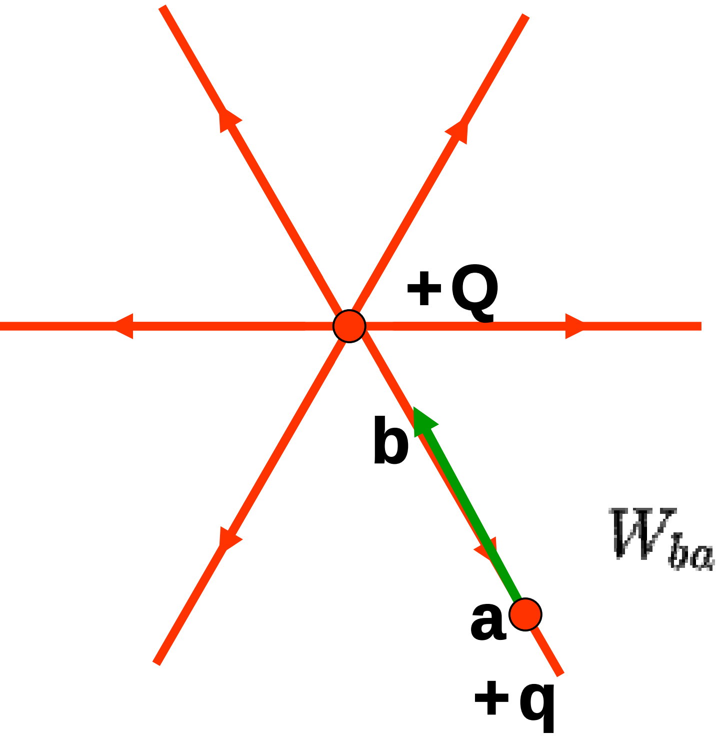
Electricity and Magnetism

- Today
  - Electric Potential Energy
  - Electric Potential
  - Example of calculation
  - Practical applications
  - Conductors, Isolators and Semi-Conductors

Recap

- Work in the Electrostatic Field
- Electrostatic Potential Energy
- Electrostatic Potential

Work and Electrostatic Force



Charge q at point a in Coulomb field of Q

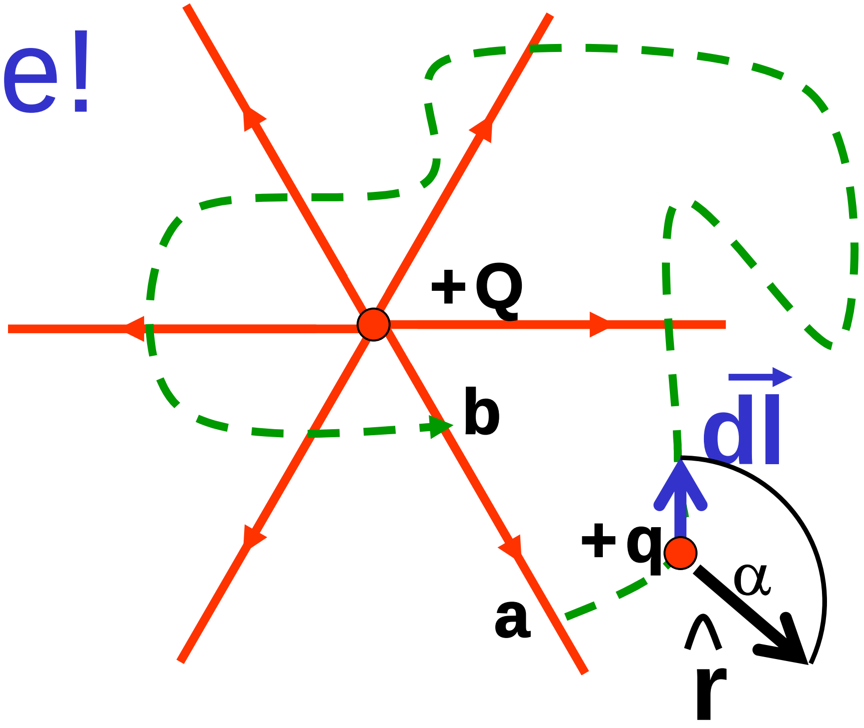
How much work to move to point b?

$$W_{ba} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b q\vec{E} \cdot d\vec{l}$$

$$= q \int_{r_a}^{r_b} E\, dr, \text{ with } E(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

$$= q \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\, dr = \boxed{\frac{1}{4\pi\epsilon_0} Qq\, (\frac{1}{r_a} - \frac{1}{r_b}).}$$

F_{ES} is conservative!



$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\,\hat{r}$$

$$W_{ba} = \int_a^b \vec{F}\cdot d\vec{l} = \frac{Qq}{4\pi\epsilon_0}\int_a^b \frac{1}{r^2}\,\hat{r}\cdot d\vec{l}$$

$$= -\frac{Qq}{4\pi\epsilon_0}\int_a^b \frac{1}{r^2}\,dr;\ \text{because } \hat{r}\cdot d\vec{l} = dl\cos(\alpha) = -dr$$

$$\Rightarrow W_{ba} = U(a) - U(b) \text{ with } U(r) = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$$

Electric Potential Energy

- Note: What we used was the fact that Coulomb Force is ***radial*** **(i.e. $\vec{F} \parallel \vec{r}$)**
  - **all radial forces are conservative (e.g. Gravity)**

Electric Potential Energy

- Potential Energy for two charges

$$U(r) = \frac{Qq}{4\pi\epsilon_0}\frac{1}{r} \text{ for } U(\infty) \equiv 0.$$

- Can only observe differences in potential
  - often set U(∞) = 0 or U(earth) = 0
  - U(r) energy needed to bring q,Q together from infinity

Electric Potential

- Electric Potential Energy proportional to q
- Define **V = U / q**

V = U / q

$$\frac{W_{ba}}{q} = \frac{U(a)}{q} - \frac{U(b)}{q} = V(a) - V(b) = -\Delta V$$

- Electric Potential **V**:
  – Units are Volt [V] = [J/C]

Electric Potential

- Note: because **V = U/q** -> **U = V q**
  - for a given **V**, **U** can be positive or negative, depending on sign of **q**
- Example: Single Charge

$$V(r) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r} \text{ for } V(\infty) \equiv 0.$$

Electric Potential for many charges

- Superposition principle....

$$V(r) = \sum 1/(4\pi\varepsilon_0)\, Q_i / r_i$$

- Sum of scalars, not vectors!
- Integral for continous distributions

Electric Potential for many charges

- Electric potential depends on charges that create field, not the test charge!

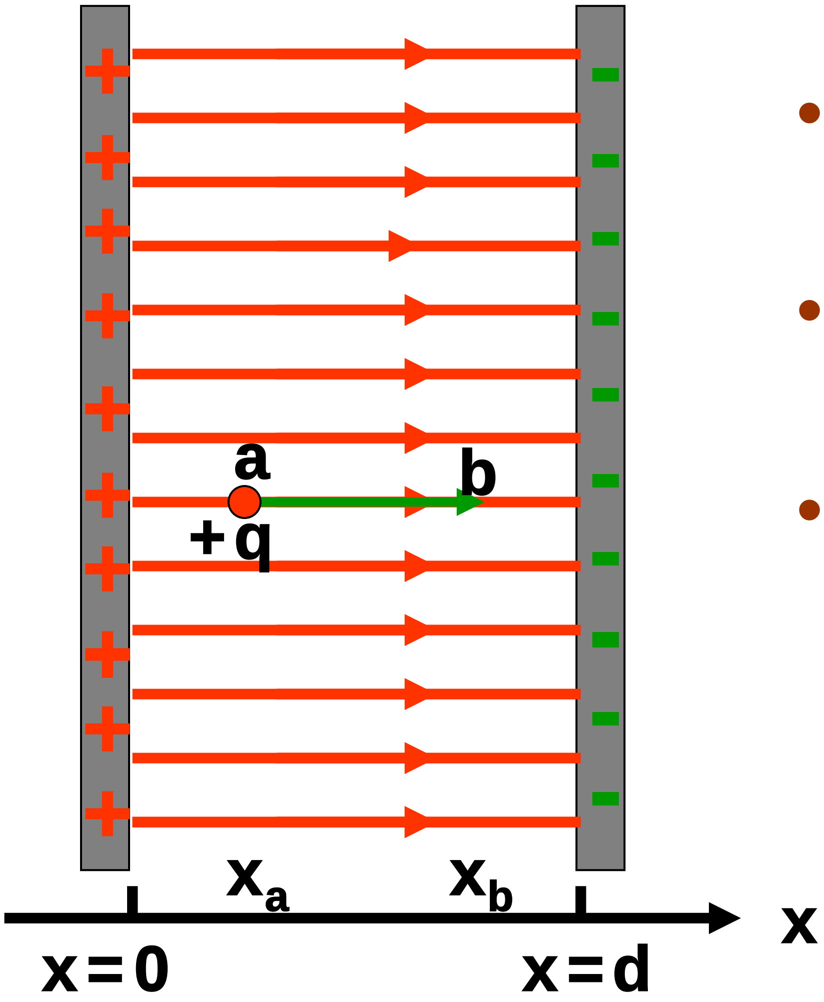
$$V(r) = \Sigma 1/(4\pi\varepsilon_0)\ Q_i/r_i$$

- V tells us how much energy a charged object can aquire when moving from a to b

Electrical potentials

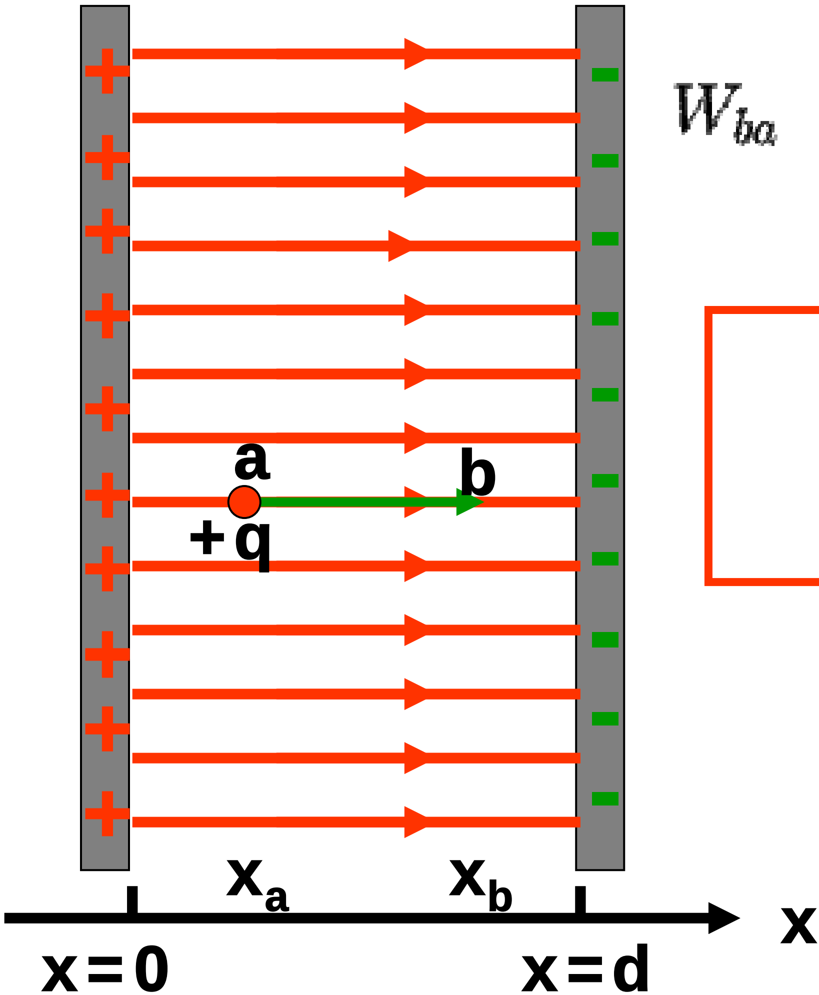
- Battery: 1.5 V
- Power outlet: 120 V
- HV power line: 10^6 V
- Accelerators: 10^8 V
- Thunderstorm: 10^8 V

Example: Capacitor plates



- Deposite opposite charges on plates
- What is the Electric Potential?
  - What does E look like?
- Move charge **+q** from **a** to **b**

Example: Capacitor plates



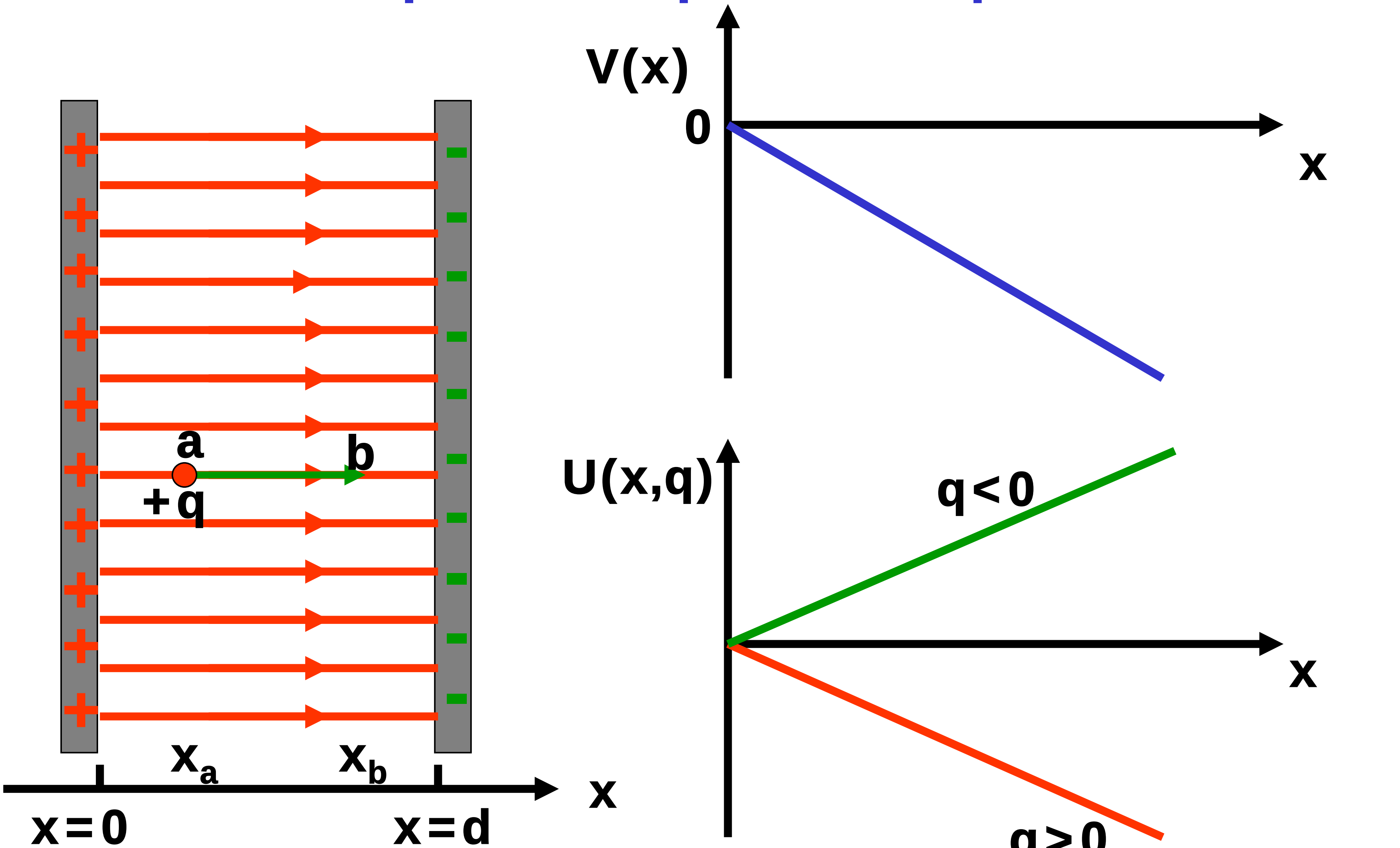
$$W_{ba} = \int_a^b q\vec{E}\cdot d\vec{l} = U(a) - U(b);\ \vec{E}||d\vec{l}$$

$$= \int_a^b qE\,dx = qE(x_b - x_a)$$

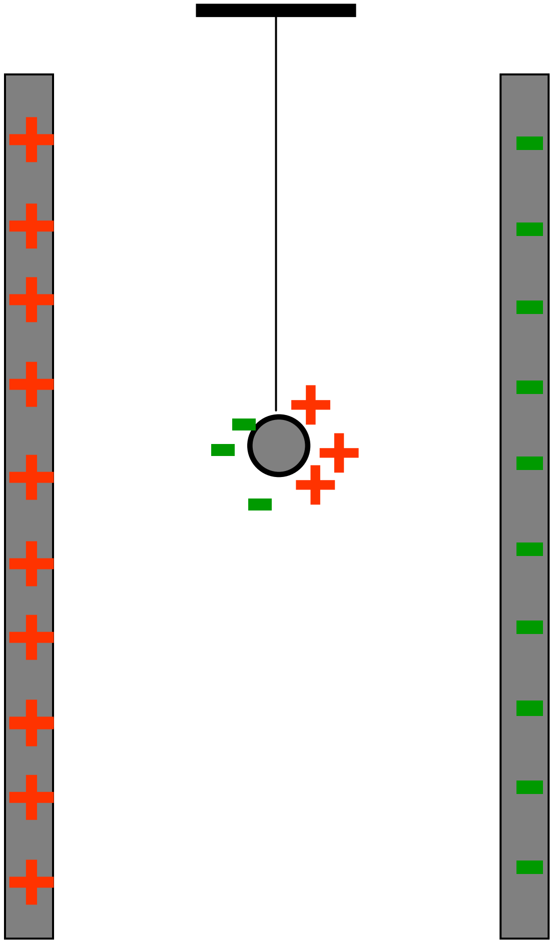
$$\Rightarrow\ U(x) = -qEx;\ (U(0) \equiv 0)$$

$$\Rightarrow\ V(x) = -Ex;\ (V(0) \equiv 0)$$

Example: Capacitor plates

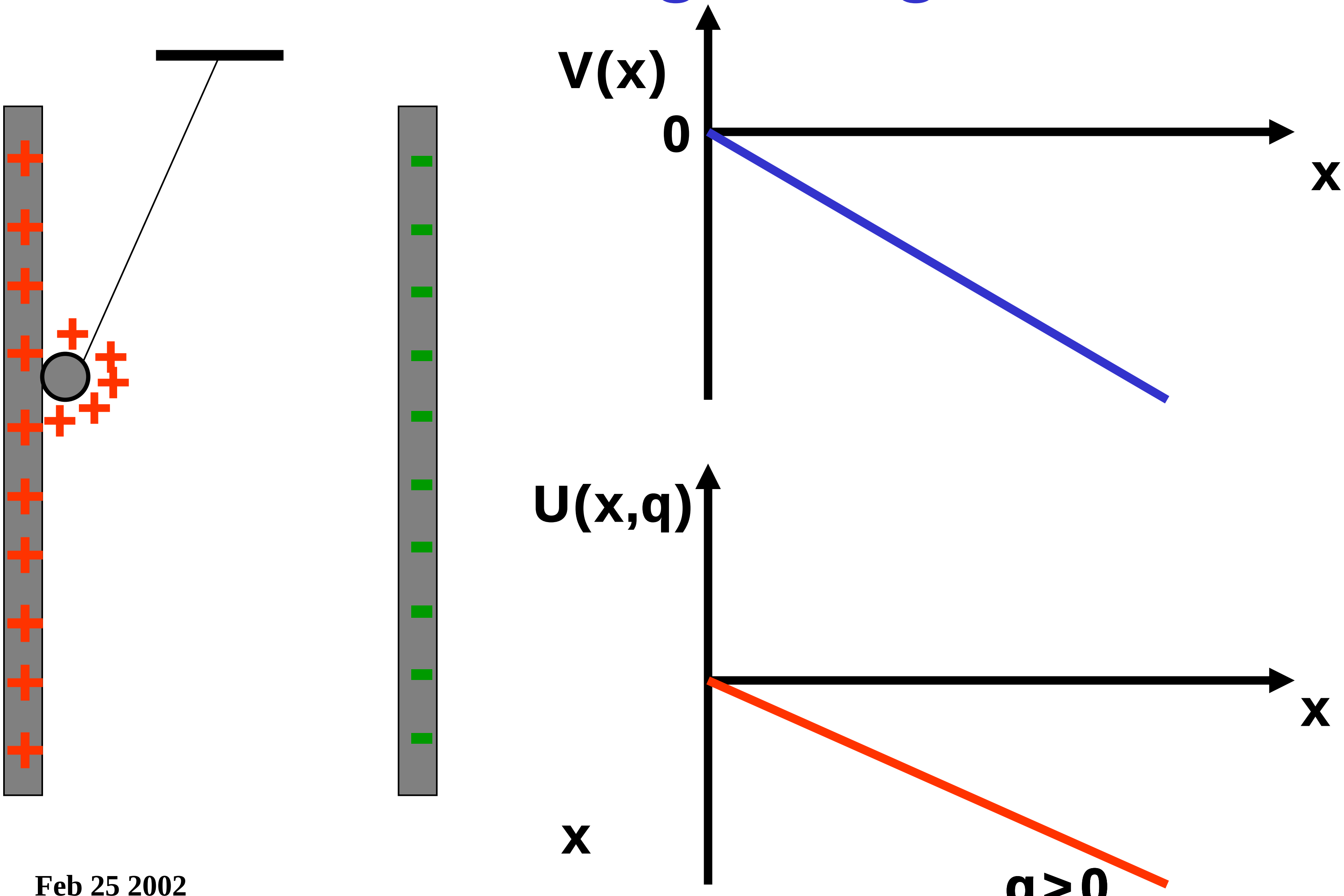


Demo: Ping-Pong Ball

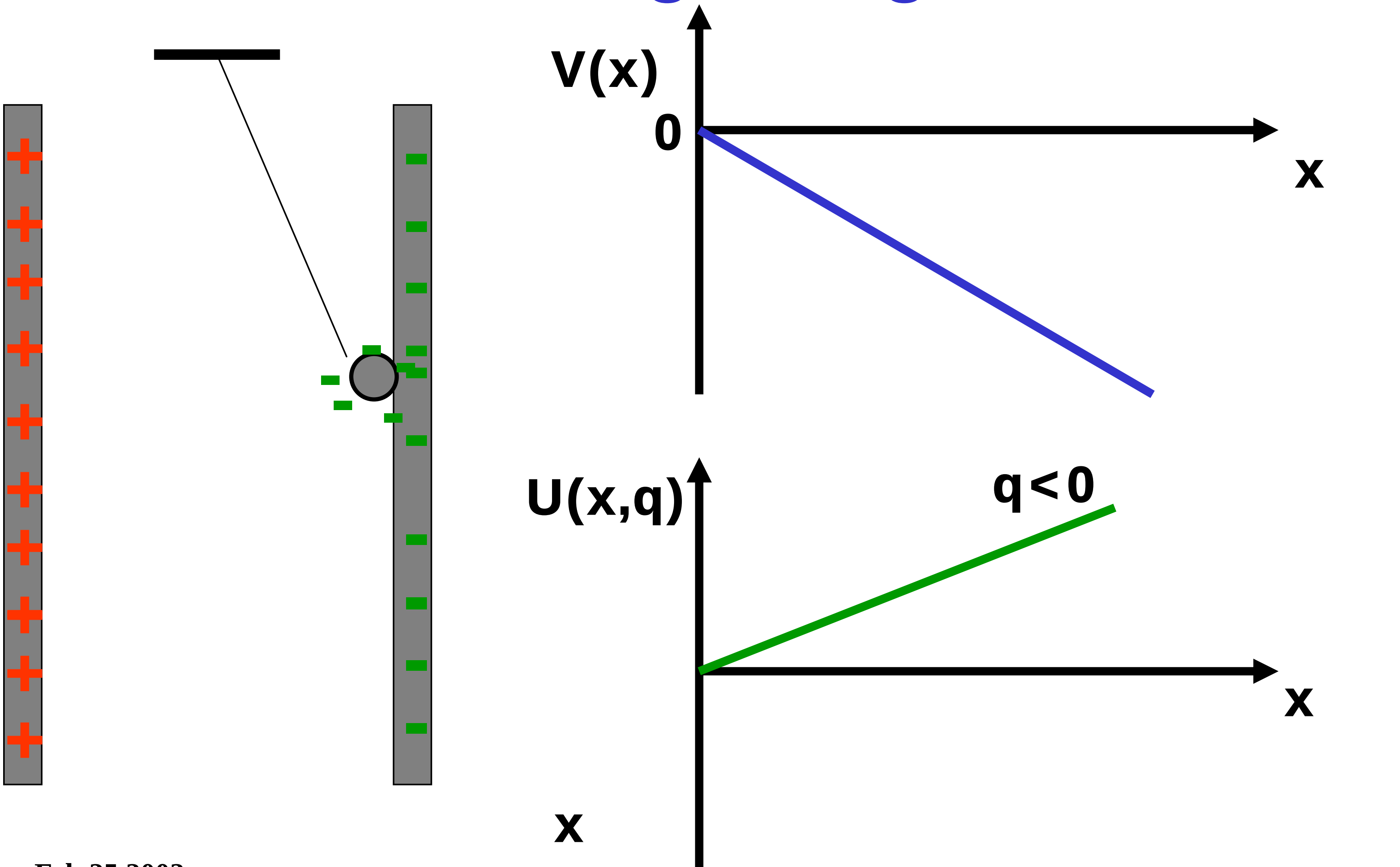


q>0

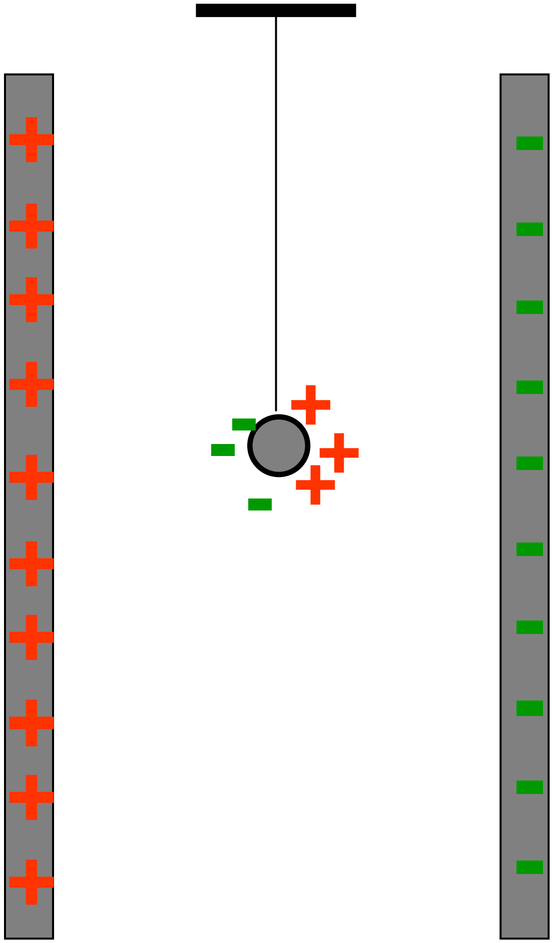
Demo: Ping-Pong Ball

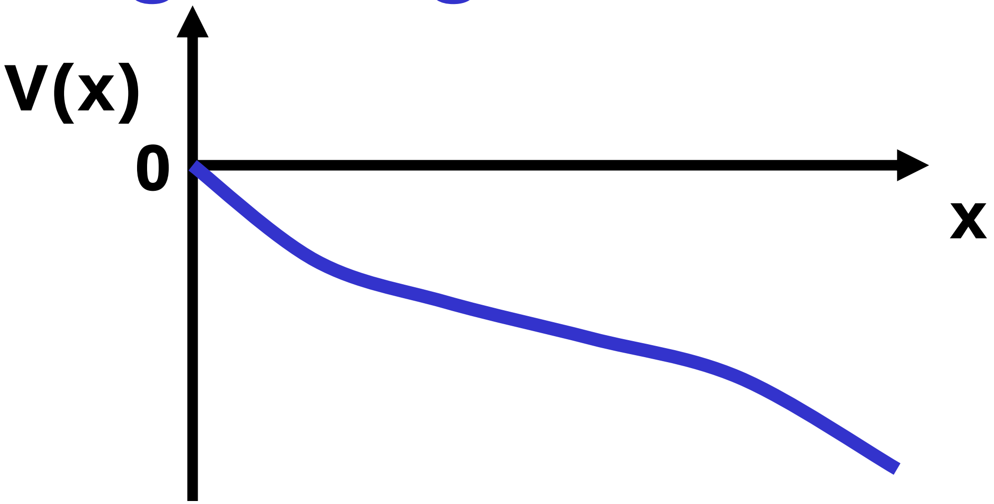


Demo: Ping-Pong Ball



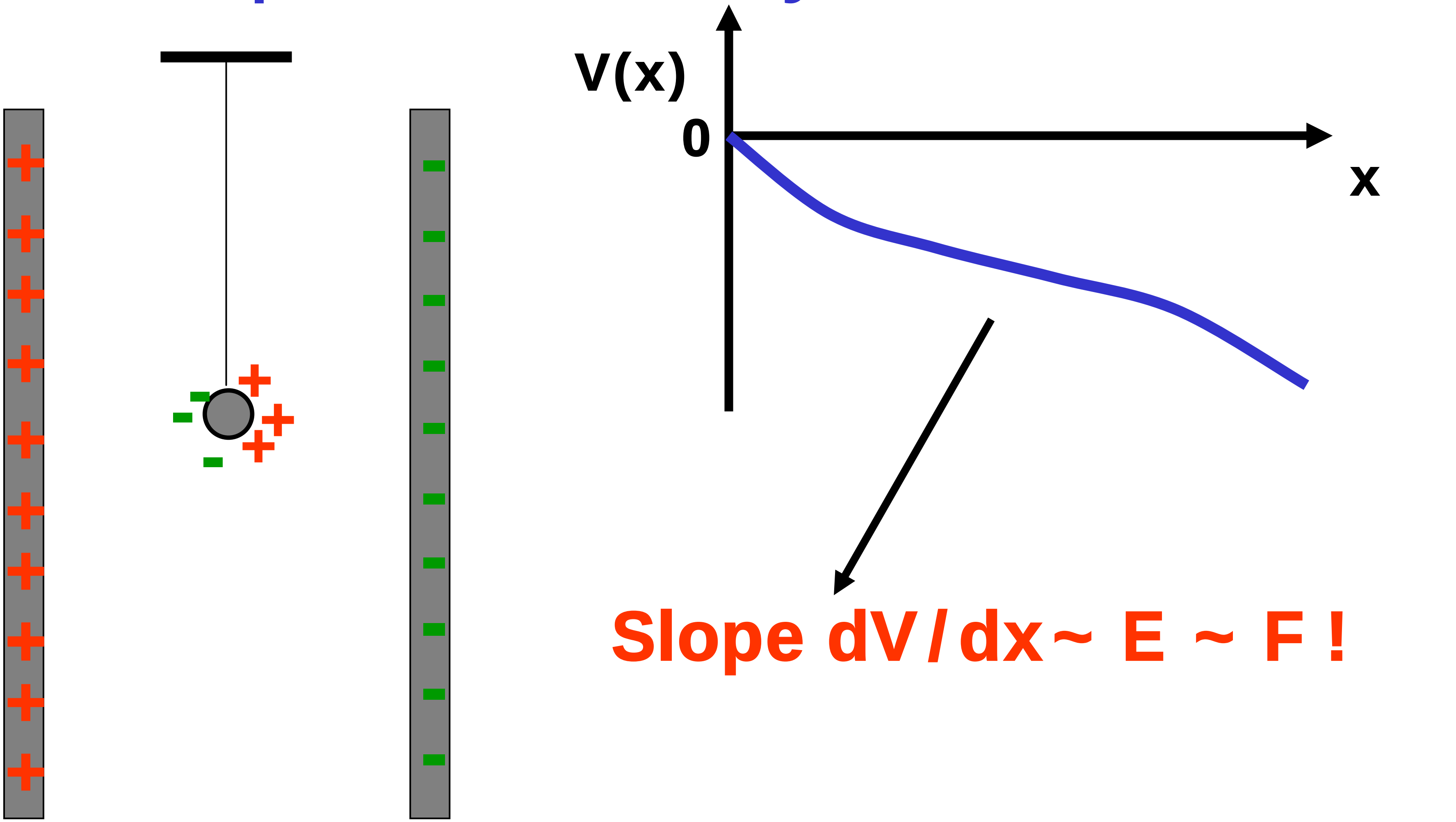
Demo: Ping-Pong Ball



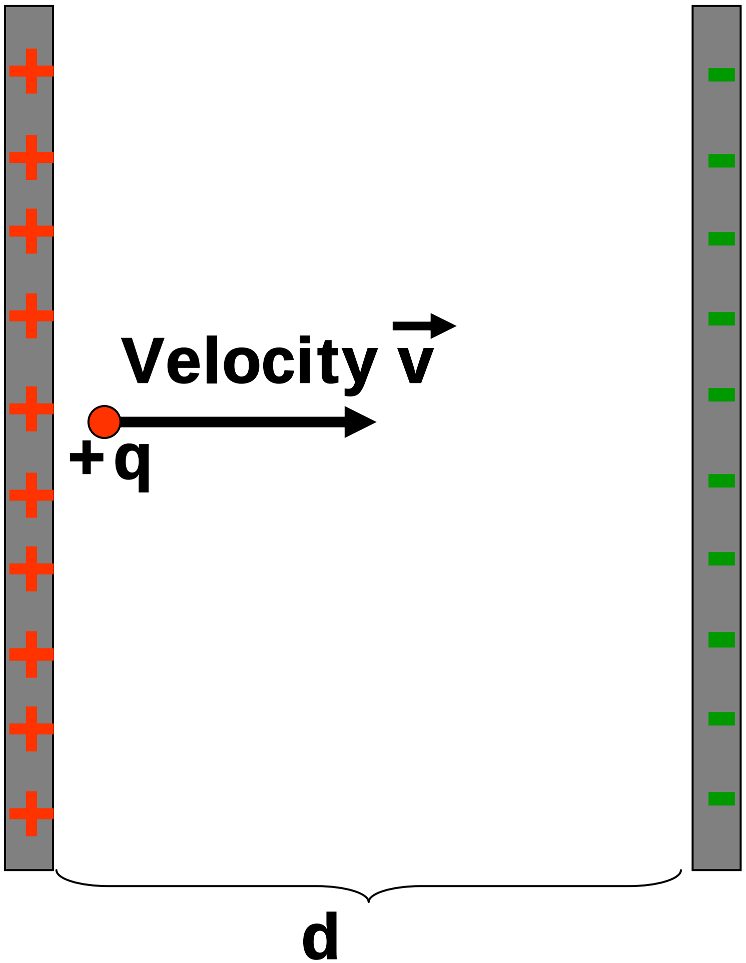


- Field not perfectly uniform:
  - **Net Force on Dipole**

Trick question: Why does it start?



Applications



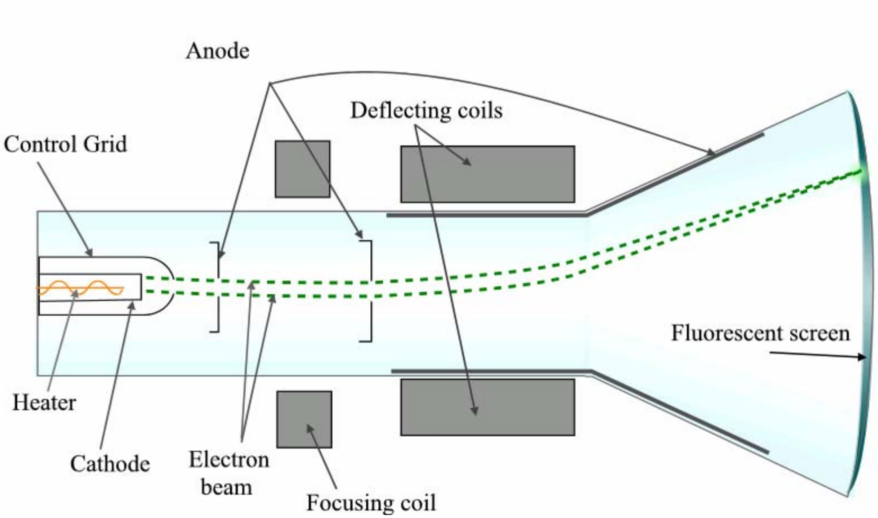
$$1/2mv_0^2 + U(0) = 1/2mv_f^2 + U(d)$$

$$\Rightarrow 1/2mv_f^2 = -q\Delta V$$

- Energy for single particle (e.g. electron) small
- Often measured in 'Electron Volt' [eV]
- Energy aquired by particle of charge 10^{-19} C going through **ΔV=1V**
- **Independent of d**

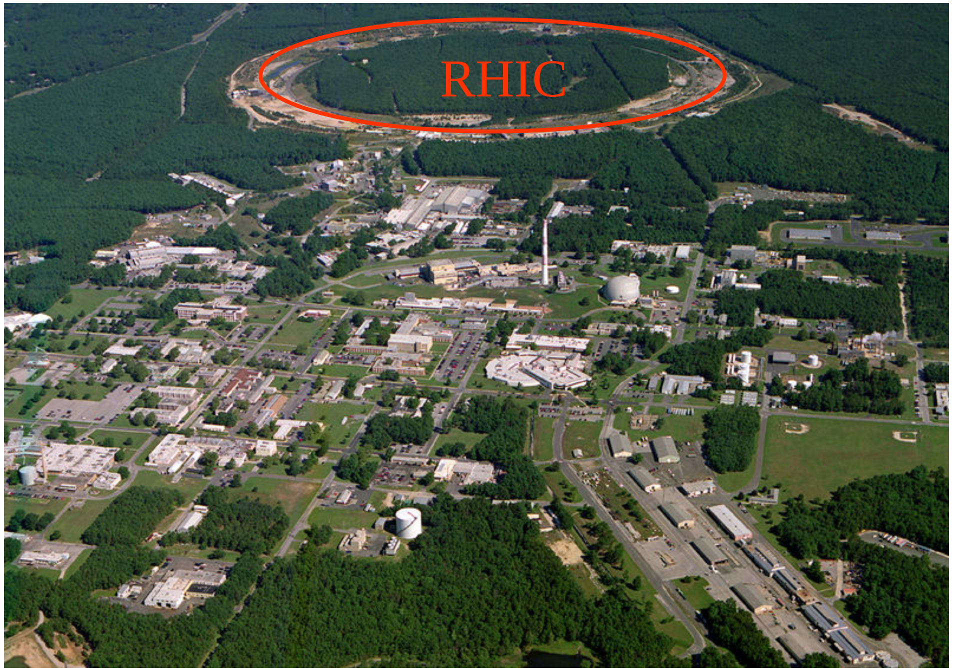
Applications

Cathode Ray Tube



Δ**V**

Relativistic Heavy Ion Collider



E_{kin} ~ 10 keV (10^4eV)

E_{kin} ~ 100 GeV (10^{11}eV)

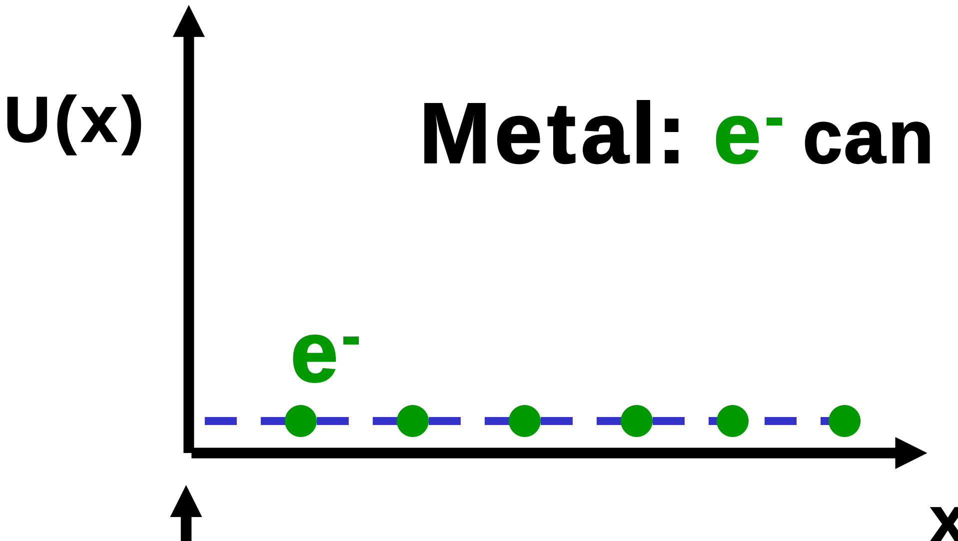
Conductors

- Important for next few weeks
  - Electrical Circuits
- Why are some materials conductive?
  - **All** materials contain electrical charges!

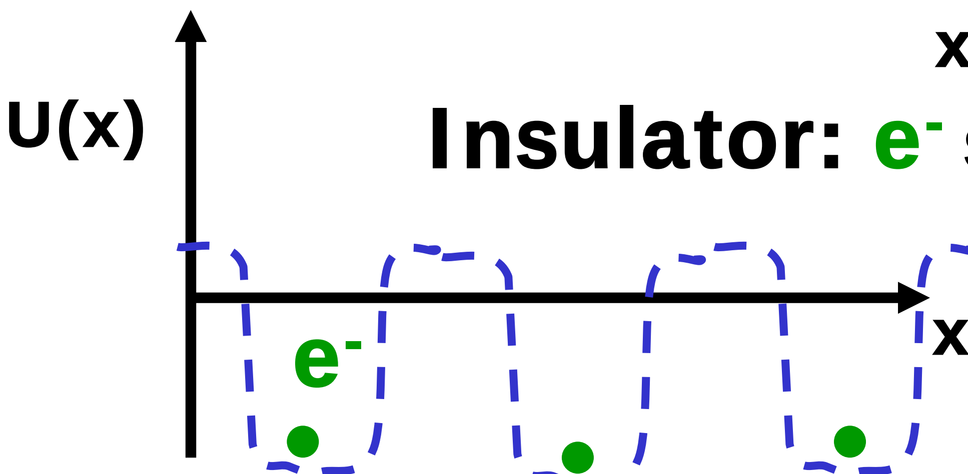
Conductivity

Microscopic view

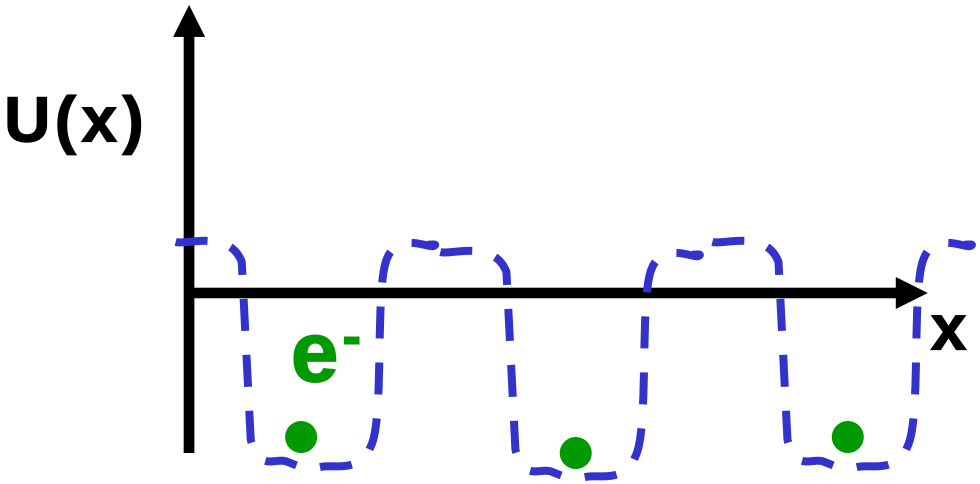
Metal: e⁻ can move around freely



Insulator: e⁻ stuck in place



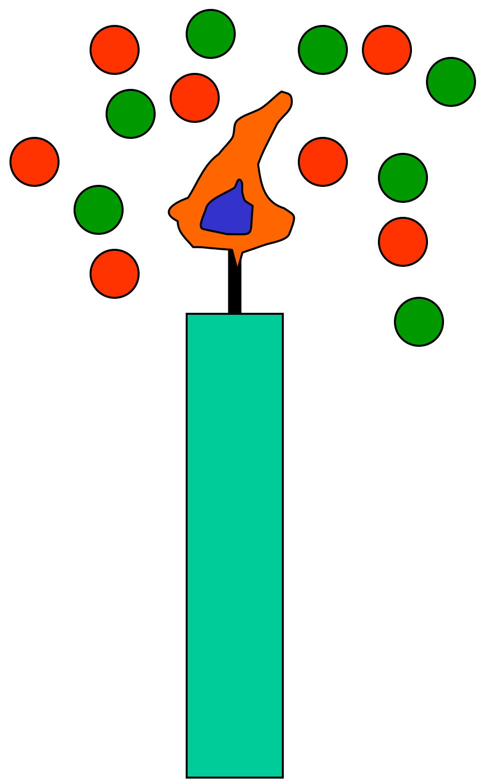
Conductivity

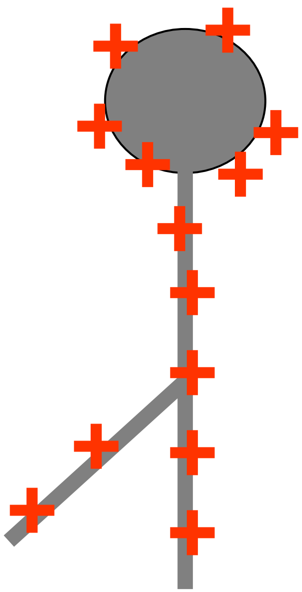


- How can we get charges ‘unstuck’?
  - Give them enough energy to jump out of potential wells

In-Class Demo

Charged Ions

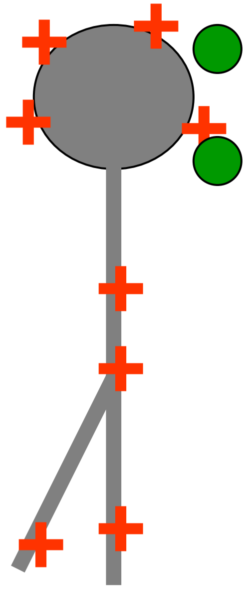




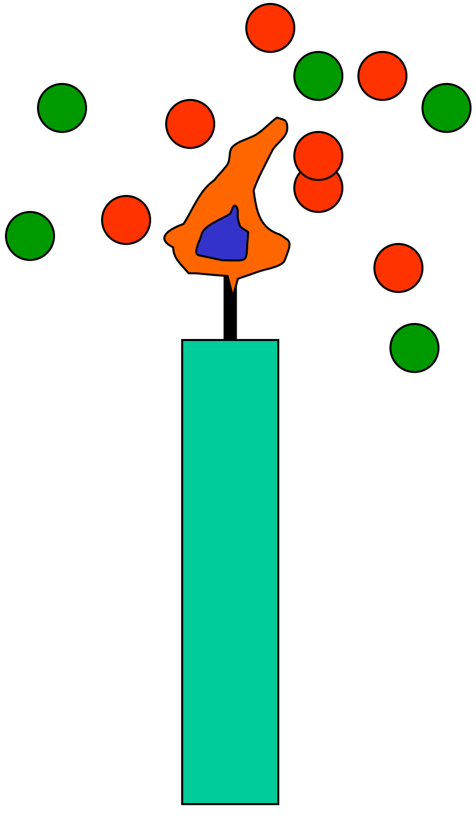
Electroscope

In-Class Demo

Ions discharge Electroscope

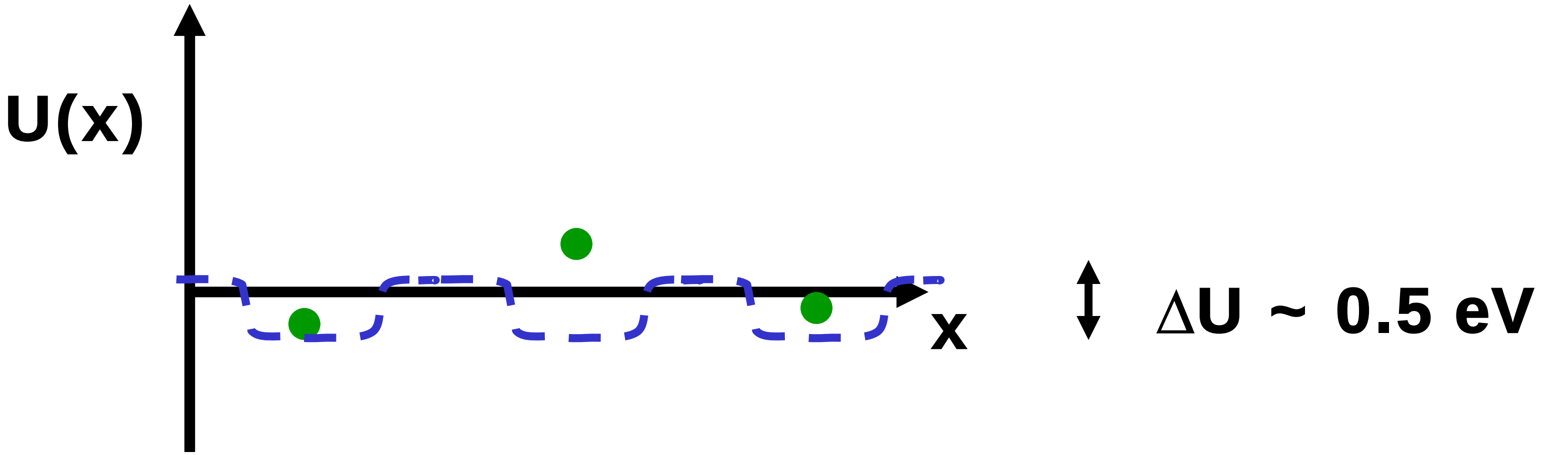


Electroscope



Charged Ions

Important application



- **Semi-conductor**
  - # of charges controllable (by T and V)
  - At T=0^{o}C E_{kin} ~ 1/40 eV
  - Basis of all Electronic Circuits (e.g. Computers)

Conductivity

- Note: Usually, charge carried by electrons, but not always
  - ‘holes’ (i.e. missing electrons) in semi-conductors

Conductors

- E = 0 inside
  - otherwise charges would move
- No charges inside
  - Gauss
- E perpendicular to surface
  - otherwise charges on surface would move
- Potential is constant on conductor

Conductors

- Potential is constant on conductor

$$V(a) - V(b) = \frac{W_{ba}}{q} = \frac{1}{q}\int_a^b \vec{E} d\vec{l}$$
$$= 0 \text{ as } E = 0 \text{ in conductors .}$$