8.022 Lecture Notes Class 6 - 09/14/2006

Show that $a^2 > 0$ implies $\delta(x^2 - a^2) = \frac{1}{2|x|}[\delta(x-a) + \delta(x+a)]$

$$\int_{-\infty}^{\infty} \delta(x^2 - a^2) d(x^2 - a^2) = 1$$

where $d(x^2 - a^2) = 2xdx$.

$$\int_{-\infty}^{\infty} 2x \cdot \delta(x^2 - a^2) dx = 1$$

Let $u = x^2 - a^2$

$du = 2xdx$

$$2 \cdot \int_{-a^2}^{\infty} \delta(u) du = 1$$

$$2 \cdot \theta(u)|_{-\infty}^{\infty} = 1$$

$$\int_{-\infty}^{\infty} f(x)\delta(x^2 - a^2)dx$$

$$\begin{aligned}
&= \int_{-\infty}^{0} f(x)\delta(x^2 - a^2)dx + \int_0^{\infty} f(x)\delta(x^2 - a^2)dx \\
&= \int_0^{\infty} f(-\sqrt{y})\delta(y - a^2)\frac{dy}{2\sqrt{y}} + \int_0^{\infty} f(\sqrt{z})\delta(z - a^2)\frac{dz}{2\sqrt{z}} \\
&= \int_0^{\infty} \frac{1}{2\sqrt{x}}[f(-\sqrt{x}) + f(\sqrt{x})]\delta(x - a^2)dx \\
&= \frac{1}{2a}[f(-a) + f(a)]
\end{aligned}$$

$$\delta(x^2 - a^2) = A\delta(x-a) + B\delta(x+a)$$

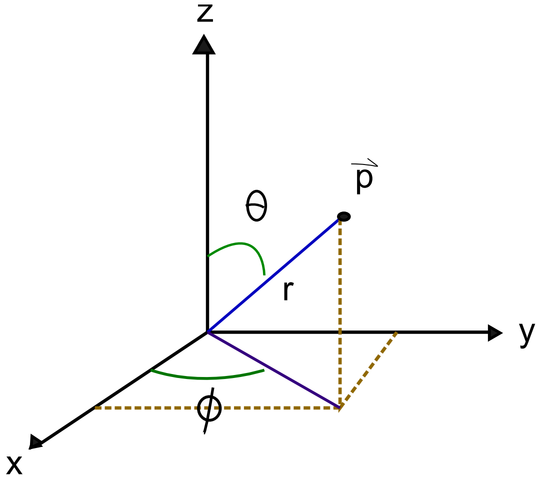
$$\int_{-\infty}^{\infty} f(x)[A\delta(x-a) + B\delta(x+a)]dx = A(a)f(a) + B(-a)f(-a)$$

Since LHS must equal RHS, so

$$\frac{1}{2a}[f(-a) + f(a)] = A(a)f(a) + B(-a)f(-a)$$

$$A = B = \frac{1}{2|x|}$$

Curvilinear Coordinates



Spherical Polar Coordinates

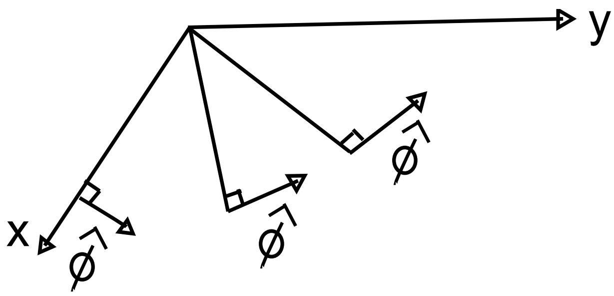
$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$$

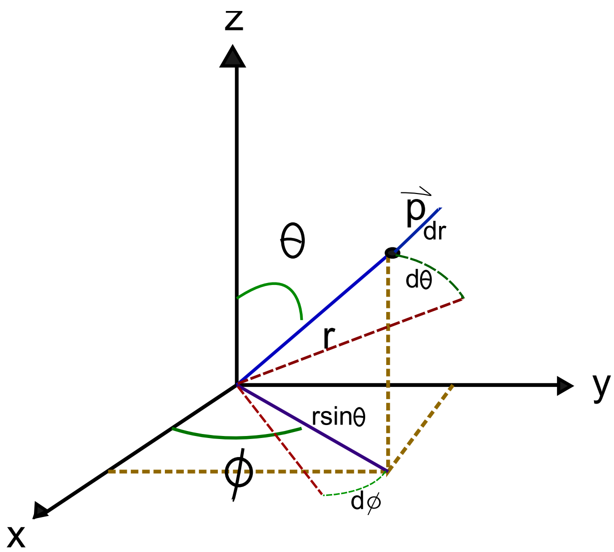
$$\hat{r} = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z}$$
$$\hat{\theta} = \cos\theta\cos\phi\hat{x} + \cos\theta\sin\phi\hat{y} - \sin\theta\hat{z}$$
$$\hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y}$$

$$\hat{r} = \hat{r}(\vec{p}) \qquad \hat{\theta} = \hat{\theta}(\vec{p}) \qquad \hat{\phi} = \hat{\phi}(\vec{p})$$

$d\vec{x} = \Sigma_i dx_i \quad \vec{e_i}\{\vec{e_i}$ are orthonormal$\}$



$d\vec{x} \cdot \vec{e_r} = dr \rightarrow$ move one unit along $\hat{r}$, total charge is dr.
$d\vec{x} \cdot \vec{e_\theta} = d\theta \cdot r \rightarrow$ move along $\hat{\theta}$, total change is $d\theta \cdot r$
$d\vec{x} \cdot \vec{\phi} = (r \sin\theta) d\phi$
$\{\vec{r}, r\hat{\theta}, r\sin\theta\hat{\phi}\}$ orthonormal basis ... set ..



$$d\vec{x} = \hat{r} dr + (\hat{\theta} \cdot r) d\theta + (\hat{\phi} \cdot r \sin\theta) d\phi$$