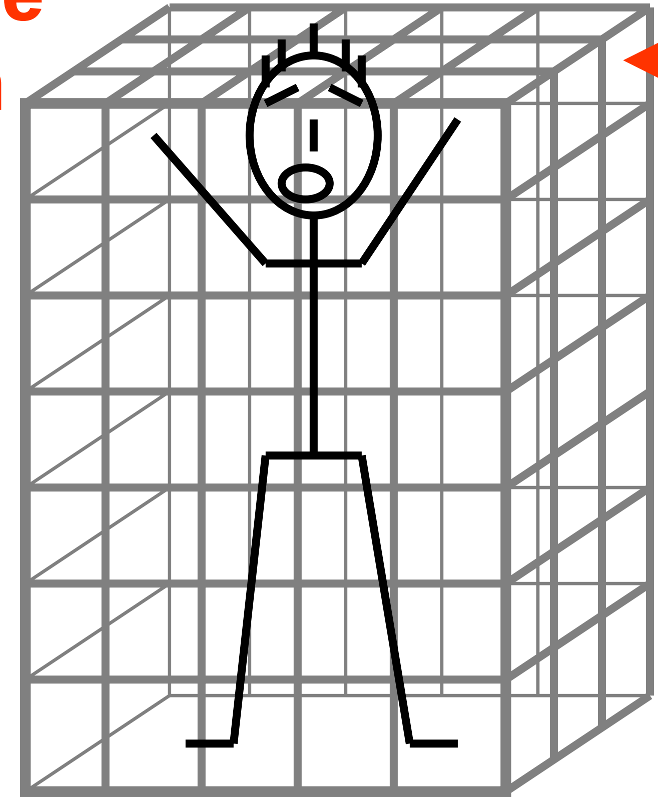
Electricity and Magnetism

- Today
  - Kelvin Water Drop Generator
  - Electric Potential Energy
  - Electric Potential

Recap: In-Class Demos

- **No field inside**
- **No charge on inside surface**

Large E; E~$1/r^2$



Faraday Cage

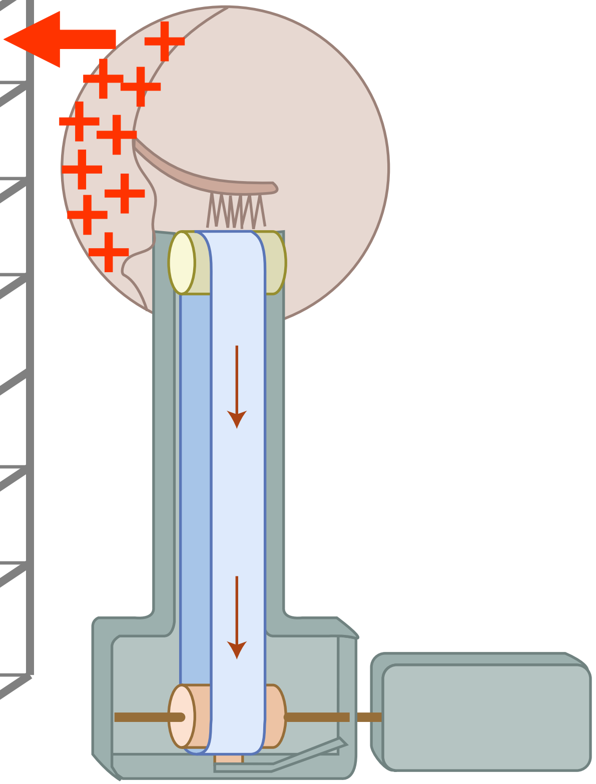
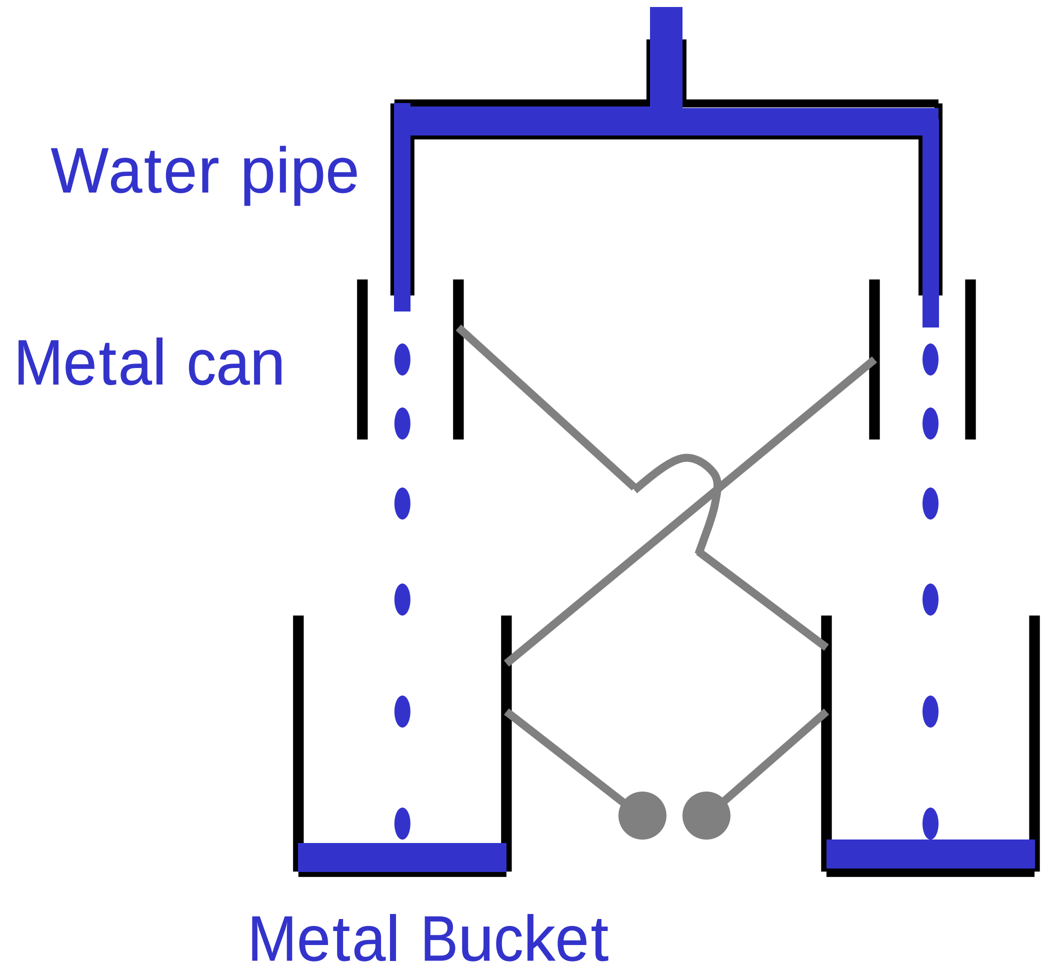


Figure by MIT OCW.

Van der Graaf Generator

Kelvin Water Drop Generator

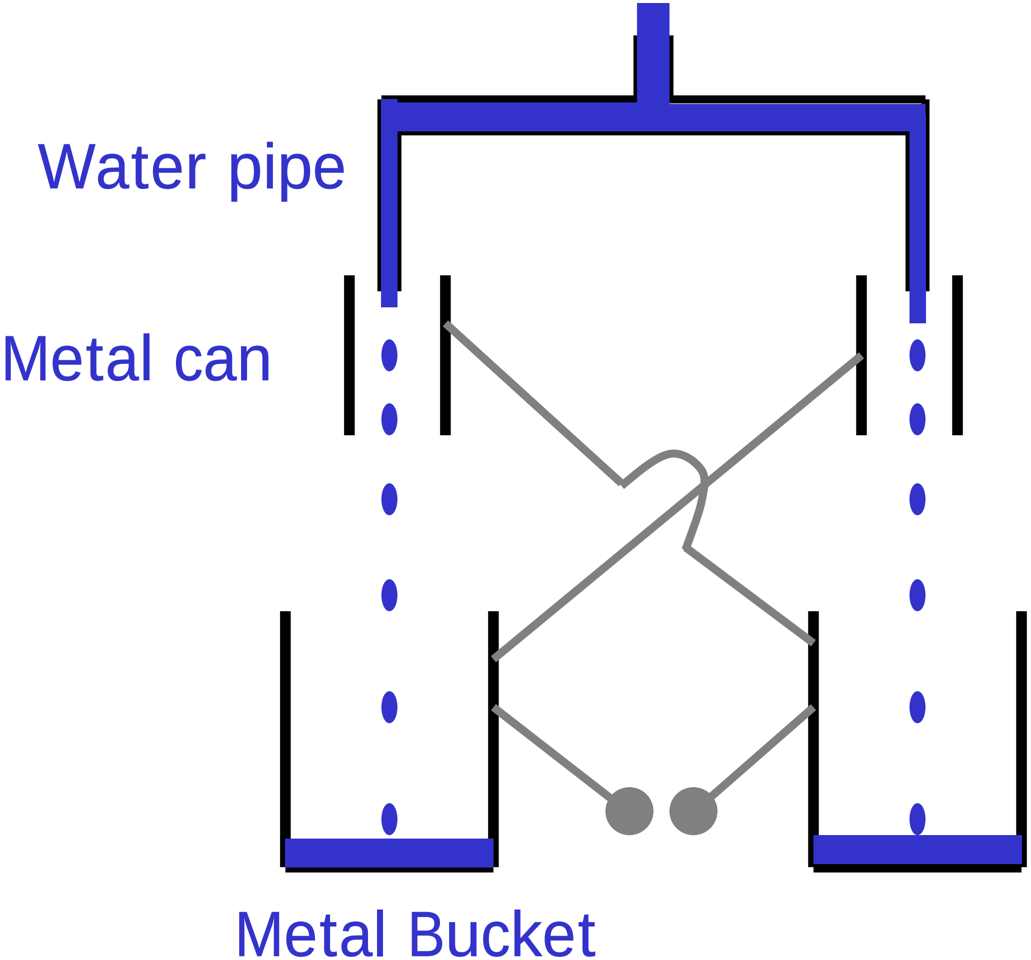


- No battery, motor, anything…
- Start water flow
- Water 'flares out'
  - like tinsel
- Spark!
- Cycle repeats…

Puzzles

- Spark
  - Spheres have opposite charge
  - Asymmetry!
  - But apparatus is symmetric!?
- Where does the energy come from?

Kelvin Water Drop Generator



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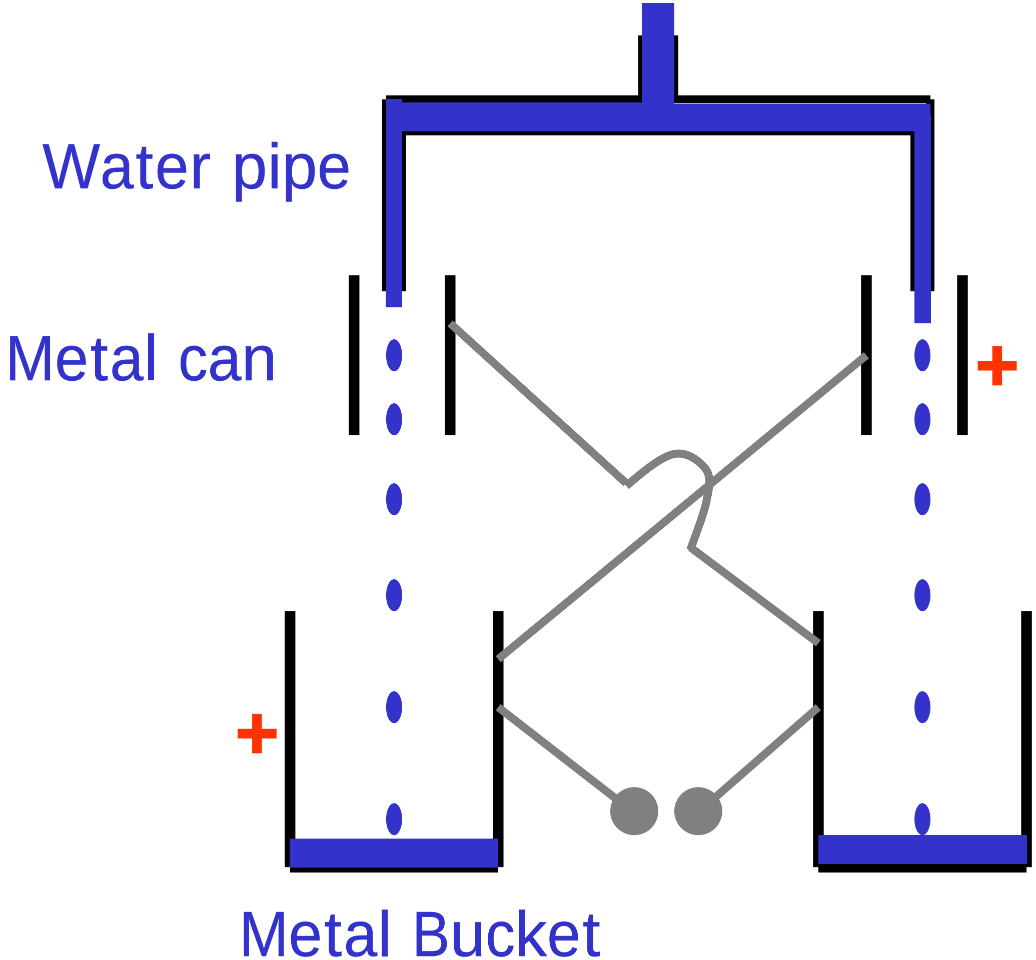
Kelvin Water Drop Generator



Symmetry is broken
by Fluctuation

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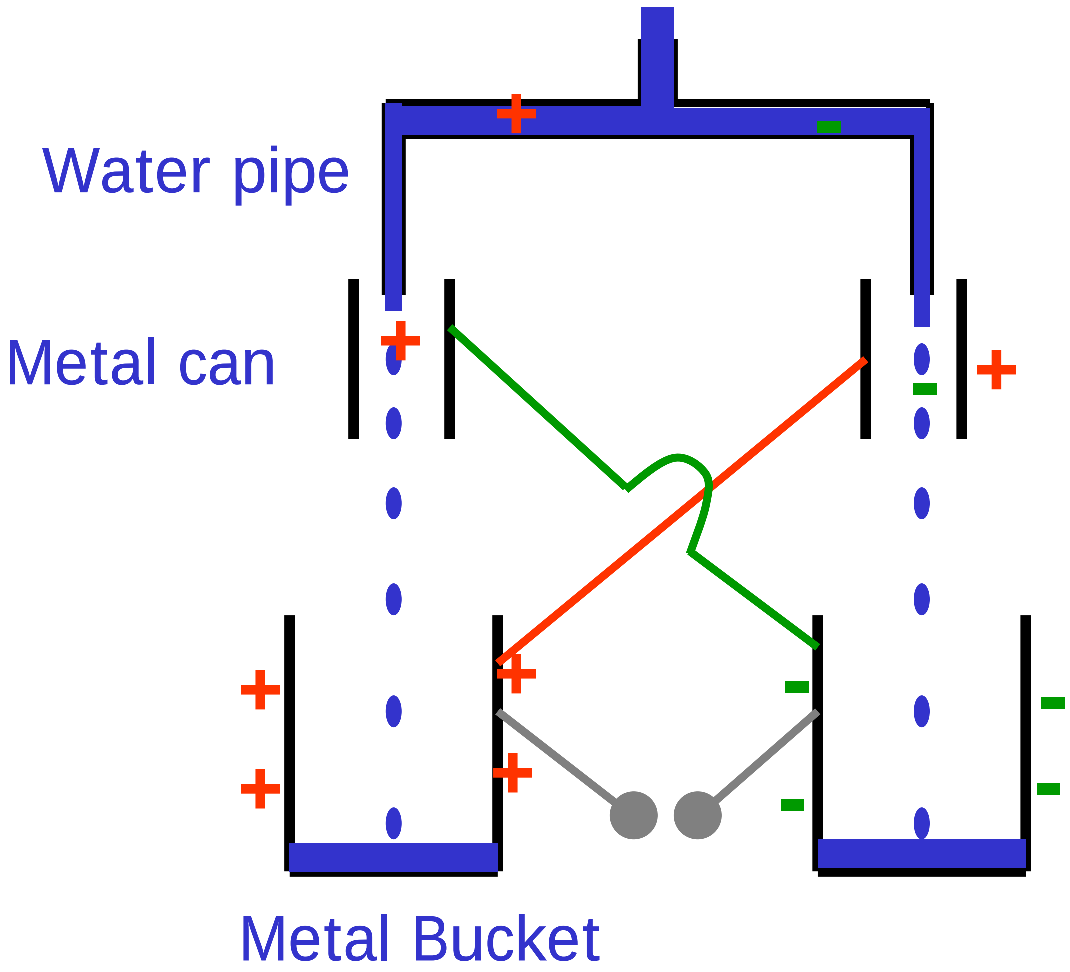
Kelvin Water Drop Generator



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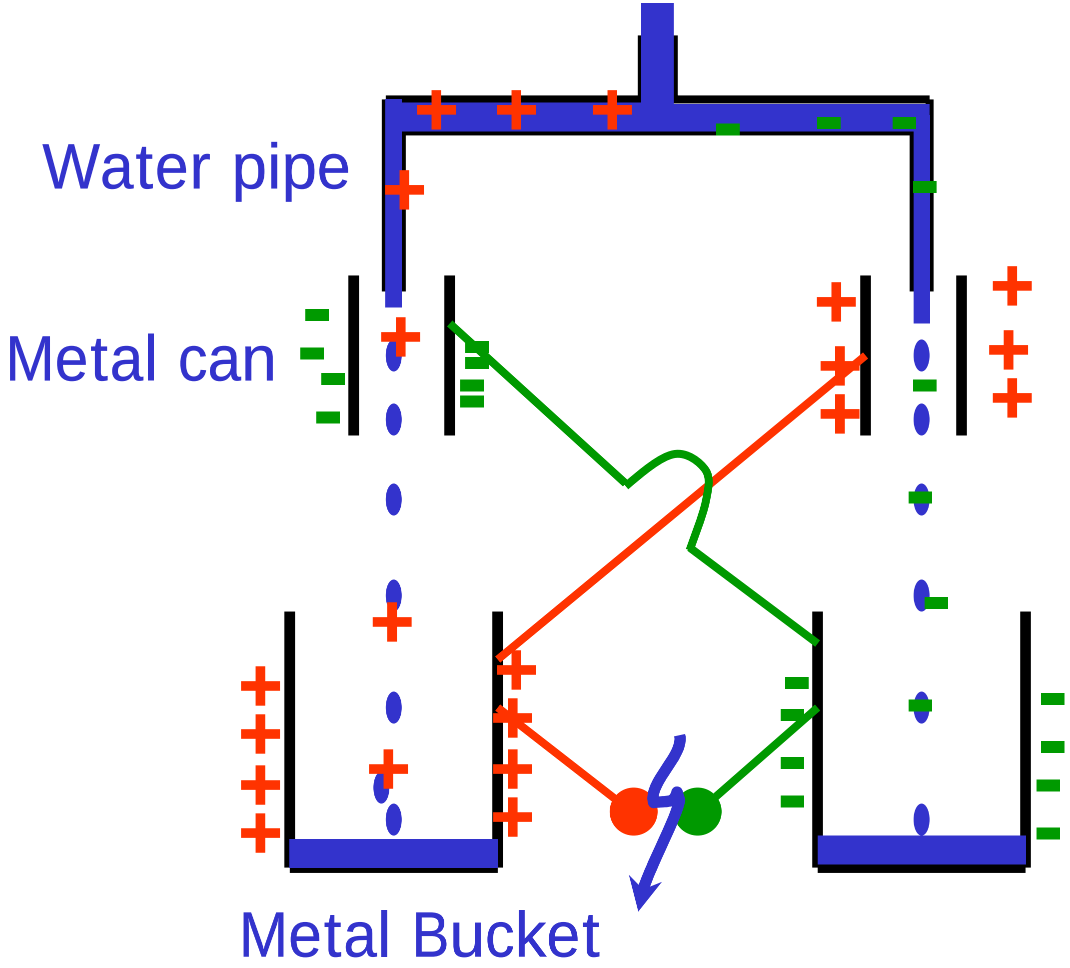
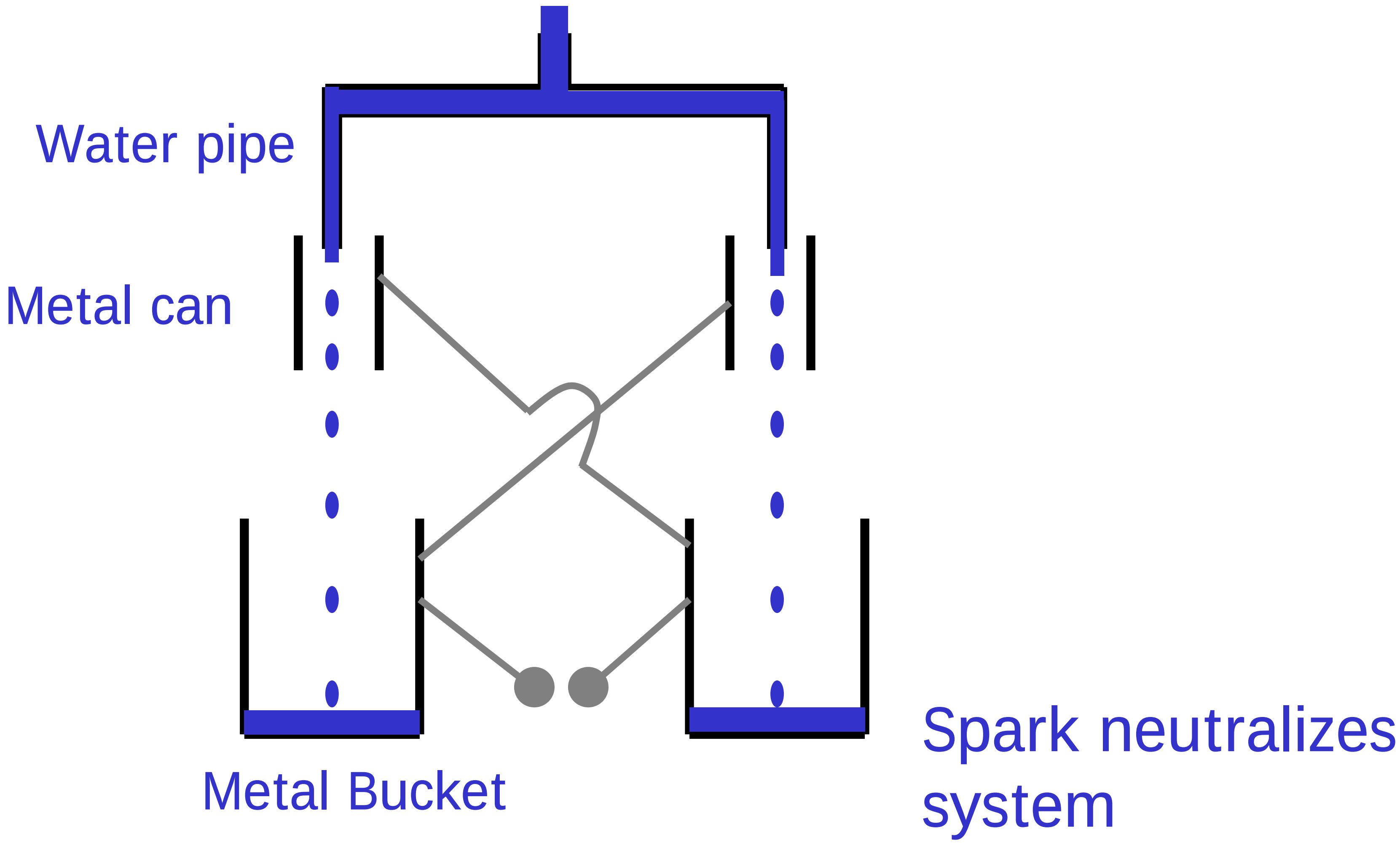
Kelvin Water Drop Generator



Positive Feedback Loop

Kelvin Water Drop Generator



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Final question

- Spark is like a small lightning
  - contains energy (like lightning)!
- Where does this energy come from?

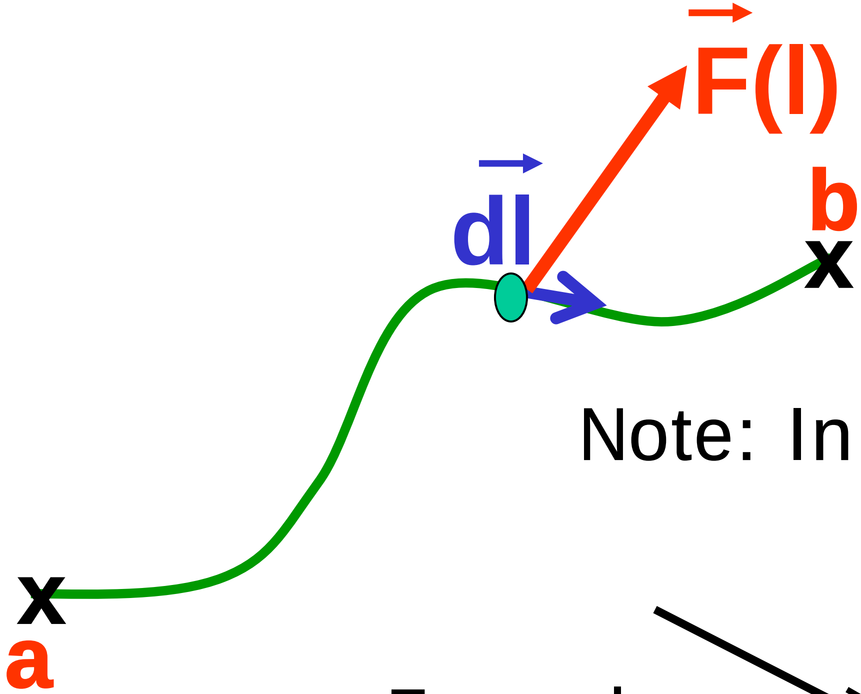
→ **Gravitational potential energy of water ($E_{pot} = m\ g\ h$)**

- Where was it before the spark?

Electric Potential Energy

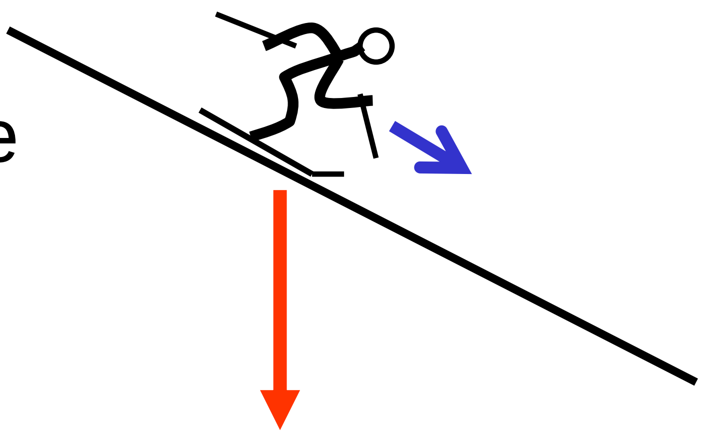
- Massive objects in Gravitational Field have potential energy
- What about charged objects in E-Field?
- Let's review some concepts from 8.01x….

Work and Potential Energy

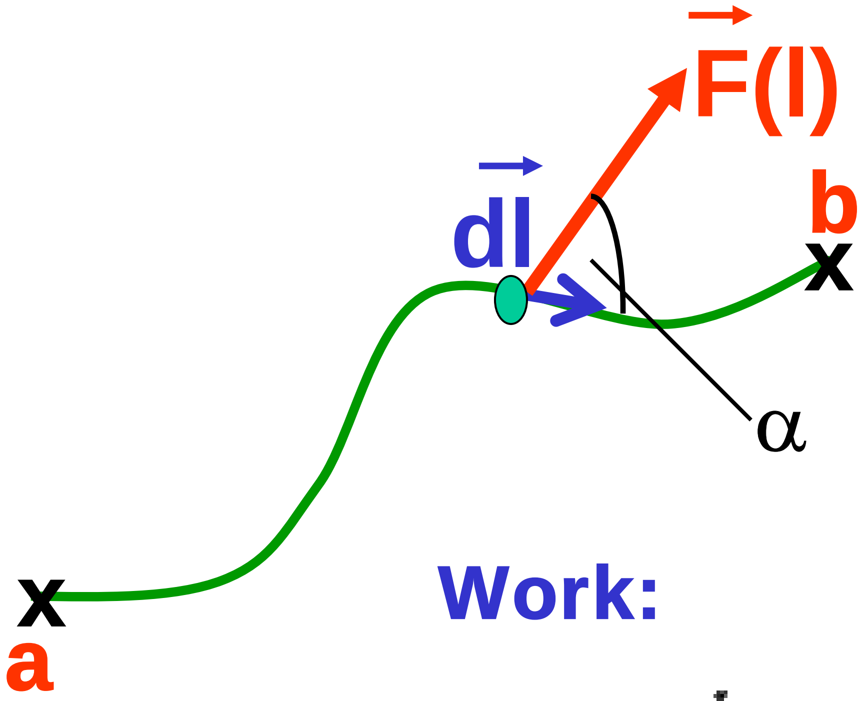


Note: In general, $\vec{F} \nparallel \vec{dl}$

Example



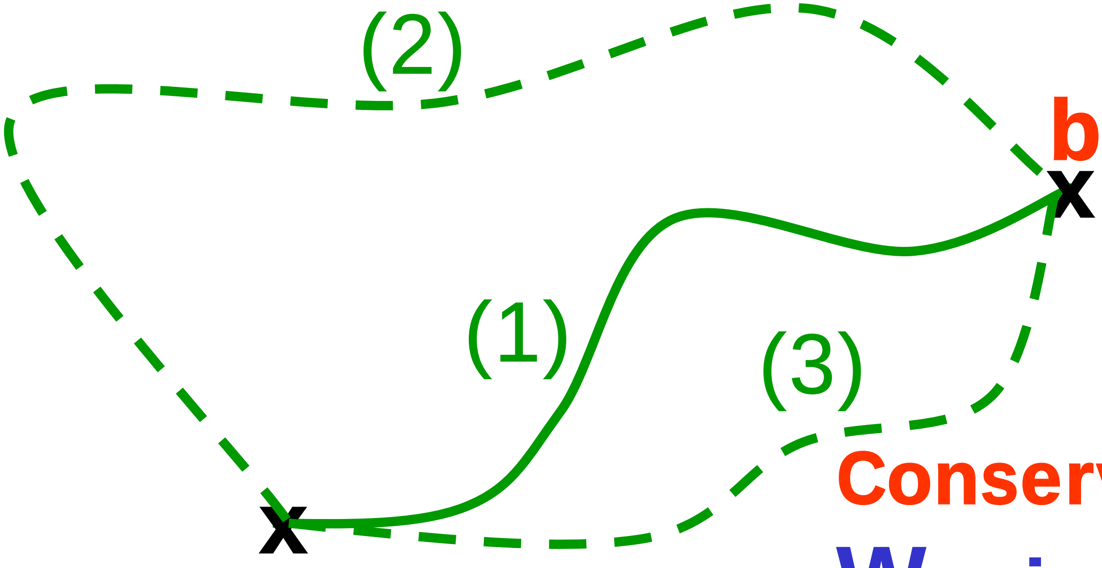
Work and Potential Energy



Work:

$$W_{ba} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b F\, dl \cos(\alpha)$$

Work and Potential Energy



Conservative Force:

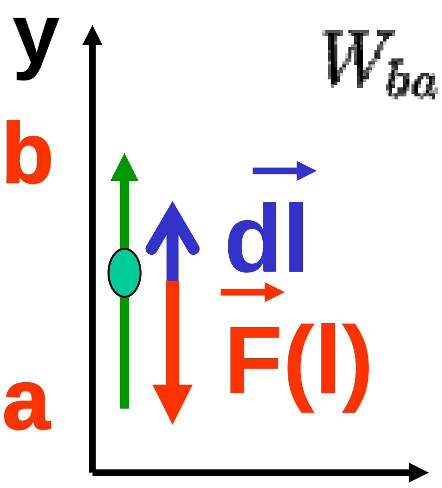
W_{ba} independent of path

$$W_{ba}^{(1)} = W_{ba}^{(2)} = W_{ba}^{(3)} = W_{ba}$$

Work and Potential Energy

- IF $\vec{F}$ conservative:
  - Define Potential Energy $W_{ba} = U(a) - U(b)$

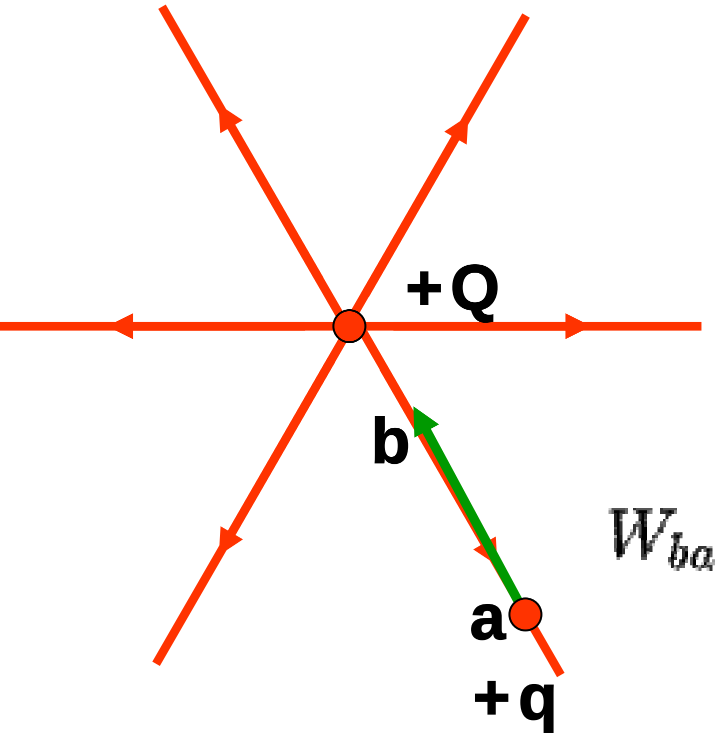
Example - Gravity



$$W_{ba} = \int_a^b \vec{F} \cdot d\vec{l} = -\int_a^b mg\,dy = mga - mgb = -\Delta U$$

$$\Rightarrow U_{grav} = mgh, \quad h \text{ is height above reference}$$

Work and Electrostatic Force



Charge q at point a in Coulomb field of Q

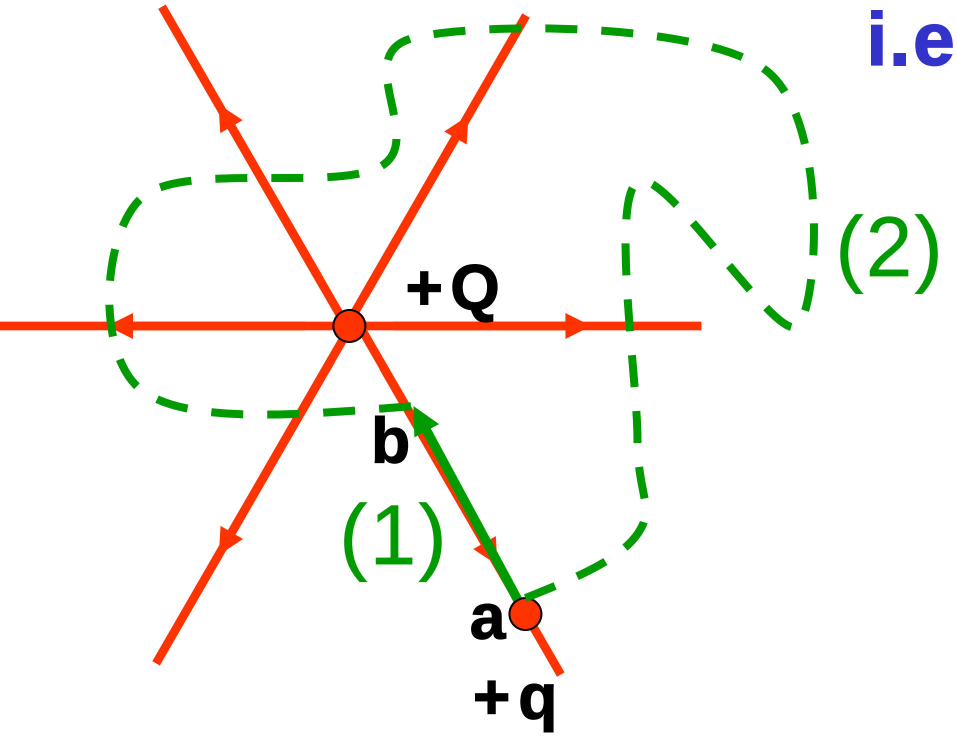
How much work to move to point b?

$$W_{ba} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b q\vec{E} \cdot d\vec{l}$$

$$= q\int_{r_a}^{r_b} E\,dr, \text{ with } E(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

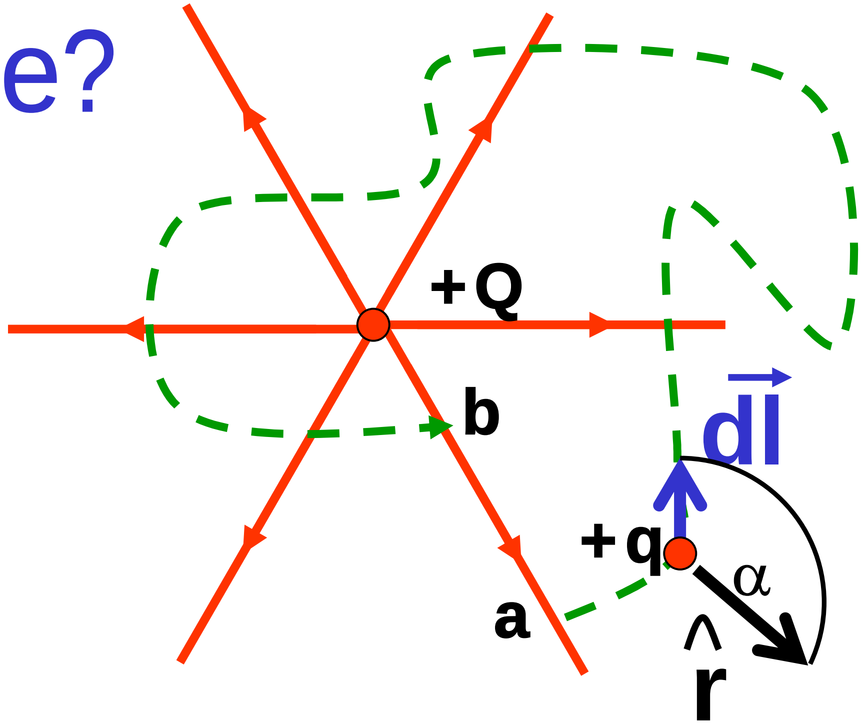
$$= q\int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,dr = \boxed{\frac{1}{4\pi\epsilon_0}Qq\,(\frac{1}{r_a} - \frac{1}{r_b}).}$$

Is F_{ES} conservative?



i.e. is $W_{ab}^{(1)} = W_{ab}^{(2)}$?

Is F_{ES} conservative?



$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\,\hat{r}$$

$$W_{ba} = \int_a^b \vec{F}\cdot d\vec{l} = \frac{Qq}{4\pi\epsilon_0}\int_a^b \frac{1}{r^2}\,\hat{r}\cdot d\vec{l}$$

$$= -\frac{Qq}{4\pi\epsilon_0}\int_a^b \frac{1}{r^2}\,dr; \text{ because } \hat{r}\cdot d\vec{l} = dl\cos(\alpha) = -dr$$

$$\Rightarrow W_{ba} = U(a) - U(b) \text{ with } U(r) = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$$

Electric Potential Energy

- Found Potential Energy associated with Electric Force between two charges

$$U(r) = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$$

- Can only observe differences in potential
  - often set $U(\infty) = 0$
  - U(r) energy needed to bring q,Q together from infinity

Electric Potential

- Electric Potential Energy proportional to q
- Again, define

$$V = U/q$$

- Electric Potential **V**:
  - Units are Volt [V] = [J/C]

Electric Potential for many charges

- Superposition principle....

$$V(r) = \sum 1/(4\pi\varepsilon_0)\, Q_i / r_i$$

- Sum of scalars, not vectors!
- Integral for continous distributions

Electric Potential for many charges

- Electric potential depends on charges that create field, not the test charge!

$$V(r) = \sum 1/(4\pi\varepsilon_0)\, Q_i / r_i$$

- V tells us how much energy a charged object can aquire when moving from a to b

Electrical potentials

- Battery: 1.5 V
- Power outlet: 120 V
- HV power line: 10^6 V
- Accelerators: 10^8 V
- Thunderstorm: 10^8 V