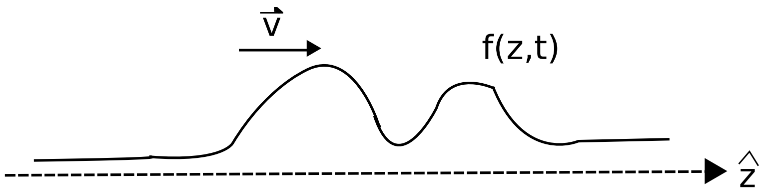
8.022 Lecture Notes Class 41 - 12/5/2006

Electromagnetic Waves

Waves

- linear
- periodic functions
- carrying "something"



$$f(z,t) = g(z - vt) = g(u) = g(u(z,t))$$

$$u = z - vt$$

$$\frac{\partial f}{\partial z} = \frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial z} = \frac{\partial g}{\partial u}$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{d^2}{dv^2}$$

$$\frac{\partial f}{\partial t} = \frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial t} = -v \cdot \frac{\partial g}{\partial u}; \frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 g}{\partial u}$$

Wave equation

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}$$

$$\begin{aligned} f(z,t) &= g(z-vt) \\ &= h(z+vt) \\ f(z,t) &= g(z-vt)+h(z+vt) \quad \text{linearity} \end{aligned}$$

Have a wave:

$$f(z,t) = A\cos[k(z-vt)+\delta]$$

δ - phase constant/shift
k - wave number
$\lambda = \frac{2\pi}{k}$, wavelength
$\nu = \frac{v}{\lambda}$, frequency
$\omega = 2\pi\nu$, angular frequency

$$f(z,t) = A\cos[kz - \omega t + \delta]$$

$$\hat{f}(z,t) = \hat{A}e^{i(kz-\omega t)} \ ; \quad f = Re(\hat{f})$$

Fourier Integral

$$\hat{f}(z,t) = \int_{-\infty}^{\infty} dk\hat{A}(a)e^{i(kz-\omega t)}$$

$$\begin{aligned} \hat{f}_I &= \hat{A}_I e^{i(kz-\omega t)} \qquad \text{incoming} \\ \hat{f}_R &= \hat{A}_R e^{i(kz-\omega t)} \qquad \text{reflected} \\ \hat{f}_T &= \hat{A}_T e^{i(kz-\omega t)} \qquad \text{transmitted} \end{aligned}$$

$$\hat{f_{total}} = \begin{cases} \hat{f}_I + \hat{f}_R, z < 0 \\ \hat{f}_T, z > 0 \end{cases}$$

Because of knot connecting two strings,

- $$\hat{f}(0^-, t) = \hat{f}(0^+, t)$$
- $$\frac{\partial \hat{f}}{\partial z}|_{0^-} = \frac{\partial \hat{f}}{\partial z}|_{0^+} \qquad \text{massless knot}$$
  if it had mass, would possibly make pushing on one side harder
- $$\hat{A}_I + \hat{A}_R = \hat{A}_T$$
  (look at $z = 0$, cancel out $e^{-i\omega t}$'s)
- $$k_l(\hat{A}_I - \hat{A}_R) = k_z \hat{A}_T$$
  (differentiate at $z = 0$)
  if strings are same material, same k's, so no reflection

Solve :

$$\text{Relationship: } \frac{k_2}{k_1} = \frac{v_1}{v_2}$$

$$\begin{aligned} \hat{A}_R &= \tfrac{k_1-k_2}{k_1+k_2}\hat{A}_I \\ &= \tfrac{v_1-v_2}{v_1+v_2}\hat{A}_I \\ \hat{A}_R &= \tfrac{2v_2}{v_1+v_2}\hat{A}_I \end{aligned}$$

Problem

Real :

$$A_R = \frac{v_1 - v_2}{v_1 + v_2}, A_T = \frac{2v_2}{v_1 + v_2}A_I$$

Why is $A_R = -\hat{A}_R$? reflected wave is upside down

Exam # 3

2)d.

$$T_{ij} = \epsilon_0(E_iE_j - \frac{1}{2}\delta_{ij}E^2) + ...''B$$

$$\vec{E} = E_x\hat{x}$$

$$\begin{pmatrix} \frac{1}{2}E^2 & & \\ & -\frac{1}{2}E^2 & \\ & & -\frac{1}{2}E^2 \end{pmatrix}$$

positive ones are passive,
negative is tension