Electricity and Magnetism

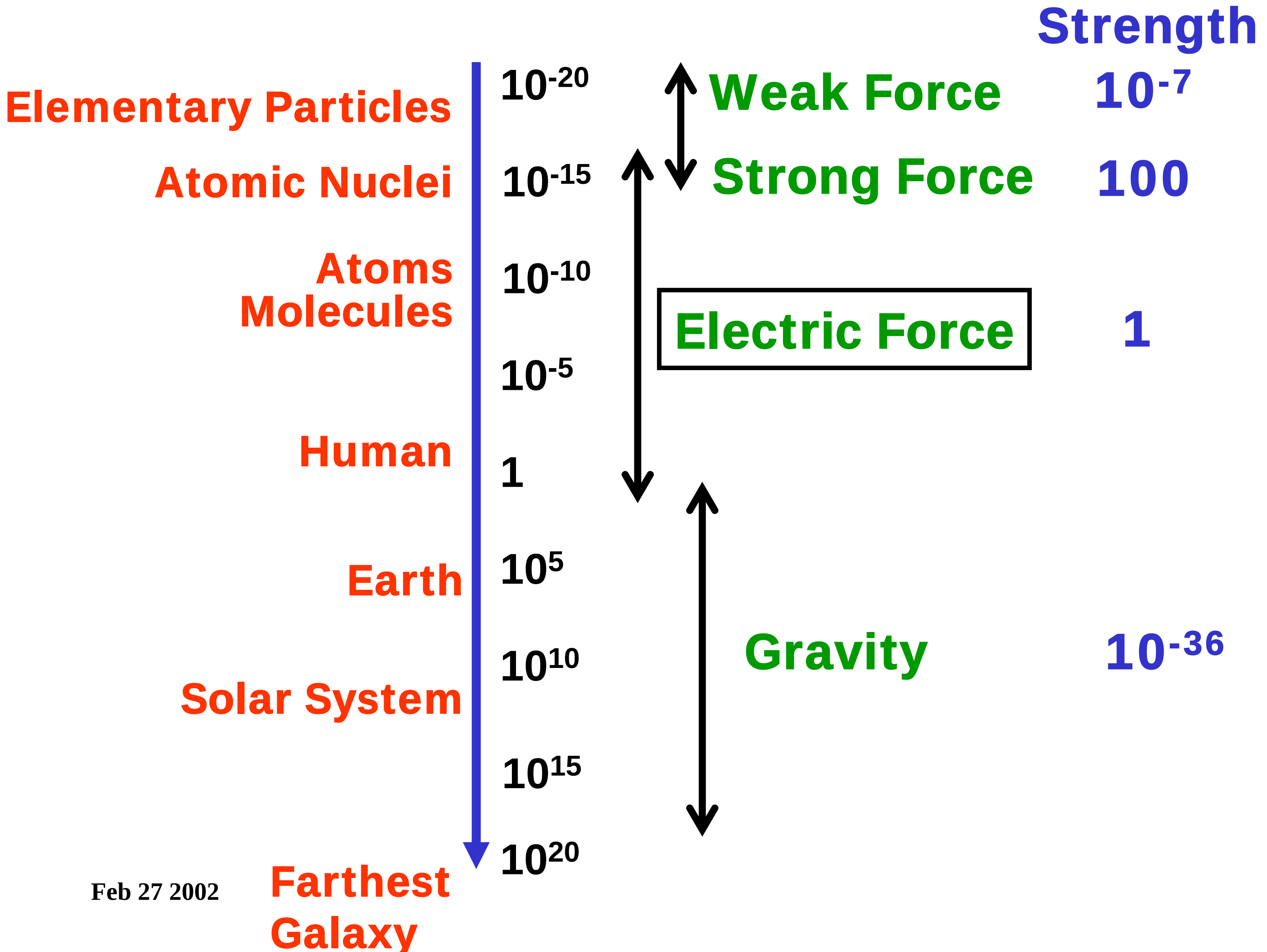
- Review
  - Electric Charge and Coulomb's Force
  - Electric Field and Field Lines
  - Superposition principle
  - E.S. Induction
  - Electric Dipole
  - Electric Flux and Gauss' Law
  - Electric Potential Energy and Electric Potential
  - Conductors, Isolators and Semi-Conductors

Today

- Fast summary of all material so far
  - show logical sequence
  - help discover topics to refresh for Friday

Electric Charge and Electrostatic Force

- New Property of Matter: **Electric Charge**
  - comes in two kinds: ‘**+**’ and ‘**-**’
- Connected to **Electrostatic Force**
  - attractive (for ‘**+-**’) or repulsive (‘**- -**’, `**++**’)
- Charge is conserved
- Charge is quantized


Strength
Elementary Particles
Atomic Nuclei
Atoms
Molecules
Human
Earth
Solar System
Farthest Galaxy
10^{-20}
10^{-15}
10^{-10}
10^{-5}
1
10^{5}
10^{10}
10^{15}
10^{20}
Weak Force 10^{-7}
Strong Force 100
Electric Force 1
Gravity 10^{-36}
Feb 27 2002

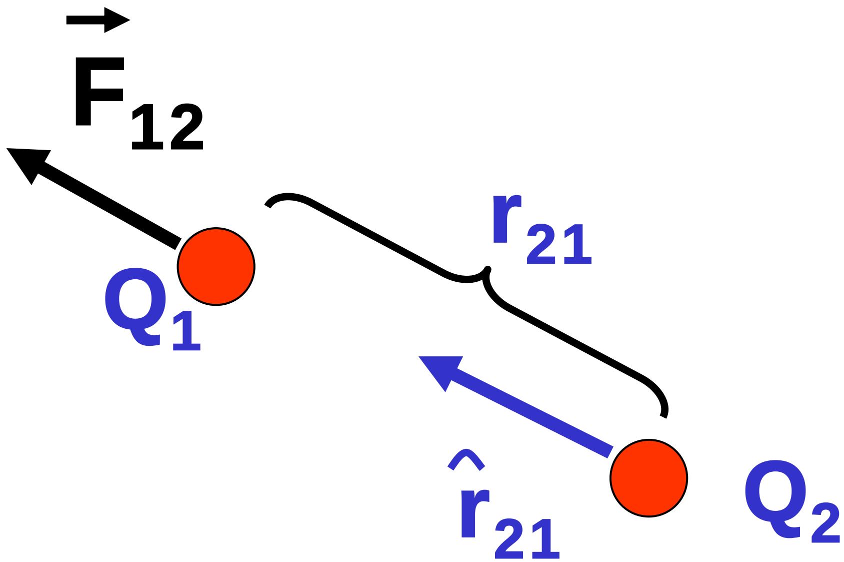
Coulomb's Law

$$\vec{F}_{12} = k\,\frac{Q_1 Q_2}{r_{21}^2}\,\hat{r}_{21} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{21}^2}\,\hat{r}_{21}$$

- Inverse square law ($F \sim 1/r^2$)
- Gives magnitude and direction of Force
- Attractive or repulsive depending on sign of Q_1Q_2

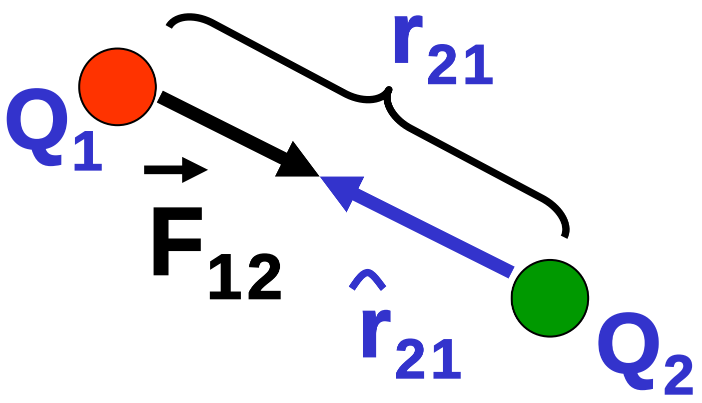
Coulomb's Law

$$\vec{F}_{12} = k\,\frac{Q_1 Q_2}{r_{21}^2}\,\hat{r}_{21} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{21}^2}\,\hat{r}_{21}$$



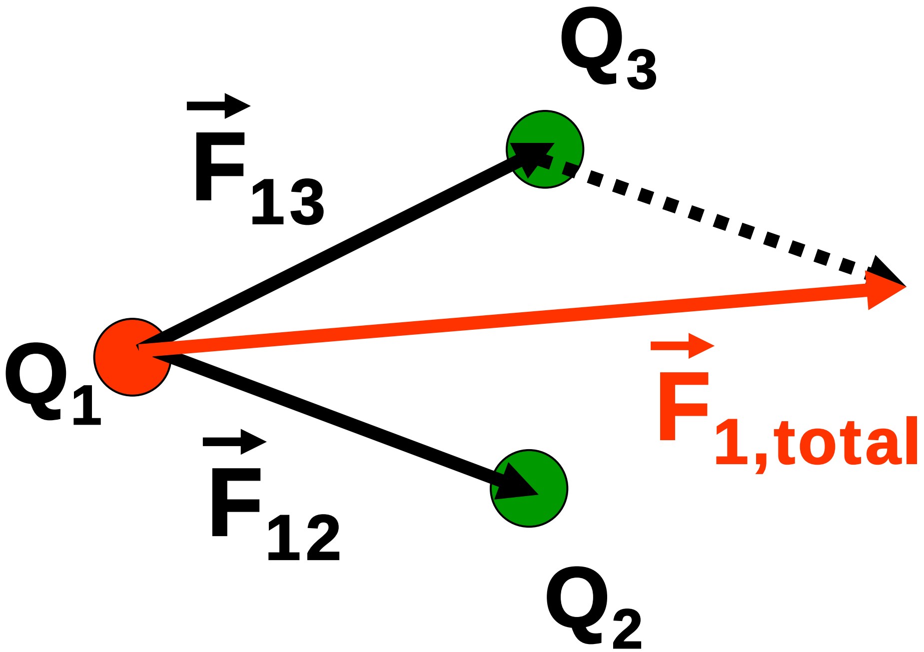
Coulomb's Law

$$\vec{F}_{12} = k\,\frac{Q_1 Q_2}{r_{21}^2}\,\hat{r}_{21} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{21}^2}\,\hat{r}_{21}$$



$$\vec{F}_{12} = -\vec{F}_{21}$$

Superposition principle



- Note:
  - Total force is given by vector sum
  - Watch out for the charge signs
  - Use symmetry when possible

Superposition principle

- If we have many, many charges
  - Approximate with continous distribution
- Replace sum with integral!

$$\vec{F}_{0,total} = \int d\vec{F}_0 = \int k \cdot \frac{Q_0 dQ}{r^2} \hat{r}$$

Electric Field

- New concept – Electric Field $\vec{E}$
- Charge Q gives rise to a Vector Field

$$\vec{E}(\vec{x}) = \vec{F}(\vec{x})/q$$

- $\vec{E}$ is defined by strength and direction of force on small test charge q

The Electric Field

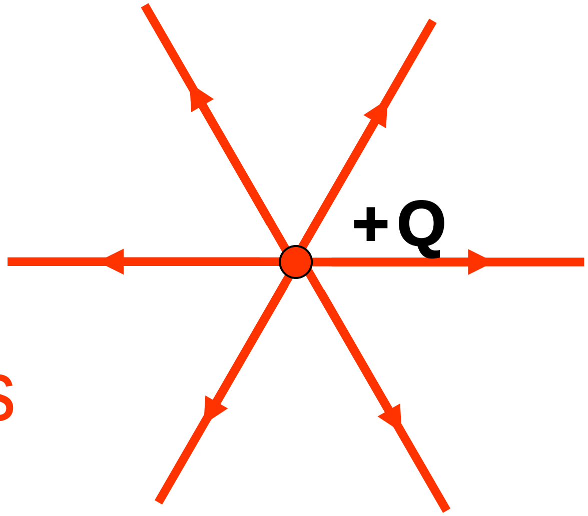
- Electric Field also exists is test charge q is not present
- The charge Q gives rise to a property of space itself – the Electric Field
- For more than one charge -> Superposition principle

Electric Field

- For a single charge

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

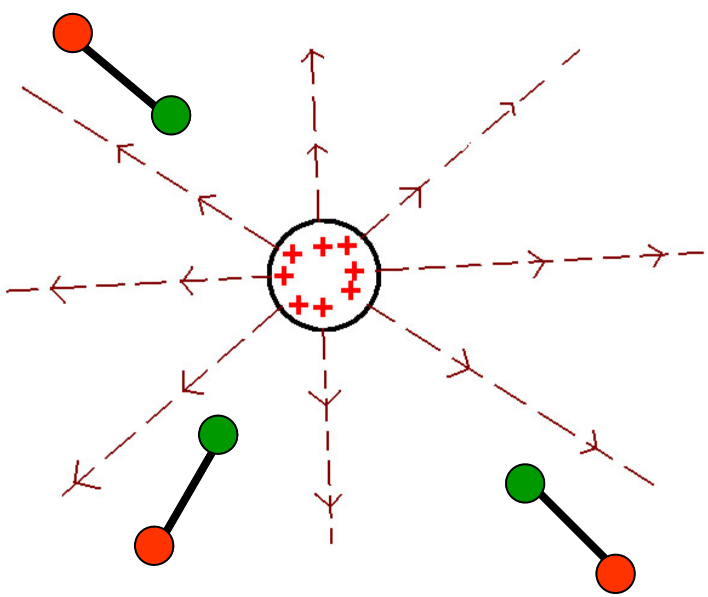
- Visualize using Field Lines

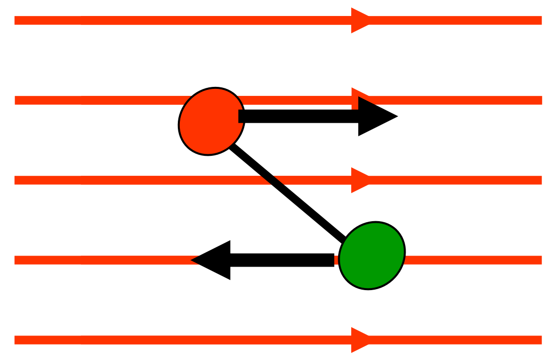


Field Lines

- Rules for field lines
  - Direction: In direction of $\vec{E}$ at each point
  - Density: Shows magnitude of **E**
  - Field Lines never cross
  - From positive to negative charge
    - i.e. show direction of force on a positive charge
  - Far away: Everything looks like point charge

Electric Dipole

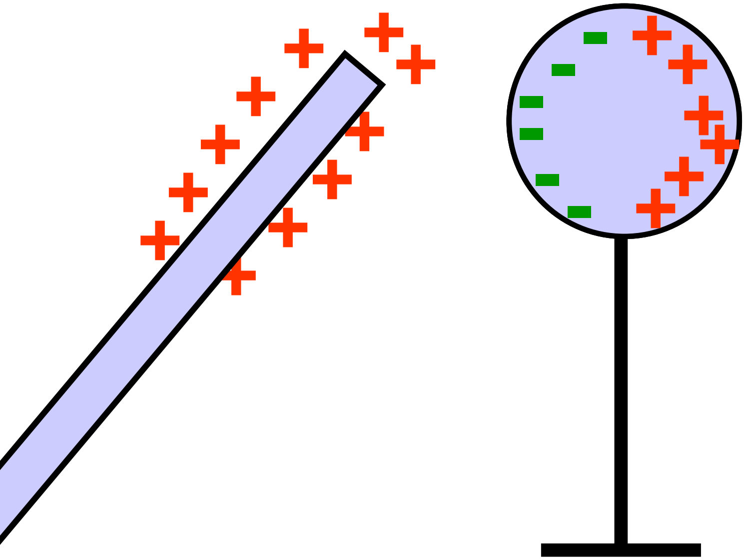




Torque $\vec{\tau} = \vec{p} \times \vec{E}$

$\vec{p} = Q\,\vec{l}$ Dipole moment

Electrostatic Induction



- Approach neutral object with charged object
- Induce charges (dipole)
- Force between charged and globally neutral object

Electric Flux

- Electric Flux: $\Phi_E = \vec{E} \cdot \vec{A}$
- Same mathematical form as water flow
- No 'substance' flowing
- Flux tells us how much field 'passes' through surface A

Electric Flux

- For ‘complicated’ surfaces and non-constant E:
  - Use integral

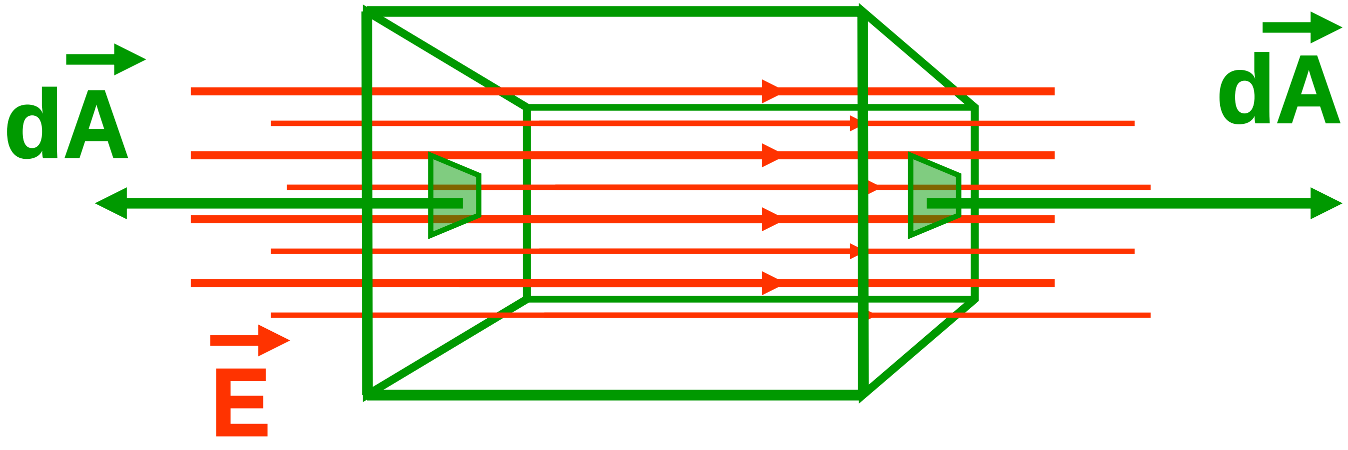
$$\Phi_E = \int_A \vec{E} \cdot d\vec{A}$$

- Often, ‘closed’ surfaces

$$\Phi_E = \oint_A \vec{E} \cdot d\vec{A}$$

Electric Flux

- Example of closed surface: Box (no charge inside)



- Flux in (left) = -Flux out (right): $\Phi_E = 0$

Gauss' Law

- How are flux and charge connected?
- Charge Q_{encl} as source of flux through closed surface

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Gauss' Law

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

- True for ANY closed surface around Q_{encl}
- Relates charges (cause) and field (effect)

Gauss' Law

- Different uses for Gauss' Law
  - Field **E** -> **Q_{encl}** (e.g. conductor)
  - **Q_{encl}** -> Field **E** (e.g. charged sphere)
- Proper choice of surface – use symmetries

$$\vec{E} \perp d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = 0$$

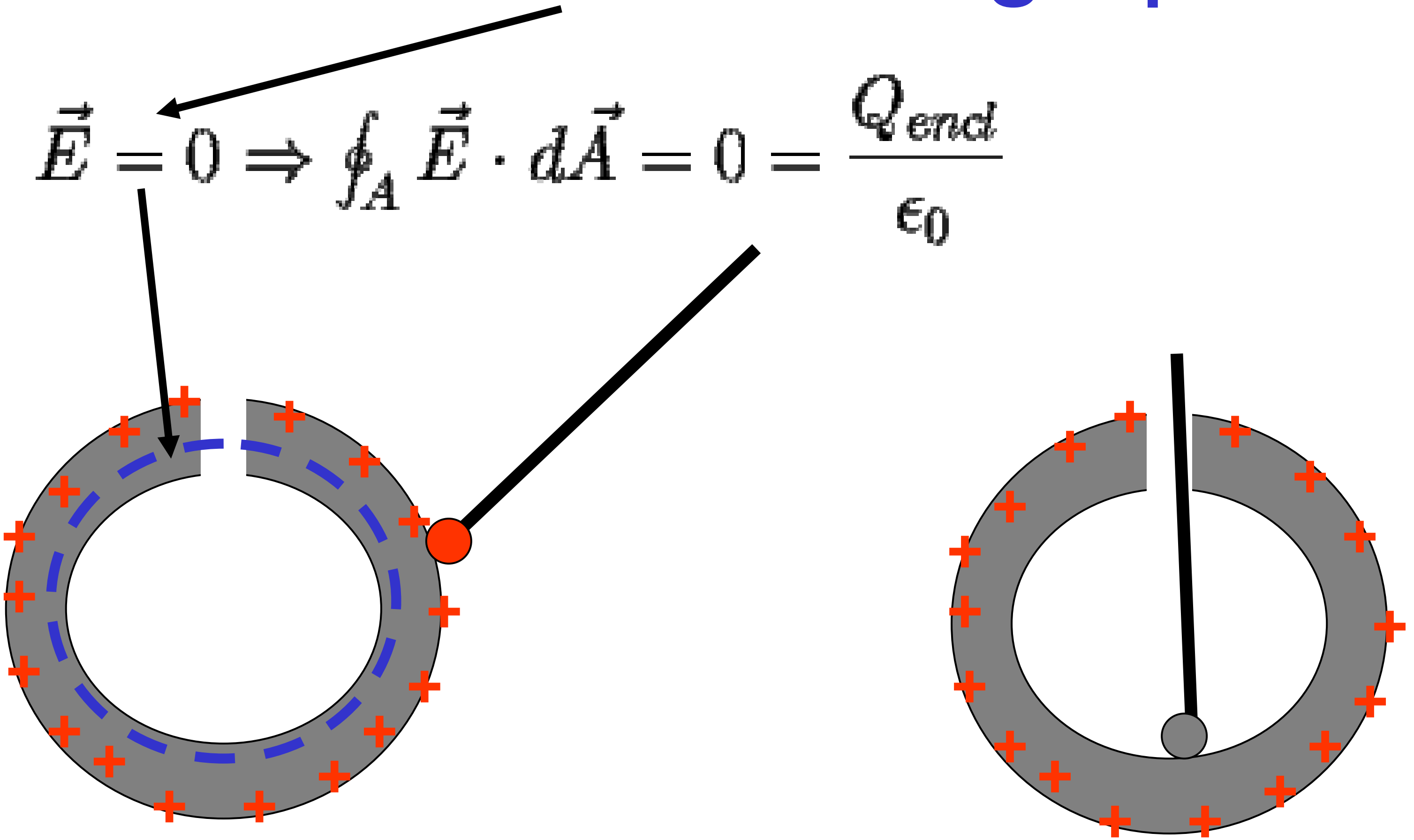
$$\vec{E} || d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = E dA$$

$$E(r) = \text{const.} \Rightarrow \oint_A E \cdot dA = E \oint_A dA = EA$$

$$\vec{E} = 0 \Rightarrow \oint_A \vec{E} \cdot d\vec{A} = 0 = \frac{Q_{encl}}{\epsilon_0}$$

Hollow conducting Sphere

$$\vec{E} = 0 \Rightarrow \oint_A \vec{E} \cdot d\vec{A} = 0 = \frac{Q_{encl}}{\epsilon_0}$$



Gauss' Law

- Charge Sphere radius r_0, charge Q, r > r_0

$$\frac{Q_{encl}}{\epsilon_0} = \oint_{sphere} \vec{E} \cdot d\vec{A} =$$

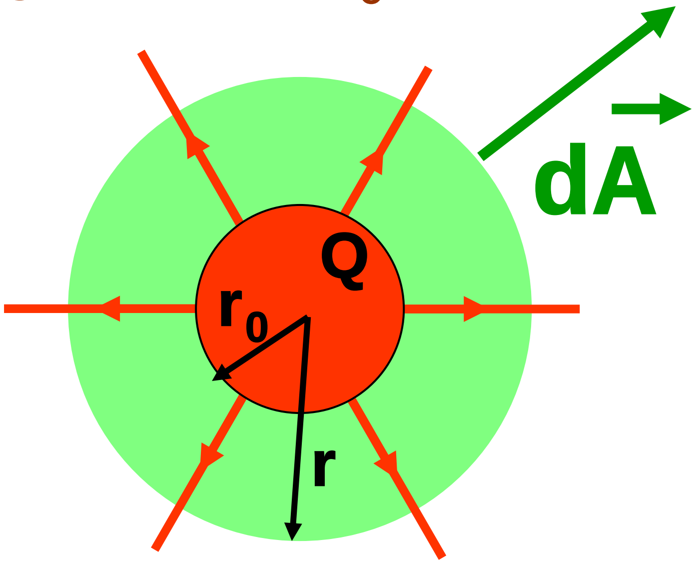
$$\oint_{sphere} E dA =$$

$$E \oint_{sphere} dA =$$

$$E(4\pi r^2) \Longrightarrow$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q_{encl}}{r^2}$$

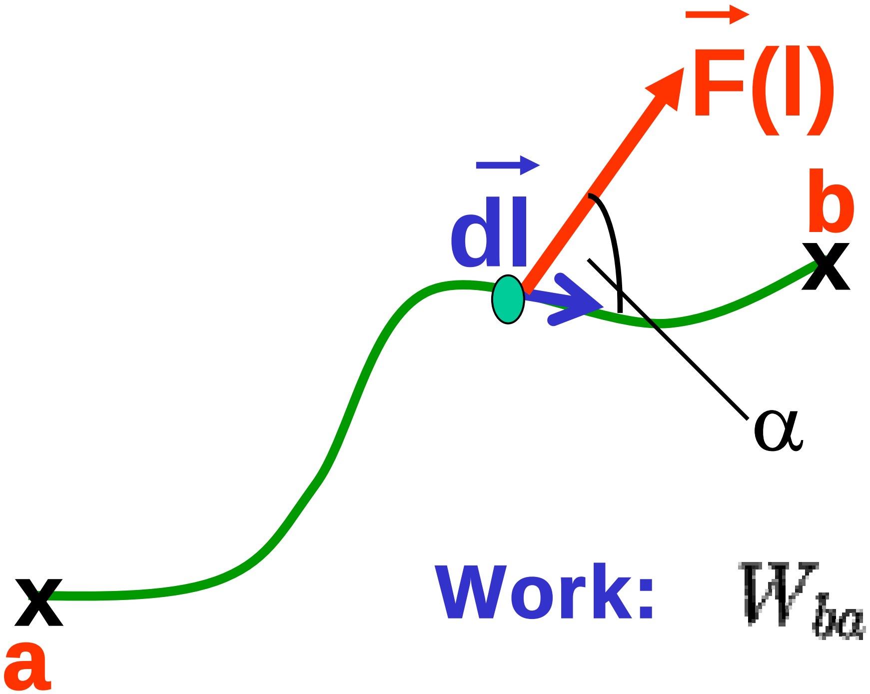
← Q_{encl} = Q



Gauss' Law

- Most uses of Gauss' Law rely on simple symmetries
  - Spherical symmetry
  - Cylinder symmetry
  - (infinite) plane
- and remember, **$E = 0$** in conductors

Work and Potential Energy



Work: $W_{ba} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b F\,dl \cos(\alpha)$

Conservative Force: $W_{ba} = -\Delta U = U(a) - U(b)$

Potential Energy

Feb 27 2002

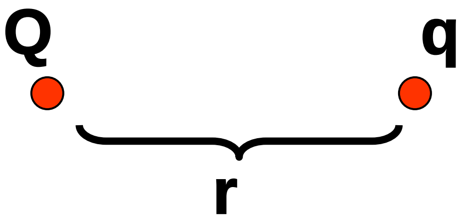
Electric Potential Energy

- Electric Force is conservative
  - all radial forces are conservative (e.g. Gravity)
- We can define Electric Potential Energy

$$W_{ba} = \int_a^b q\vec{E} \cdot d\vec{l} = U(a) - U(b) = -\Delta U$$

$\vec{\mathbf{F}}$

Example: Two charges



$$U(r) = \frac{Qq}{4\pi\epsilon_0}\frac{1}{r} \text{ for } U(\infty) \equiv 0.$$

- If q,Q same sign:
  - U > 0; we have to do work ‘pushing’ charges together
- If q,Q unlike sign:
  - U < 0; Electric force does work ‘pulling’ charges together

Electric Potential

- Electric Potential Energy proportional to q
- Define **V = U / q**

$$\frac{W_{ba}}{q} = \frac{U(a)}{q} - \frac{U(b)}{q} = V(a) - V(b) = -\Delta V$$

- Electric Potential **V**:
  - Units are Volt [V] = [J/C]

Electric Potential

- Note: because **V = U/q** -> **U = V q**
  - for a given **V**: **U** can be positive or negative, depending on sign of **q**
- **V** :Work per unit charge to bring **q** from a to b

$$\frac{W_{ba}}{q} = \frac{U(a)}{q} - \frac{U(b)}{q} = V(a) - V(b) = -\Delta V$$

- Ex.: Single Charge $V(r) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$ for $V(\infty) \equiv 0.$

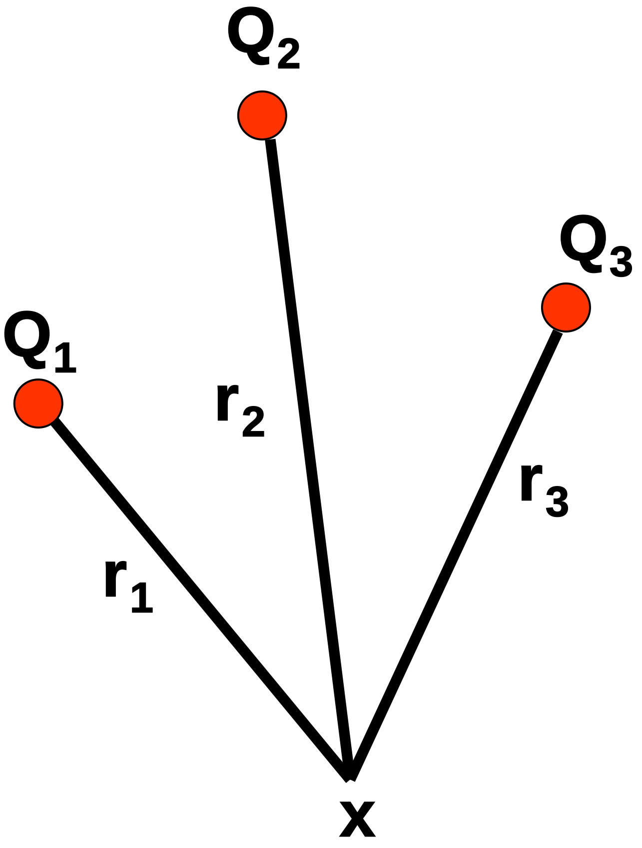
Electric Potential for many charges

- Superposition principle….

$$V(x) = \sum 1/(4\pi\varepsilon_0)\, Q_i/r_i$$

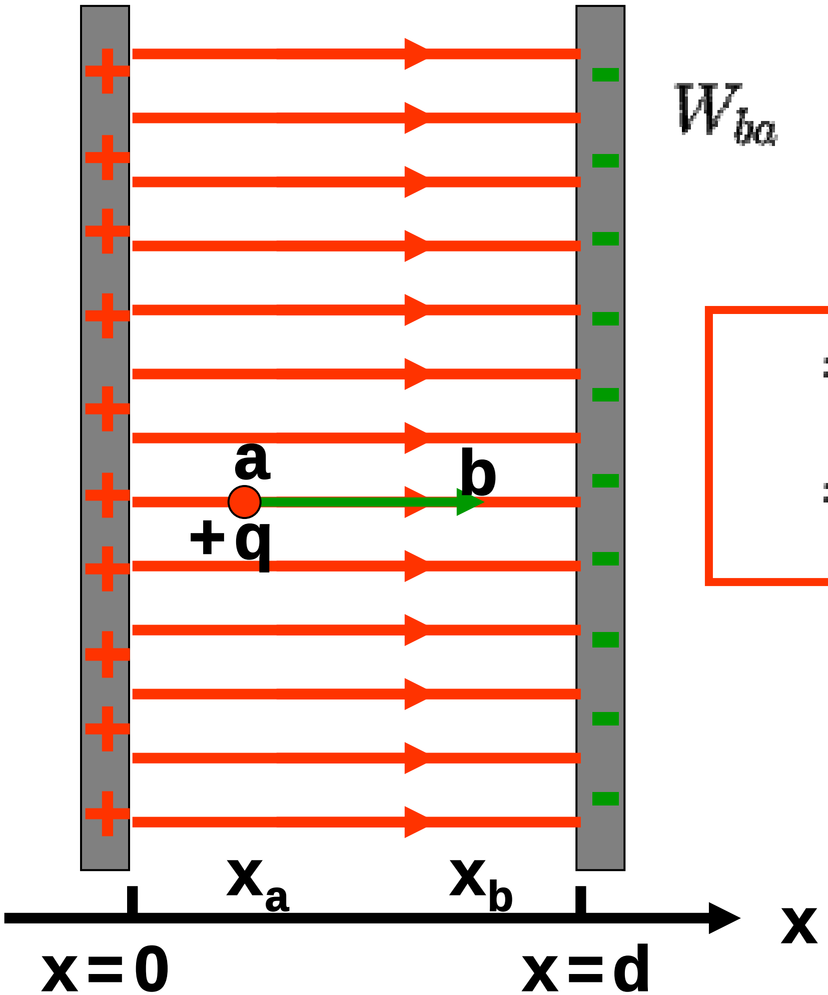
- Sum of scalars, not vectors!
- Integral for continous distributions

Example: Three charges



$$V(x) = \sum 1/(4\pi\varepsilon_0)\, Q_i / r_i$$

Example: Capacitor plates



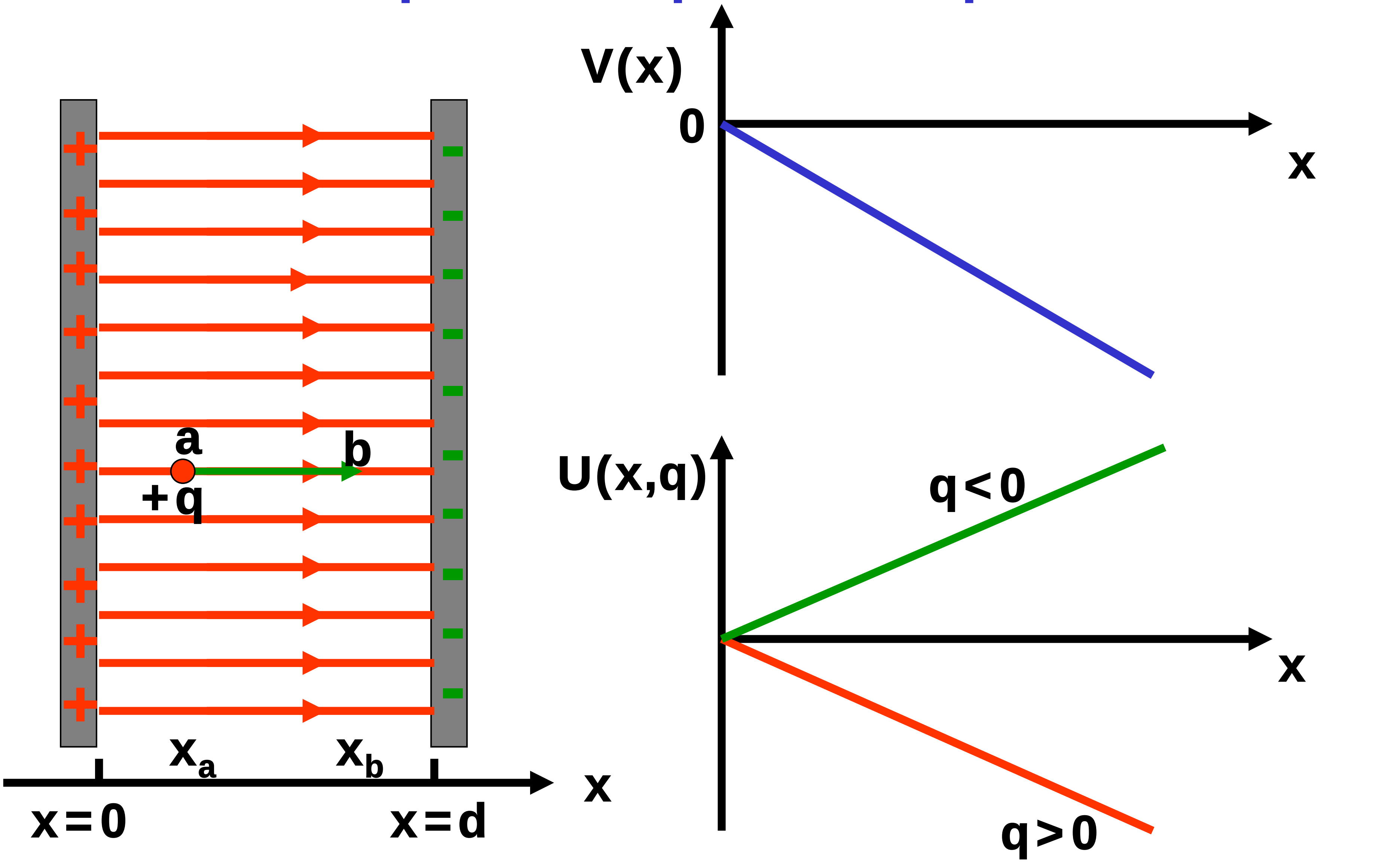
$$W_{ba} = \int_a^b q\vec{E}\cdot d\vec{l} = U(a) - U(b);\ \vec{E}||d\vec{l}$$

$$= \int_a^b qE dx = qE(x_b - x_a)$$

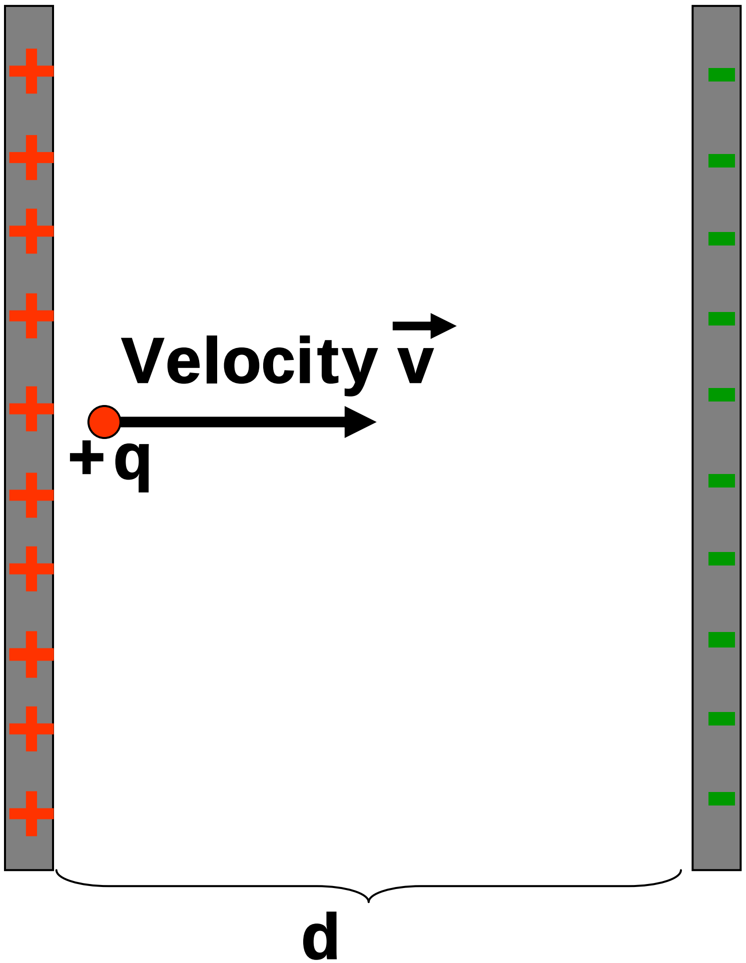
$$\Rightarrow U(x) = -qEx;\ (U(0) \equiv 0)$$

$$\Rightarrow V(x) = -Ex;\ (V(0) \equiv 0)$$

Example: Capacitor plates



Applications



$$1/2mv_0^2 + U(0) = 1/2mv_f^2 + U(d)$$

$$\Rightarrow 1/2mv_f^2 = -q\Delta V$$

- Energy for single particle (e.g. electron) small
- Often measured in 'Electron Volt' [eV]
- Energy aquired by particle of charge 10^{-19} C going through **ΔV=1V**

Conductors

- E = 0 inside
  - otherwise charges would move
- No charges inside
  - Gauss
- E perpendicular to surface
  - otherwise charges on surface would move
- Potential is constant on conductor