nabla}\times\vec{E})$$

$$\vec{E}\times(\vec{\nabla}\times\vec{E}) = \tfrac{1}{2}\nabla(E^2) - (\vec{E}\cdot\vec{\nabla})\vec{E} \qquad \partial_i B = \nabla\cdot B \neq B\cdot\nabla = B_i\partial_i$$

$$f = \epsilon_0[(\vec{\nabla}\cdot\vec{E})\vec{E} - (\vec{E}\cdot\vec{\nabla})\vec{E}] + \frac{1}{\mu_0}[(\vec{\nabla}\cdot\vec{B})\vec{B} - (\vec{B}\cdot\vec{\nabla})\vec{B} - \frac{1}{2}\nabla(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2) - \epsilon_0\frac{\partial}{\partial t}(\vec{E}\times\vec{B})$$

(Represents stresses and strains!)

Maxwell Stress Tensor - describes stresses and strains of electromagnetic fields

$$T = \begin{pmatrix} T_{xx} & & Txz \\ & T_{yy} & \\ T_{zx} & & T_{zz} \end{pmatrix}$$

$$T_i = \epsilon_0(E_iE_j - \frac{1}{2}\delta ij E^2 + \frac{1}{\mu_0}(B_iB_j - \frac{1}{2}\delta ij B^2$$

$$(\vec{a}\cdot T)_j = \Sigma a_i T_{ij}$$

$$(\vec{\nabla}\cdot T)_j = \epsilon_0[(\nabla\cdot E)E_j + (E\cdot\nabla)E_j - \frac{1}{2}\delta_{ij}E^2] + \frac{1}{\mu_0}[(\nabla\cdot B)B_j + (B\cdot\nabla)B_j - \frac{1}{2}\delta_{ij}B^2]$$

So,

$$f = \vec{\nabla}\cdot T - \epsilon_0\mu_0\frac{\partial S}{\partial t}$$

$$\vec{F} = \oint \vec{T}\cdot d\vec{a} - \epsilon_0\mu_0\frac{d}{dt}\int_V \vec{\delta}\delta^3 x$$