Electricity and Magnetism

- More on
  - Electric Flux
  - Gauss’ Law

More on Electric Flux and Gauss' Law

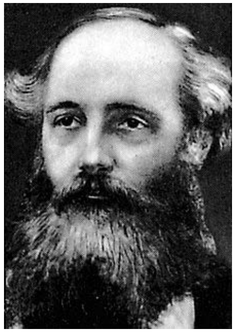
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell Equations (1873)



Electric Flux

Note absence of ' $\rightarrow$ '

Electric Flux $\Phi_E = \vec{E} \cdot \vec{A}$

'Φ_E' is a Scalar: How much?

I.e. how much field passes through surface A?

$\vec{A}$?

- Direction
  - Normal to surface
- Magnitude
  - Surface Area
- For closed surface
  - Pointing outwards

Electric Flux

- What if $\vec{E}$ not constant on surface A?
- Use integral

$$\Phi_E = \int_A \vec{E} \cdot d\vec{A}$$

- Often, 'closed' surfaces

$$\Phi_E = \oint_A \vec{E} \cdot d\vec{A}$$

Gauss' Law

- Connects Flux through closed surface and charge inside this surface:

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

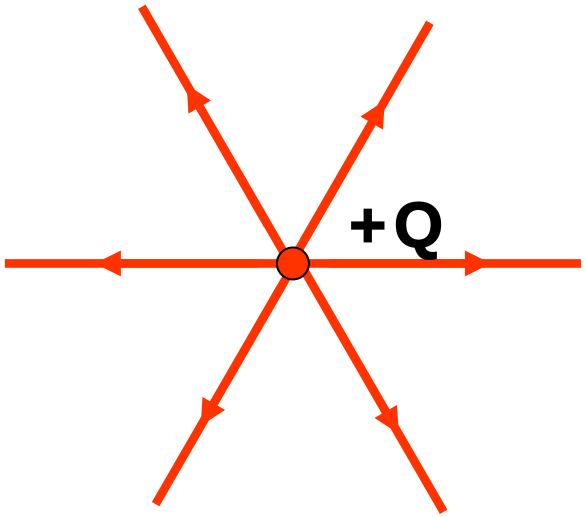
Note: k = 1/ 4 $\pi\, \varepsilon_0$

Gauss' Law

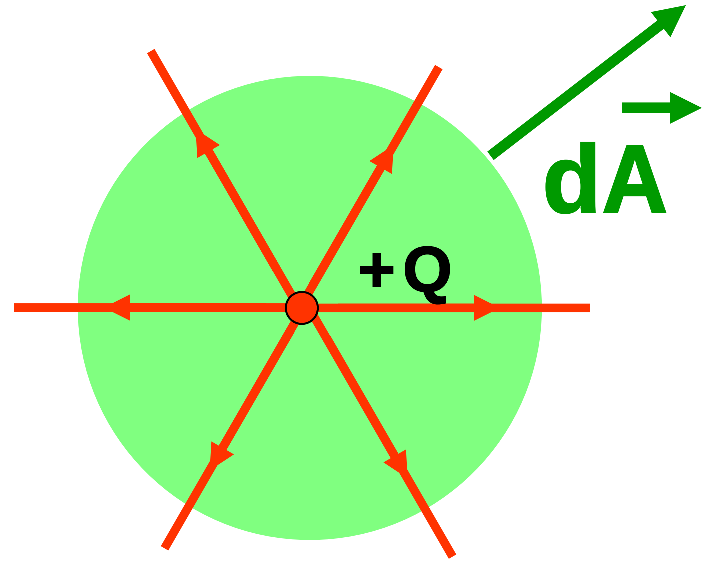
$$\oint_A \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

- True for ANY closed surface around Q_{encl}
- Suitable choice of surface A can make integral very simple

Use the Symmetry!



Point Charge



Spherical Surface

(1) $r > r_0$:

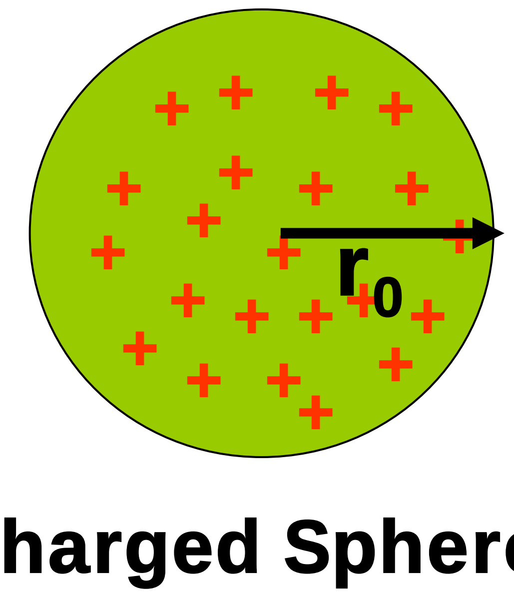
$$\oint_{A_1} \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{encl}}{\epsilon_0}$$

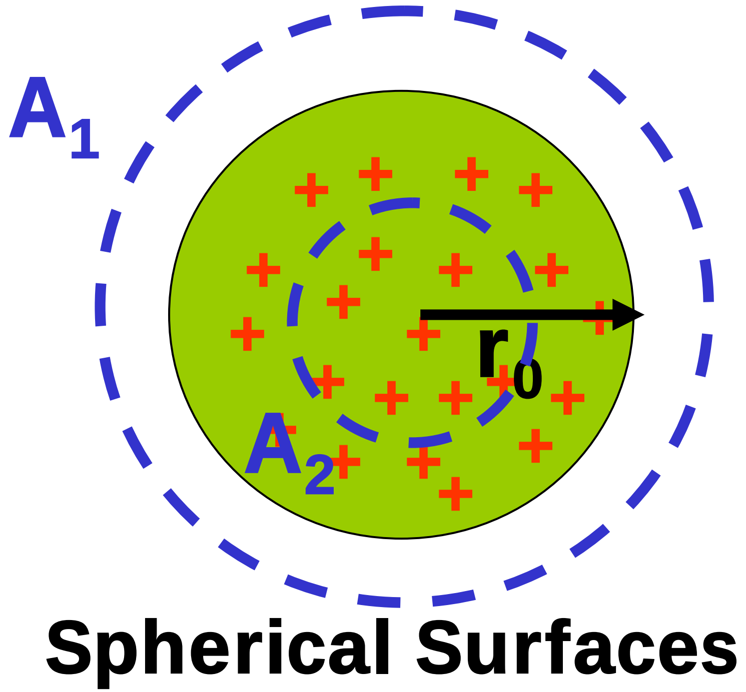
$$\Longrightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

$$\vec{E} || d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = E dA$$

$$E(r) = \text{const.} \Rightarrow \oint_A E \cdot dA = E \oint_A dA = EA$$

Use the Symmetry!





$$\vec{E} || d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = E dA$$

$$E(r) = \text{const.}$$

Use the Symmetry!

(1) $r > r_0$:

$$\oint_{A_1} \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{encl}}{\epsilon_0}$$

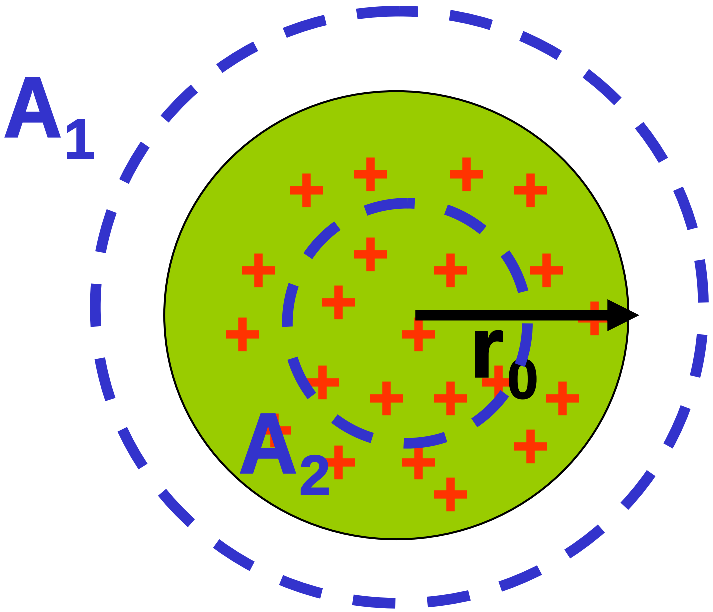
$$\Longrightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

(2) $r < r_0$:

$$\oint_{A_2} \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{encl}}{\epsilon_0}$$

$$Q_{encl} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi r_0^3} Q = \frac{r^3}{r_0^3} Q$$

$$\Longrightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^3}\, r$$

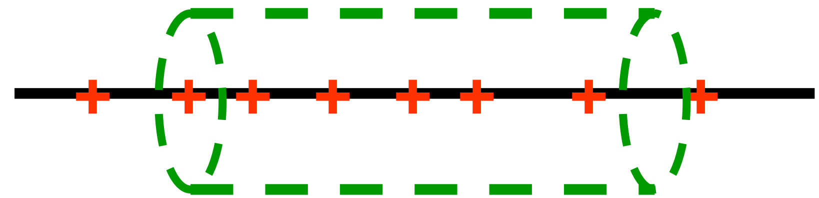


Spherical Surfaces

$$\vec{E} || d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = E dA$$

$$E(r) = \text{const.}$$

Use the Symmetry!



Charged Line **Cylindrical Surface**

$$\oint_{cyl} \vec{E} \cdot d\vec{A} \quad = \quad E(2\pi r l) = \frac{Q_{encl}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$\Longrightarrow \quad E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

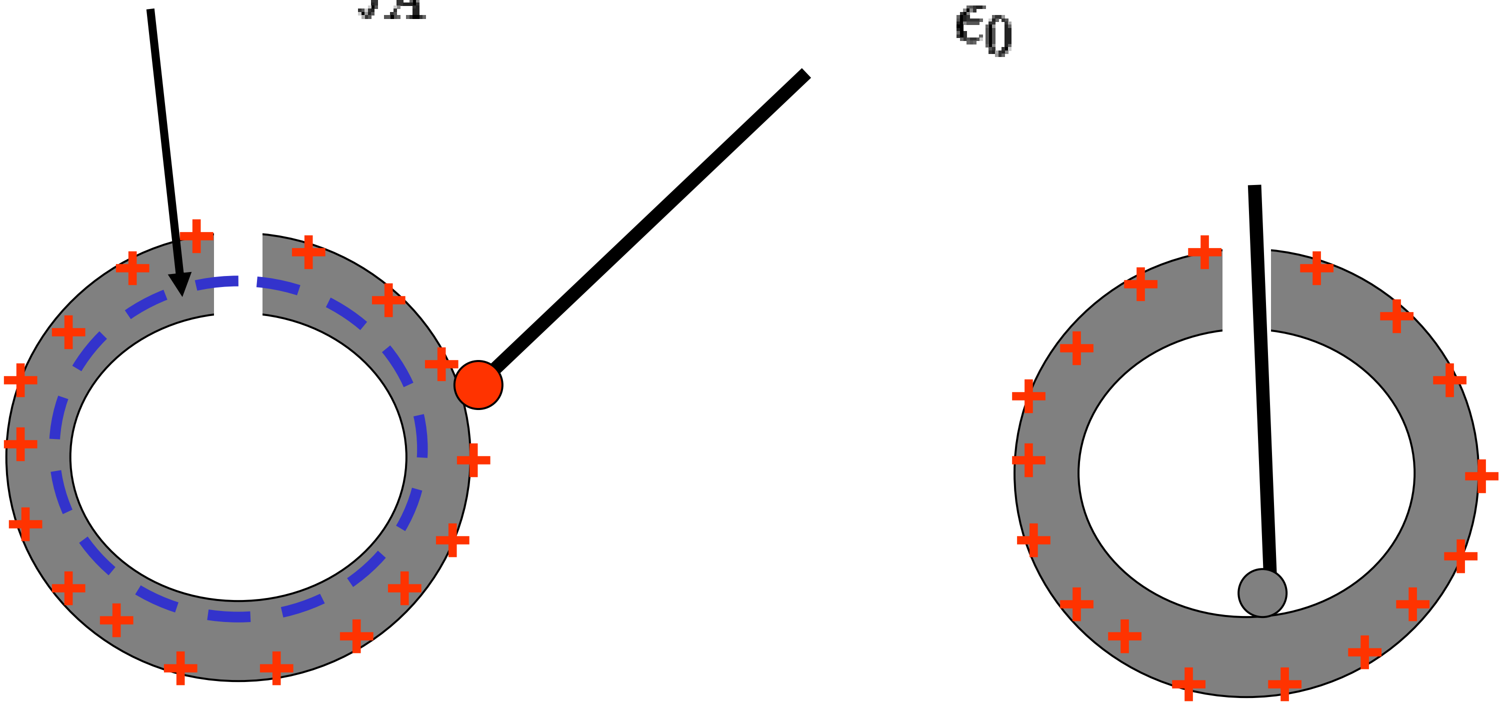
$$\vec{E} \perp d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = 0$$

$$\vec{E} || d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = E dA$$

$$E(r) = \text{const.}$$

Hollow conducting Sphere

$$\vec{E} = 0 \Rightarrow \oint_A \vec{E} \cdot d\vec{A} = 0 = \frac{Q_{encl}}{\epsilon_0}$$



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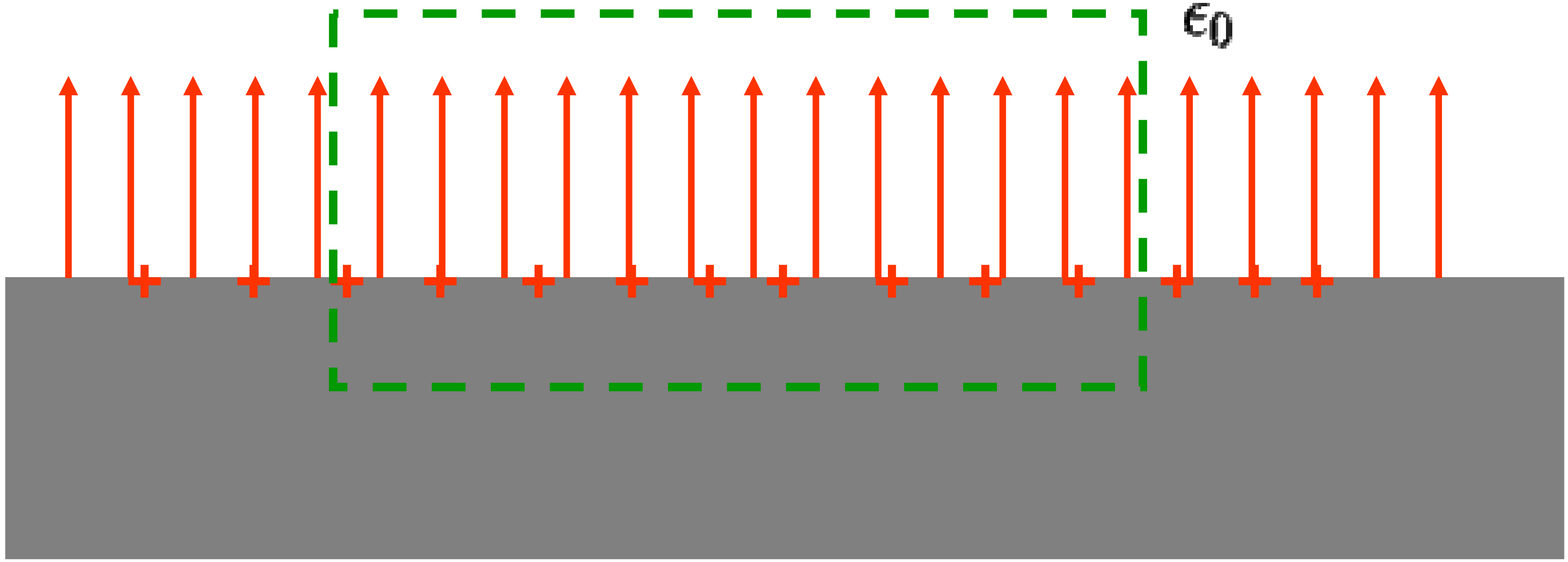
Last example

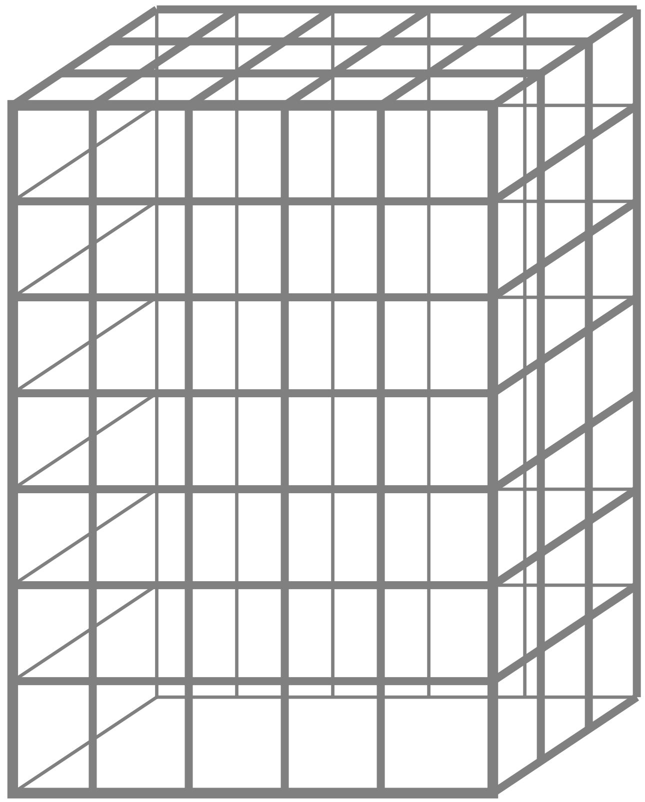
$$\sigma = Q/A$$

$$\oint_A \vec{E} \cdot d\vec{A} = EA = \frac{Q_{encl}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$\Longrightarrow E = \frac{\sigma}{\epsilon_0}$$



Faraday Cage



Hollow Metal Sphere

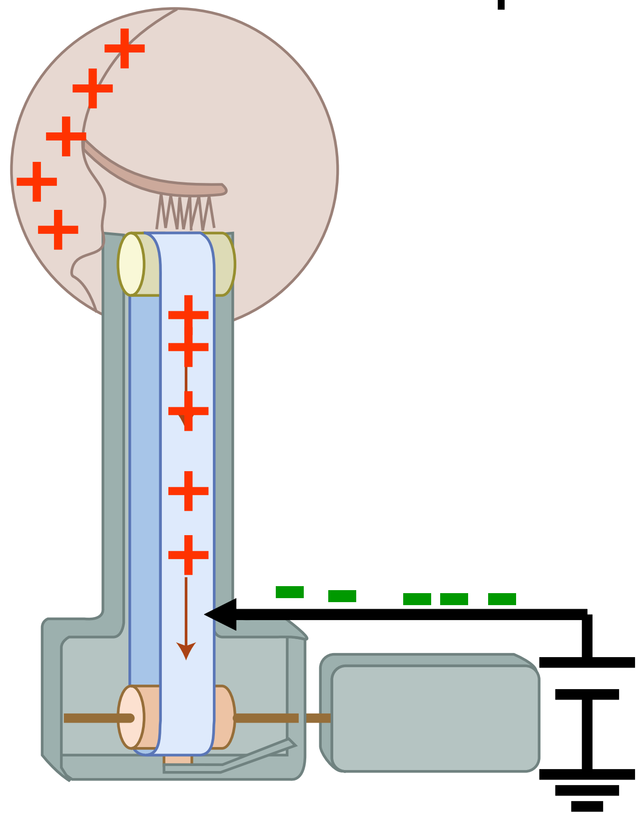
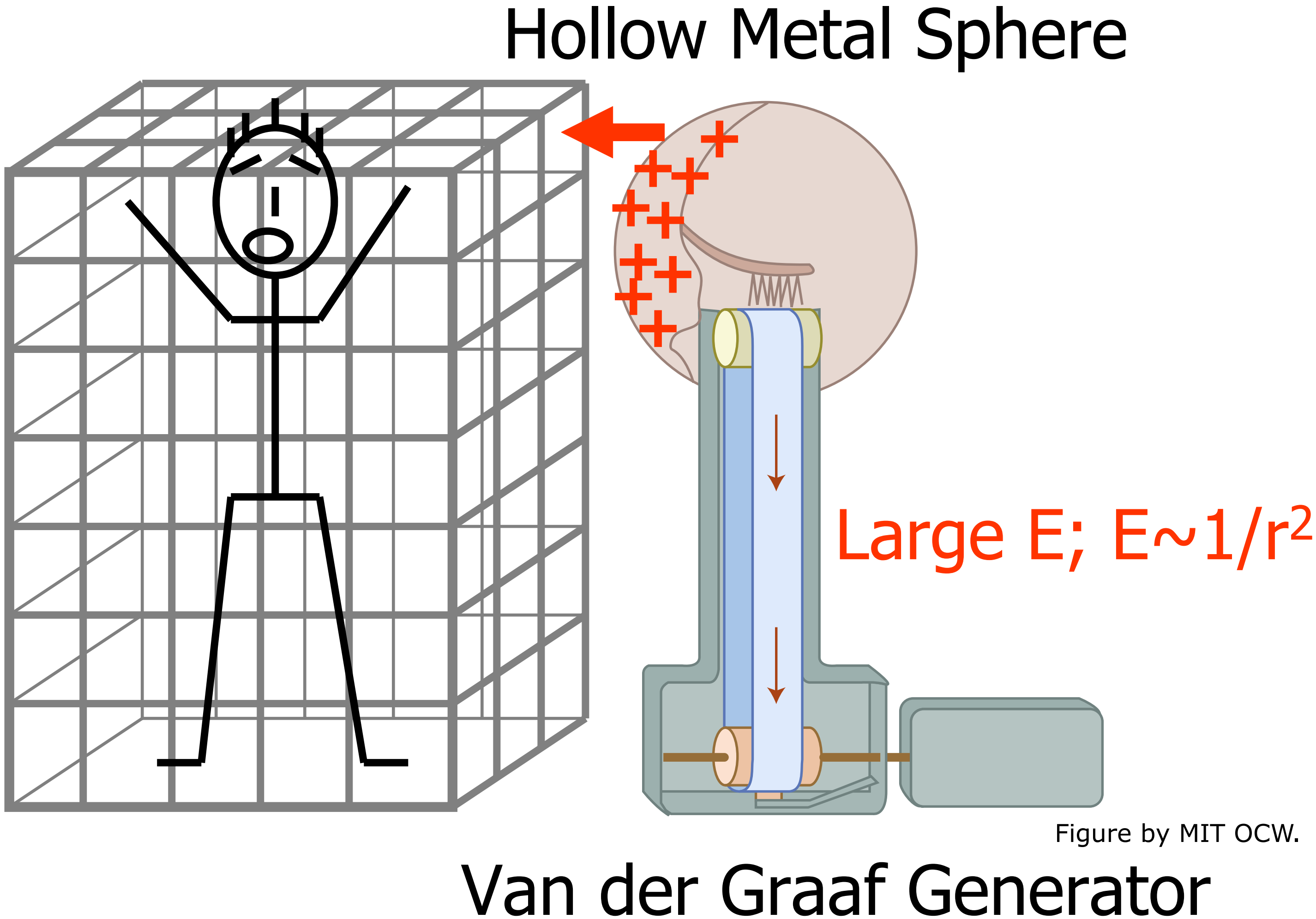


Figure by MIT OCW.

Van der Graaf Generator

Faraday Cage



‘Challenge’ In-Class Demo