**8.022 Lecture Notes Class 5 - 09/13/2006**

$$\int_a^b \vec{v}\cdot d\vec{l} \qquad \iint_S \vec{v}\cdot d\vec{a} \qquad \iiint_V T d\vec{\tau}$$

$$\begin{aligned}\int_a^b \frac{df}{dx}dx =&\ f(b)-f(a)\\ \int_a^b (\vec{\nabla}T)\cdot d\vec{l} =&\ T(b)-T(a)\\ (\vec{\nabla}T)\cdot d\vec{l} =&\ \qquad dT\end{aligned}$$

$\int_a^b \vec{v}(\vec{x})\cdot d\vec{l}$ is path-independent iff $\vec{v}(\vec{x}) = \vec{\nabla}f(\vec{x})$ .

$$\frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{x'})\vec{r}}{r^3}d^3x = \vec{E}(\vec{x})$$

$$\begin{aligned}\vec{\nabla}_{\vec{x}'}\cdot\vec{E} =&\ \tfrac{1}{4\pi\epsilon_0}\nabla\cdot\int\rho(\vec{x}')\cdot\tfrac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}d^3x'\\ =&\ \tfrac{1}{4\pi\epsilon_0}\cdot\int\rho(\vec{x}')\nabla_{\vec{x}'}(\tfrac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3})d^3x\\ =&\ \tfrac{1}{\epsilon_0}\int\rho(\vec{x}')\delta^3(\vec{x}-\vec{x}')d^3x'\\ =&\ \qquad 0\end{aligned}$$

$(\frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3})$ is zero unless $\vec{x}=\vec{x}'$,
$\int\delta(x)$ picks out where $\vec{x}=\vec{x}'$ its zero elsewhere
$(\int\delta(x)=1$ when $\vec{x}=\vec{x}')$
So $\vec{E}(\vec{x})$ is conservative.

Divergence Theorem

$$\iiint_V(\vec{\nabla}\cdot\vec{v})d\tau = \oint\!\!\oint_S \vec{v}\cdot d\vec{\mu}$$

Think of this as measuring flow of fluid through a volume. If no sources/sinks inside, Right Hand Side = 0.

Stokes' Theorem

$$\iint_S (\vec{\nabla} \cdot \vec{v}) \cdot da = \oint_C \vec{v} \cdot d\vec{l}$$

Fundamental Theorem of Line Integrals

$$\int_C \cdots dl = T(b) - T(a)$$

$\delta(x)$- Dirac Delta Function (Not really a function)

$$\int_{-\infty}^{\infty} \delta(x) = 1 = \int_{-\epsilon}^{\epsilon} \delta(x) dx, \forall \epsilon > 0$$

$$\theta(x) = \begin{cases} 0 & , x < 0 \\ 1 & , x \geq 0 \end{cases}$$

$$\frac{d\theta}{dx} = \delta(x)$$

$$\int_{-\epsilon}^{\epsilon} \frac{d\theta}{dx} dx = 1 = \theta(\epsilon) - \theta(-\epsilon)$$

$$\frac{d\epsilon}{dx}|_x = 0, \ \forall x \neq 0$$

Rectangle Form

$$R_\epsilon(x) = \begin{cases} \frac{1}{2\epsilon}, & |x| < \epsilon \\ 0 & \text{else} \end{cases}$$

Fourier Integral Form

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}dk$$

$\delta(x - x')$
$\delta^3(\vec{x} - \vec{x}') = \delta(x - x')\delta(y - y')\delta(z - z')$ 3D delta function

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$$

$\delta(x)$ picks out value at 0
$\delta(-x) = \delta(x)$
$\delta(bx) = \frac{1}{|b|}\delta(x)$
$\delta(g(x)) = \Sigma_i \frac{\delta(x-x_i)}{|g(x_i)|}$ where $g(x) = 0$ for $x \in \{x_i\}$

$$\begin{aligned}
\vec{\nabla}_{\vec{x}} \cdot \vec{E} &= \qquad ..... \\
&= \tfrac{1}{4\pi\epsilon_0}\int \rho(x')\nabla_{\vec{x}}(\tfrac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3})d^3x' \\
&= \tfrac{1}{\epsilon_0}\int \rho(x')\delta^3(\vec{x}-\vec{x}')d^3x' \\
\vec{\nabla} \cdot \vec{E} &= \tfrac{1}{\epsilon_0}\rho(\vec{x}) \text{ Gauss's Law}
\end{aligned}$$

also happens to be true for moving charges.