8.022 Lecture Notes Class 4 - 09/12/2006

Dot Operator

$$\begin{aligned} dT &= \tfrac{\partial T}{\partial x}dx + \tfrac{\partial T}{\partial y}dy + \tfrac{\partial T}{\partial z}dz \\ &= \nabla T \cdot dl \\ \nabla T &= 0 \text{ stationary point} \\ \nabla T &= (\tfrac{\partial T}{\partial x}, \tfrac{\partial T}{\partial y}, \tfrac{\partial T}{\partial z}) \end{aligned}$$

What's T ? A scalar function of a vector (usually $\Re^3$, sometimes $\Re^2$)

$$\begin{aligned} T = T(x,y,z) \rightarrow \quad & \rho(x,y,z) \text{ charge density} \\ V(x,y,z) = \ & \phi(x,y,z) \text{ electric potential} \end{aligned}$$

$\vec{E}$,$\vec{B}$ are vectors.
$\vec{\nabla}\phi = \vec{E}$
Griffiths:

Taylor Series

$$\begin{aligned} f(x+a) &= \Sigma_{n=0}^{\infty} \tfrac{a^n}{n!} \cdot \tfrac{\partial^n f(x)}{\partial x^n} \\ f(\vec{x}+\vec{a}) &= f(\vec{x}) + \vec{a}\cdot(\vec{\nabla}f) + \tfrac{\vec{a}\cdot\vec{a}}{2!}(\vec{\nabla}\cdot\vec{\nabla})f \\ &= f(\vec{x}) + (\vec{a}\cdot\vec{\nabla})f + \tfrac{(\vec{a}\cdot\vec{\nabla})^2}{2!}f \\ &= \Sigma_{n=0}^{\infty}\tfrac{1}{n!}(\vec{a}\cdot\vec{\nabla})^n f(\vec{x}) \\ \vec{E}(\vec{x}+\vec{a}) &= \Sigma_{n=0}^{\infty}\tfrac{1}{n!}(\vec{a}\cdot\vec{\nabla})^n E(\vec{x}) \end{aligned}$$

Index Notation

$$\vec{A} \times \vec{B} = \epsilon_{ijk} A_j B_k$$

↑ What Griffiths uses later on... Don't Use It!
$||\vec{r}|| = \sqrt{r_i r_i}$

<u>More Notation</u>

$\frac{\partial}{\partial x_i}\vec{e_j} = 0$, TRUE ONLY IN CARTESIAN COORDINATES

$\frac{\partial}{\partial x_i} = \partial_i$

<u>Divergence</u>

$$\vec{\nabla} \cdot \vec{f} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\oint \vec{F} \cdot d\vec{A} = (\vec{\nabla} \cdot \vec{F})\epsilon^3$$

<u>Curl</u>

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ F_x & F_y & F_z \end{vmatrix} .$$

<u>Laplacian</u>

$$\begin{aligned} \nabla \cdot (\nabla T) &= \tfrac{\partial^2 T}{\partial x^2} + \tfrac{\partial^2 T}{\partial y^2} + \tfrac{\partial^2 T}{\partial z^2} = \vec{\nabla}^2 T \\ &= \tfrac{\partial^2 T}{\partial x_i \partial x_i} = \partial_i \partial_i T \end{aligned}$$

$$\begin{aligned} \nabla(\nabla\phi) &= \nabla E = \rho \\ \vec{0} = \nabla \times (\nabla T) &= \epsilon_{ijk}\partial_i\partial_j T \\ &\quad \partial_x\partial_y T - \partial_y\partial_x T = 0 \end{aligned}$$

$\vec{\nabla}(\vec{\nabla} \cdot \vec{v})$ gradient of divergence

$\vec{\nabla} \times (\vec{\nabla} \times \vec{v}) = \nabla(\nabla \cdot v) - \nabla^2 v$ curl of curl

$$\nabla \times \left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} \right) = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial T}{\partial x} & \frac{\partial T}{\partial y} & \frac{\partial T}{\partial z} \end{vmatrix}.$$

$= (\frac{\partial^2 T}{\partial y \partial z} - \frac{\partial^2 T}{\partial z \partial y})\vec{e_x} + (\frac{\partial^2 T}{\partial x \partial z} - \frac{\partial^2 T}{\partial z \partial x})\vec{e_y} + (\frac{\partial^2 T}{\partial x \partial y} - \frac{\partial^2 T}{\partial y \partial x})\vec{e_z}$

$= \vec{0}$