Electricity and Magnetism

- Last time: Electric Field
- Today:
  - Electric Flux
  - Gauss' Law

Gauss' Law

(1830)

- Today: Gauss' Law
  - Not so many demos, some math
- Electricity and Magnetism in 4 equations:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

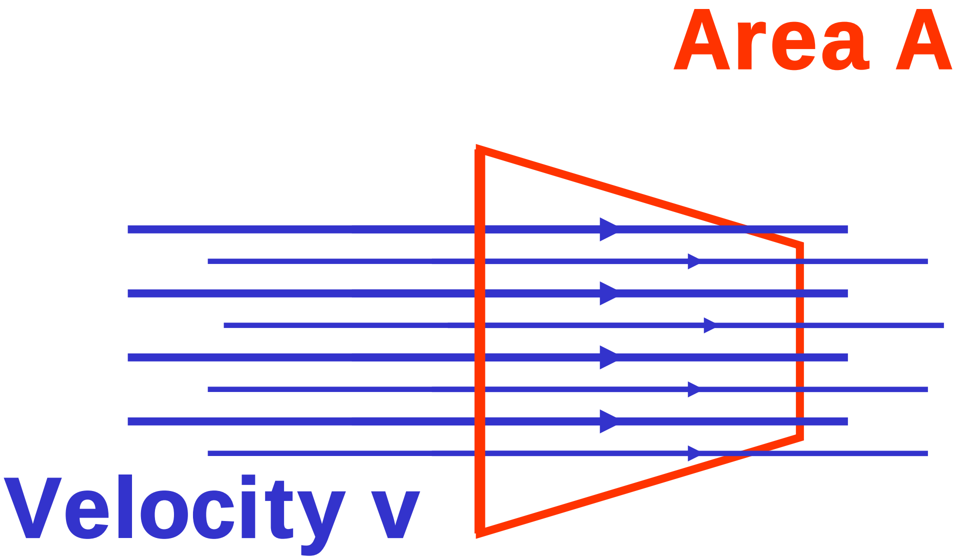
Electric Flux

- Definition (simple case):

Electric Flux $\Phi_E = \vec{E} \cdot \vec{A}$

- What does that mean?
  - Analogy with flow of e.g. water

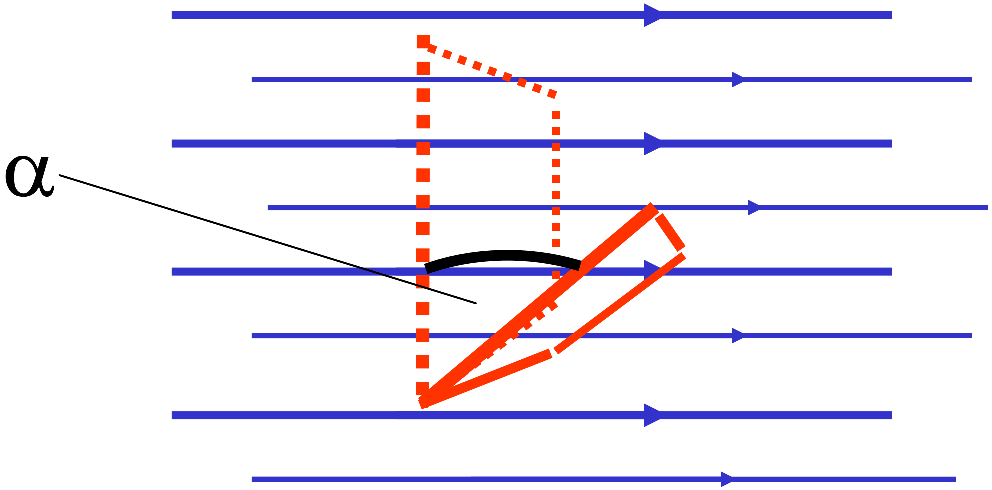
‘Flux’ of water



Flow: $dV/dt = A\,dx/dt = Av$

‘Flux’ of water

Area A



Velocity v

Flow: $A v \cos(\alpha) = \vec{A} \cdot \vec{v}$

Electric Flux

- Electric Flux: $\Phi_E = \vec{E} \cdot \vec{A}$
- Same mathematical form as water flow
- But there is no ‘substance’ flowing
- Took almost a century to accept
- Flux tells us how much field ‘passes’ through surface A

Electric Flux

- For ‘complicated’ surfaces:
  - Use integral

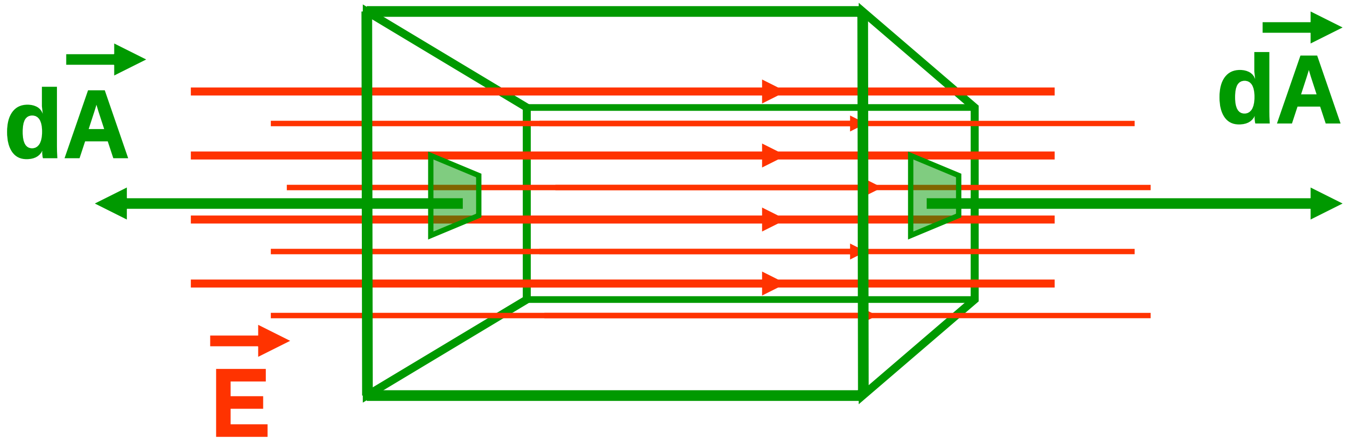
$$\Phi_E = \int_A \vec{E} \cdot d\vec{A}$$

- Often, ‘closed’ surfaces

$$\Phi_E = \oint_A \vec{E} \cdot d\vec{A}$$

Electric Flux

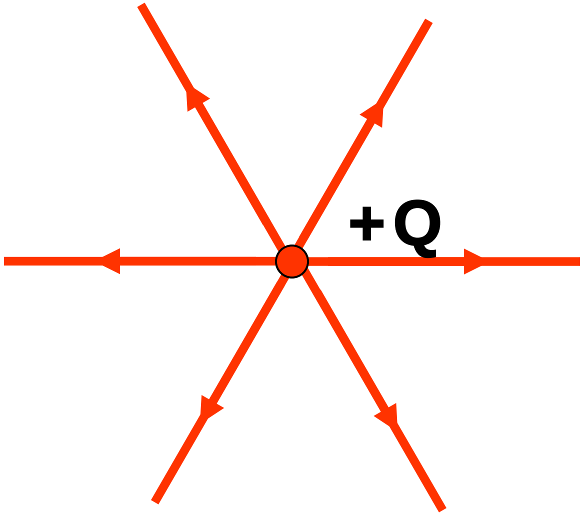
- Example of closed surface: Box



- Flux in (left) = -Flux out (right): $\Phi_E = 0$
- No ‘source’ of flux in this box

Electric Flux

- How to make Φ_E non-zero?
- Remember:



- Put Charge Q inside 'box'!

Gauss' Law

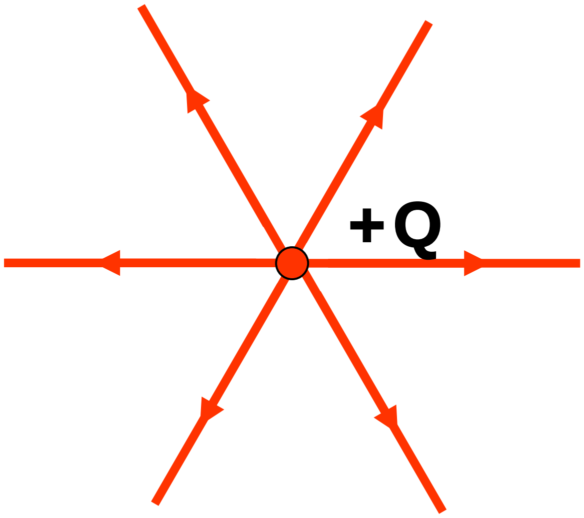
- How are flux and charge connected?
- Charge Q_{encl} as source of flux through closed surface

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Gauss' Law

- Gauss vs Coulomb

$$E = k\frac{Q}{r^2}$$
$$= \frac{1}{4\pi\epsilon_0}\ \frac{Q}{r^2} \text{ with } \epsilon_0 = \frac{1}{4\pi k}$$



Gauss' Law

- Gauss vs Coulomb

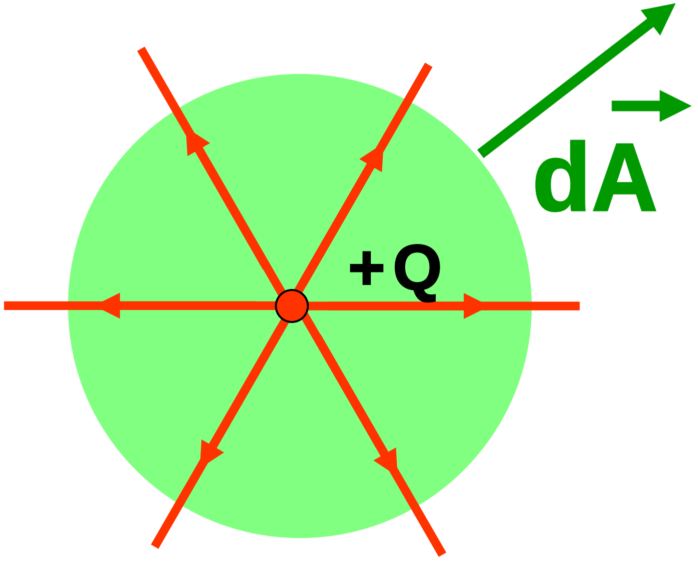
$$\frac{Q_{encl}}{\epsilon_0} = \oint_{sphere} \vec{E} \cdot d\vec{A} =$$

$$\oint_{sphere} E dA =$$

$$E \oint_{sphere} dA =$$

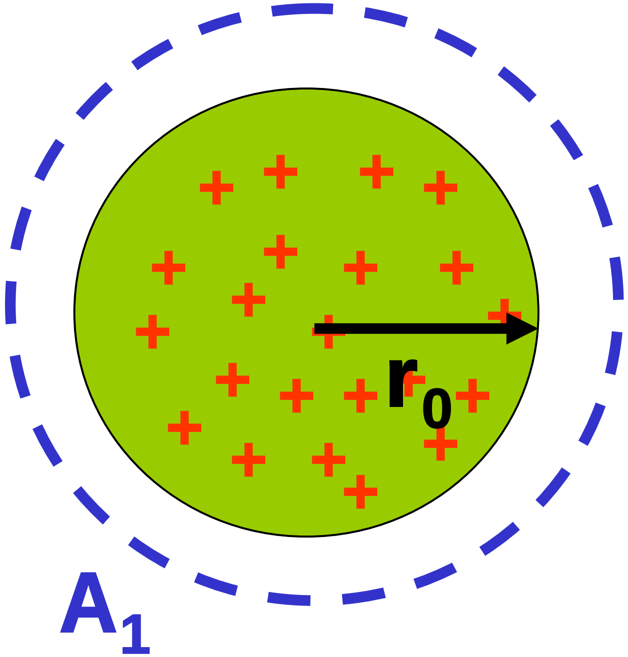
$$E(4\pi r^2) \Longrightarrow$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q_{encl}}{r^2}$$



Example

- Solid charged sphere (non-conducting)



$$(1)\ r > r_0 :$$

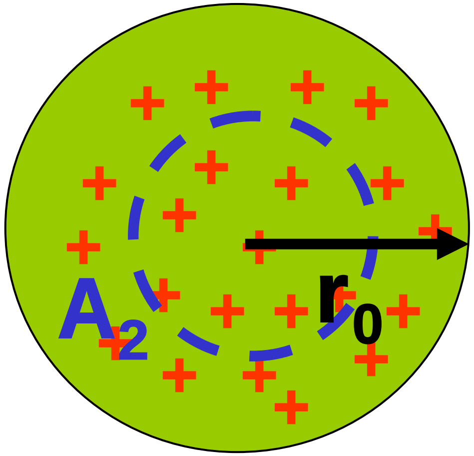
$$\oint_{A_1} \vec{E} \cdot d\vec{A} =$$

$$E(4\pi r^2) = \frac{Q_{encl}}{\epsilon_0}$$

$$\Longrightarrow \quad E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

Example

- Solid charged sphere (non-conducting)



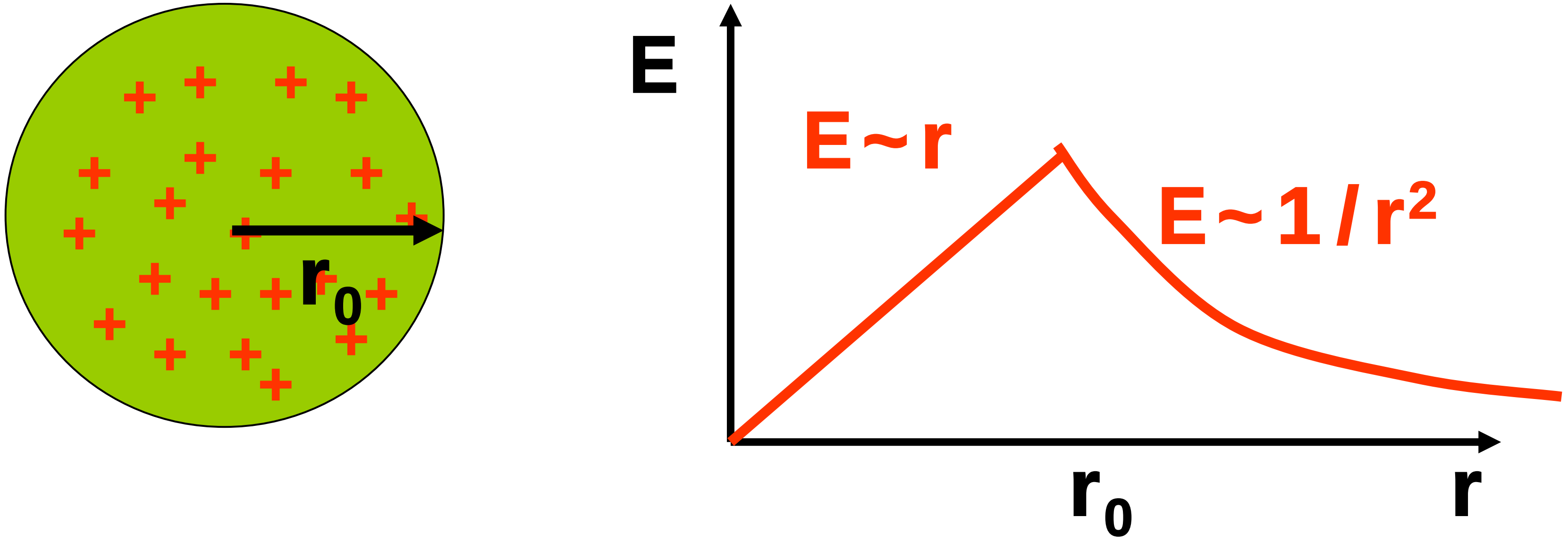
$$(2)\ r > r_0:$$

$$\begin{aligned} \oint_{A_2} \vec{E} \cdot d\vec{A} &= \\ E(4\pi r^2) &= \frac{Q_{encl}}{\epsilon_0} \\ Q_{encl} &= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi r_0^3} Q = \frac{r^3}{r_0^3} Q \end{aligned}$$

$$E(4\pi r^2) = \frac{r^3}{r_0^3} Q \implies E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_0^3}\ r$$

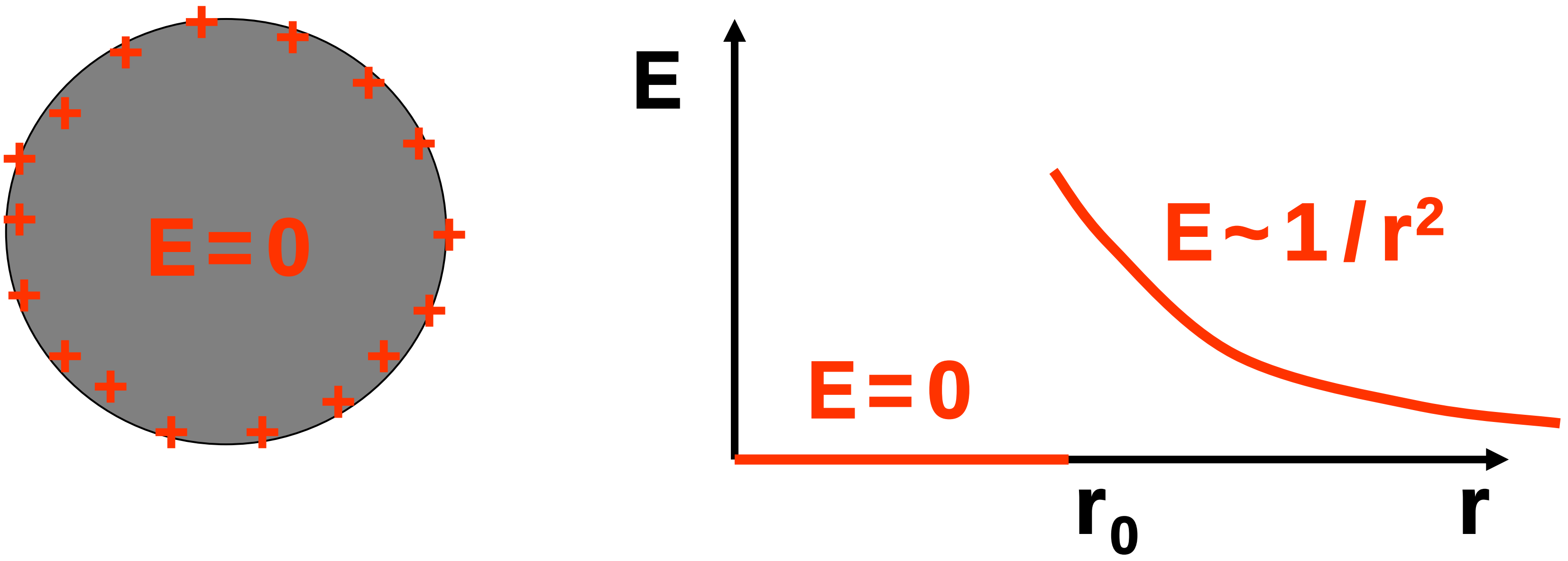
Example

- Solid charged sphere (non-conducting)



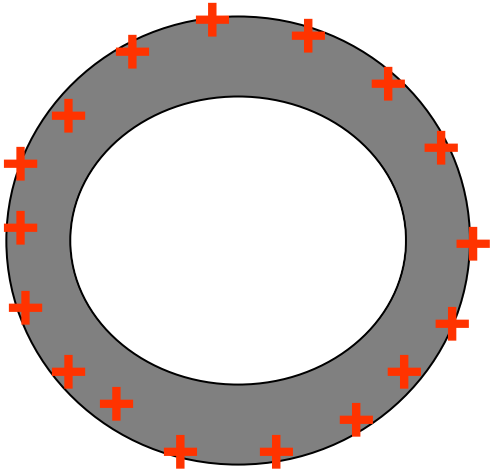
Example II

- Conducting Sphere



Example IIa

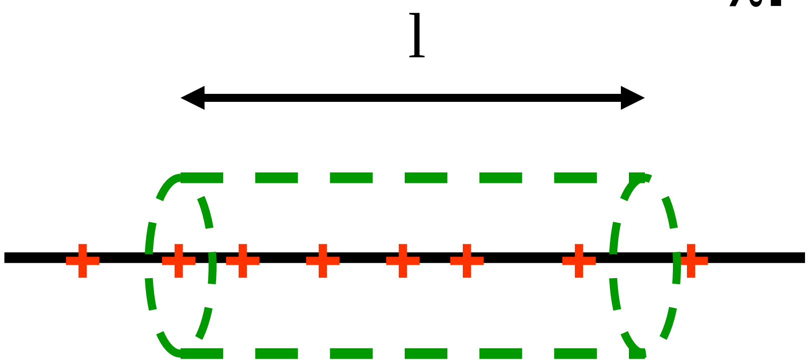
- Conducting Sphere



Example III

- Line of Charge

λ: Charge density dQ/dl



$$\oint_{cyl} \vec{E} \cdot d\vec{A} = E(2\pi r l) = \frac{Q_{encl}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$\Longrightarrow E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$