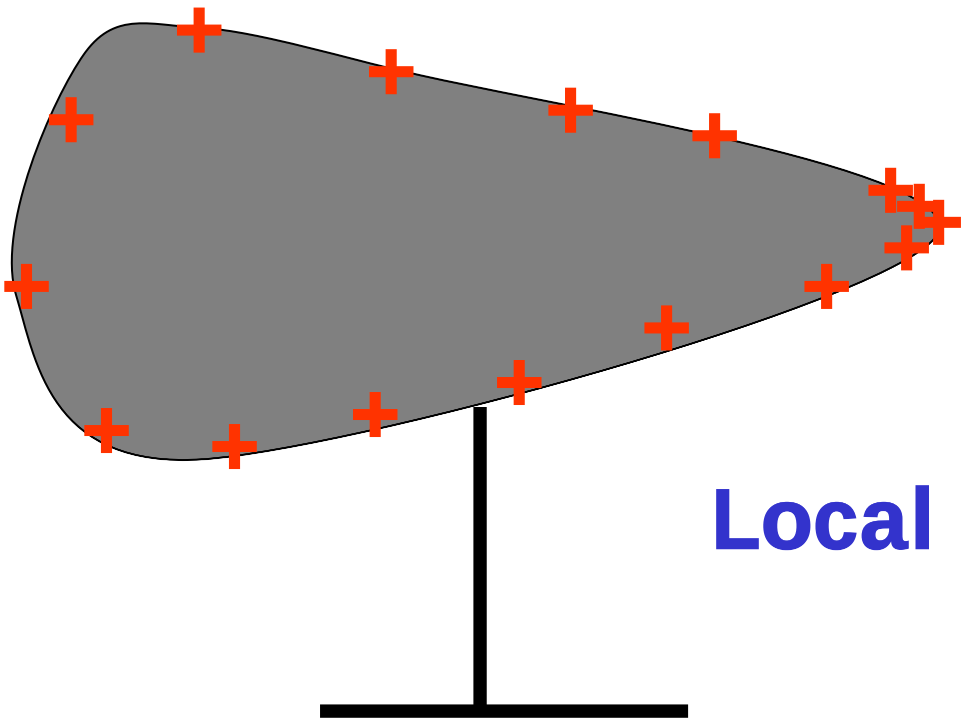
Electricity and Magnetism

- Capacitors
  - Energy storage
  - Electric circuits

Charge Density

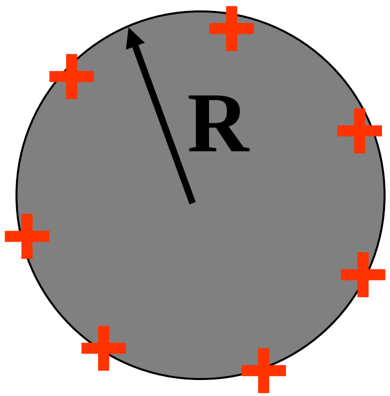
In-Class Demo: **Application: Lightning rod - Biggest E near pointy tip!**



$$\Rightarrow \quad E \propto \frac{1}{r} \propto \sigma$$

Local radius of curvature

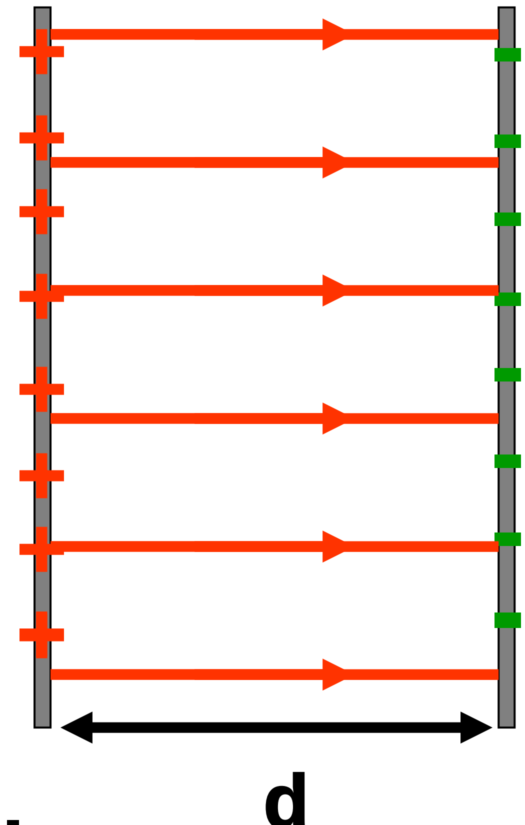
Charge and Potential



Charged Sphere

$$V = 1/(4\pi\varepsilon_0 R)\, Q$$

Geometry!



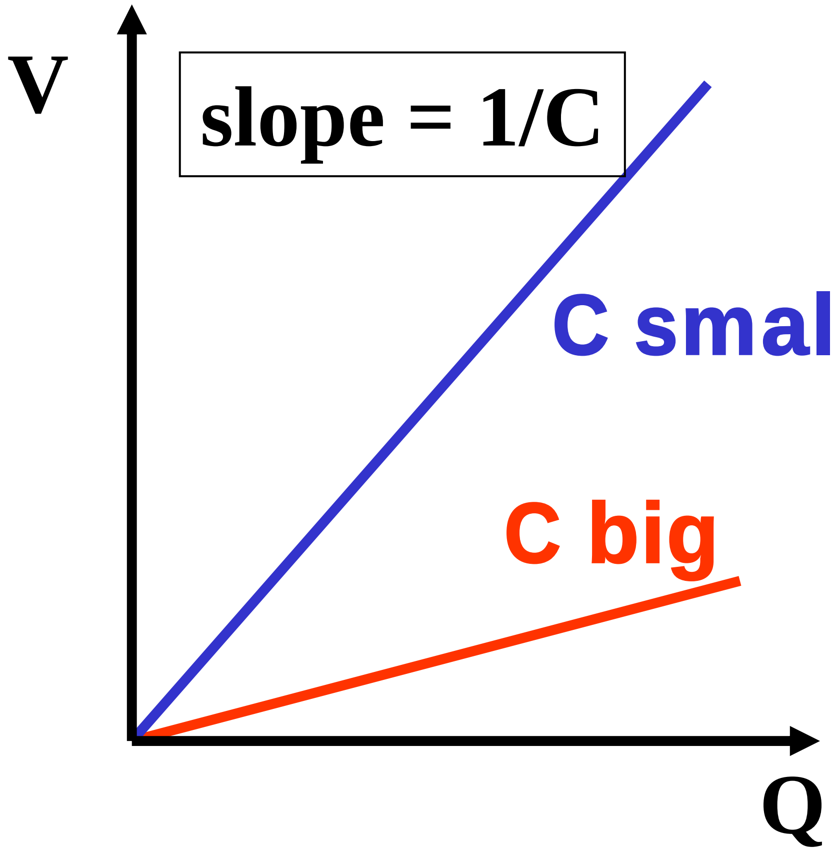
Parallel Plate Capacitor

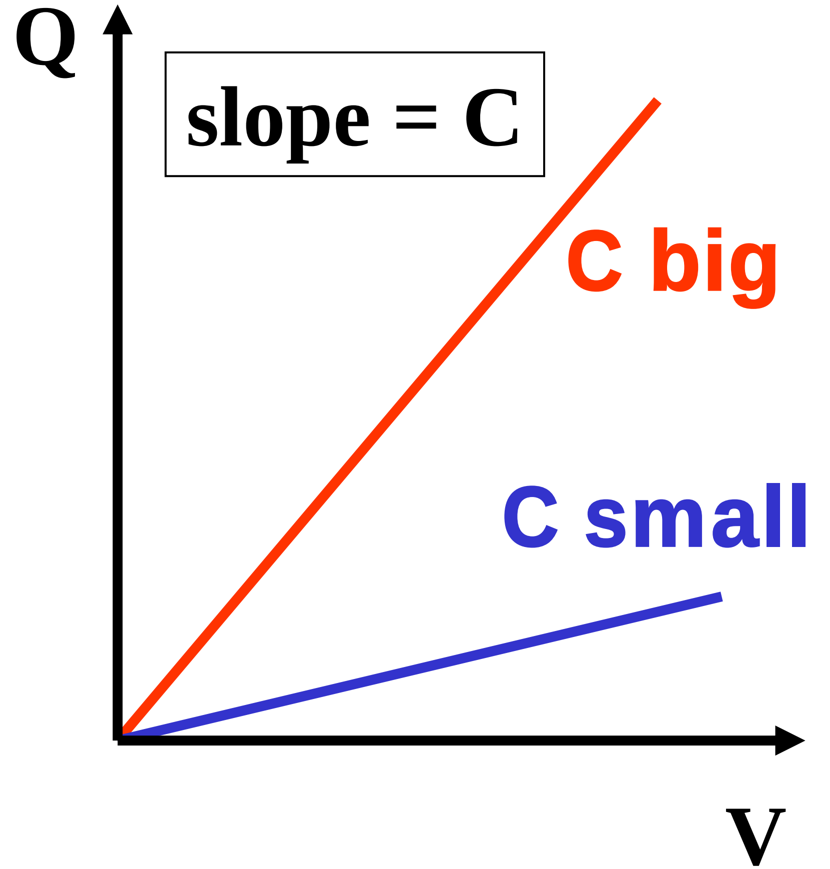
$$V = d/(A\,\varepsilon_0)\, Q$$

Charge and Potential

- For given geometry, Potential and Charge proportional
- Define
  - **Q = C V** **-> C is Capacitance**
- Measured in [F] = [C/V] : Farad

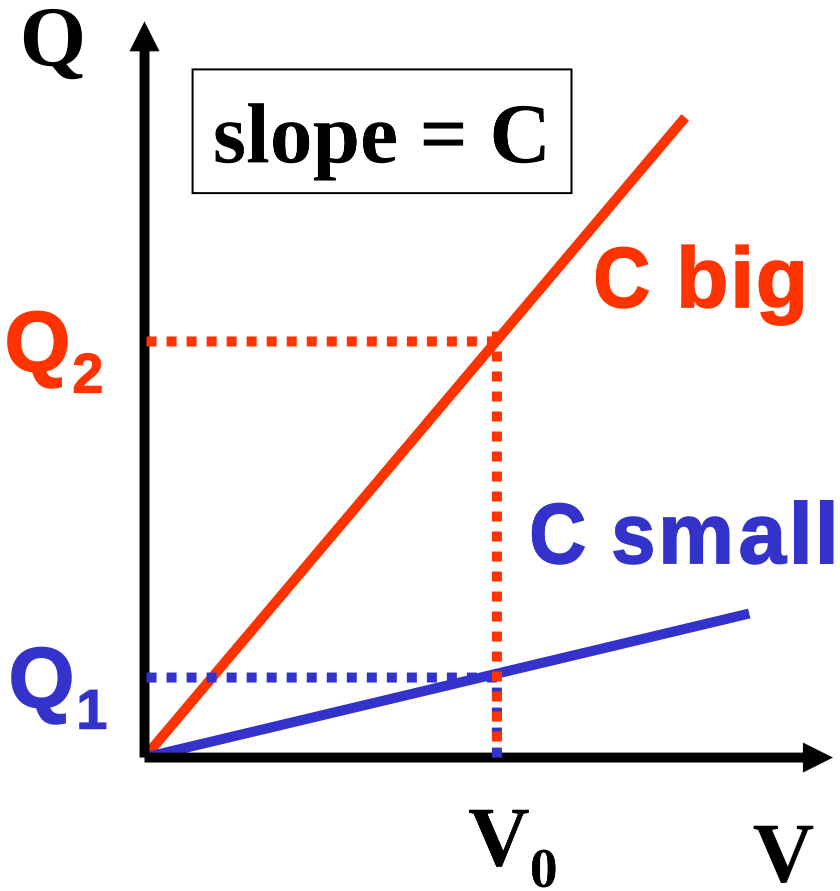
Capacitance





$$Q = C*V \Rightarrow V = Q/C$$

Capacitance

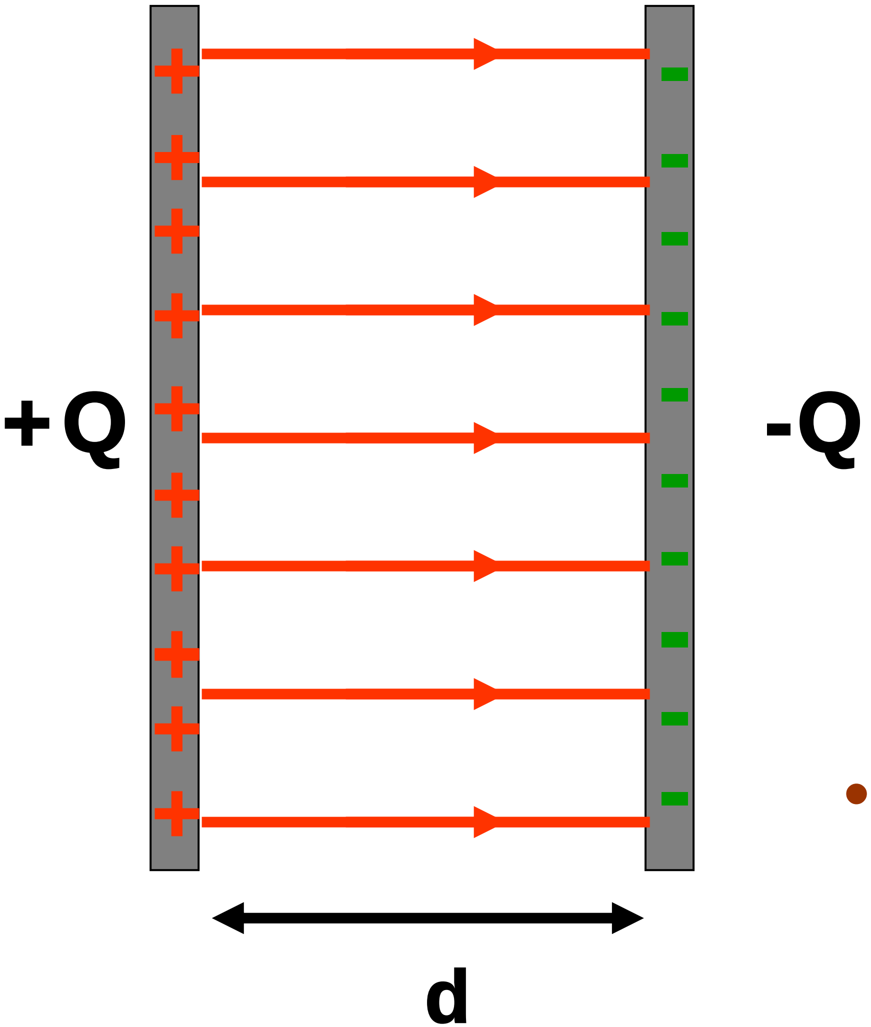


C bigger -> Can store more Charge!

Capacitor

- Def: Two conductors separated by insulator
- Charging capacitor:
  - take charge from one of the conductors and put on the other
  - separate **+** and **-** charges

Parallel Plate Capacitor



$$C = \frac{Q}{V(a) - V(b)} = \frac{Q}{E\,d}$$

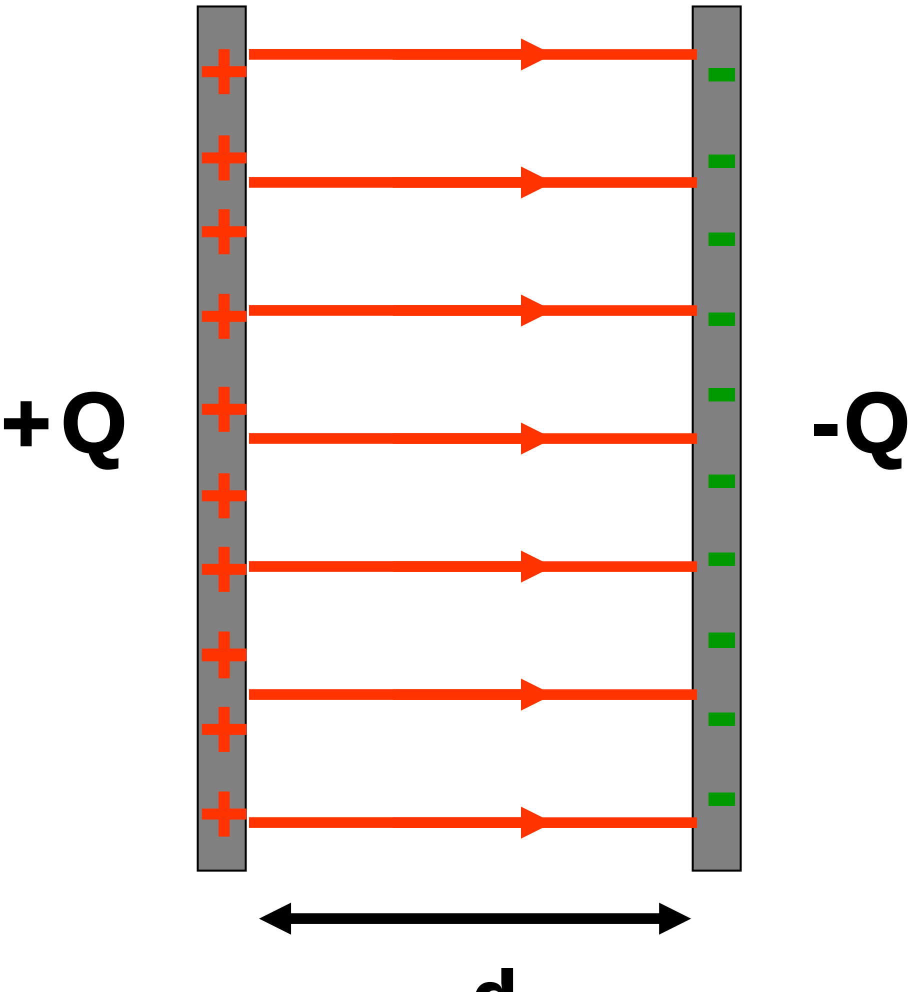
$$= \frac{Q}{\frac{\sigma}{\epsilon_0}\,d} = \frac{Q}{\frac{Q}{A\epsilon_0}\,d} =$$

$$= \epsilon_0 \frac{A}{d}$$

- To store lots of charge
  – make A big
  – make d small

Parallel Plate Capacitor

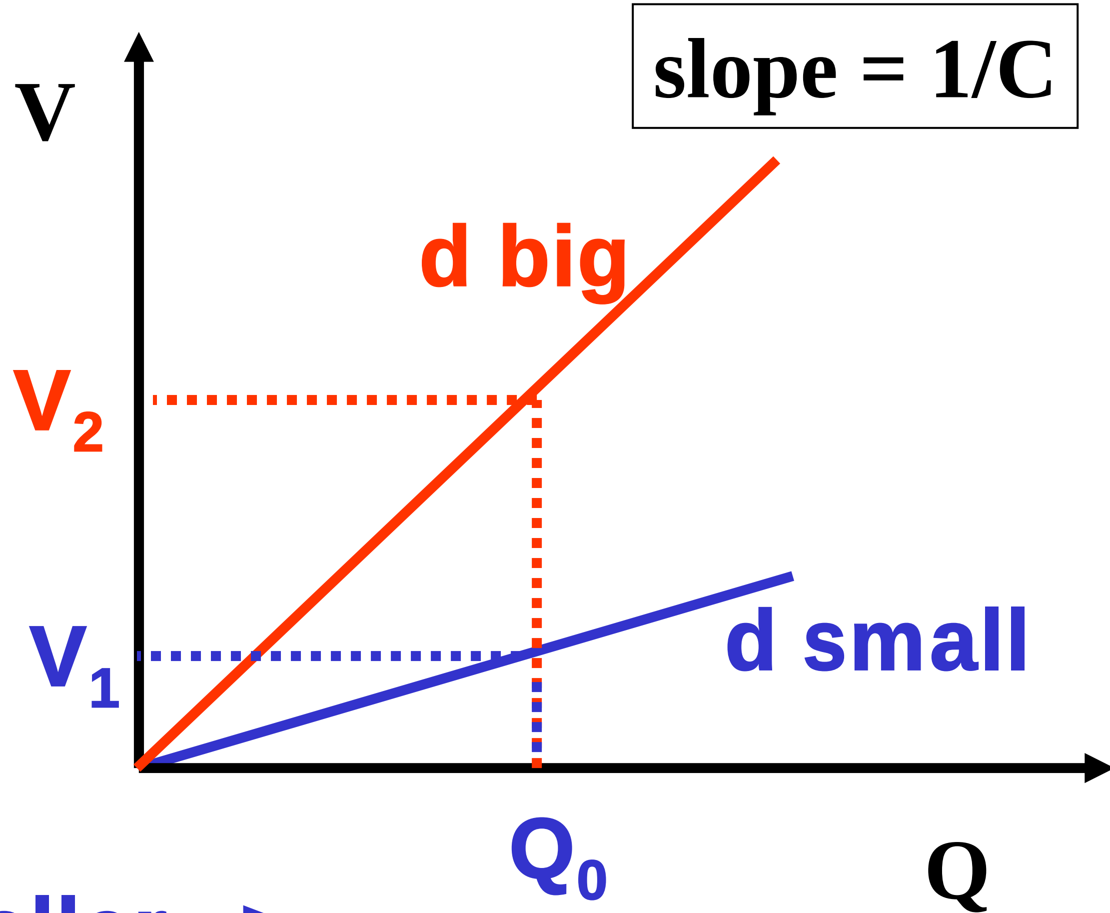
- In-Class Demo
  - Charge Capacitor
  - Change **d**



Parallel Plate Capacitor

$C = \varepsilon_0 A/d$

- Change d
  - change C
- Q constant



d bigger -> C smaller -> V bigger for fixed Q

Capacitor

- By increasing **d**, **V** increased
  - Where does energy come from?
- I have to do work to separate plates!
- Let's look at energy stored on Capacitor…

Energy stored in Capacitor

$$\frac{\text{Work}}{\text{Unit Charge}} = \frac{dW}{dq} \equiv V$$
$$\Rightarrow dW = V\,dq$$

$$W_{tot} = \int_{Q_{initial}}^{Q_{final}} V\ dq = \int_0^Q V\ dq$$
$$= \int_0^Q q/C\ dq = \frac{1}{C}\int_0^Q q\ dq$$
$$= \frac{1}{C}\frac{Q^2}{2}$$

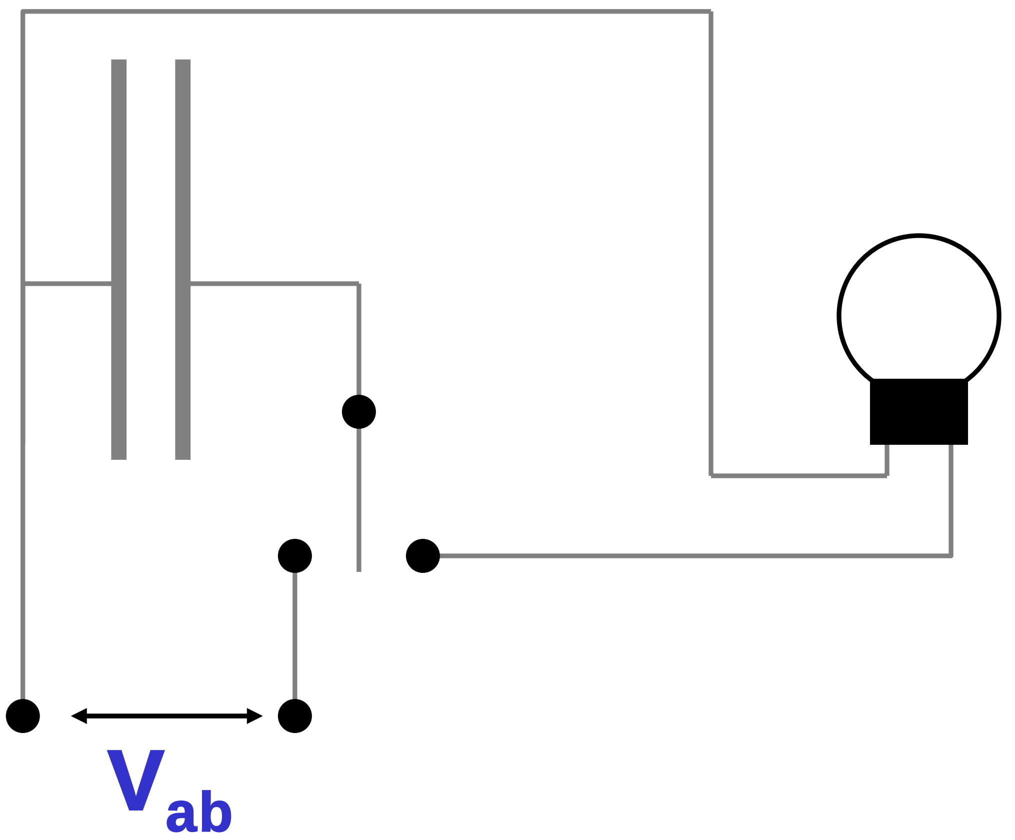
$$W_{tot} = \frac{1}{C}\frac{Q^2}{2} = \frac{Q^2}{2\frac{Q}{V}}$$
$$= \frac{1}{2}QV = \frac{1}{2}CV^2,\ \text{b/c}\ Q = C\ V$$

Energy stored in Capacitor

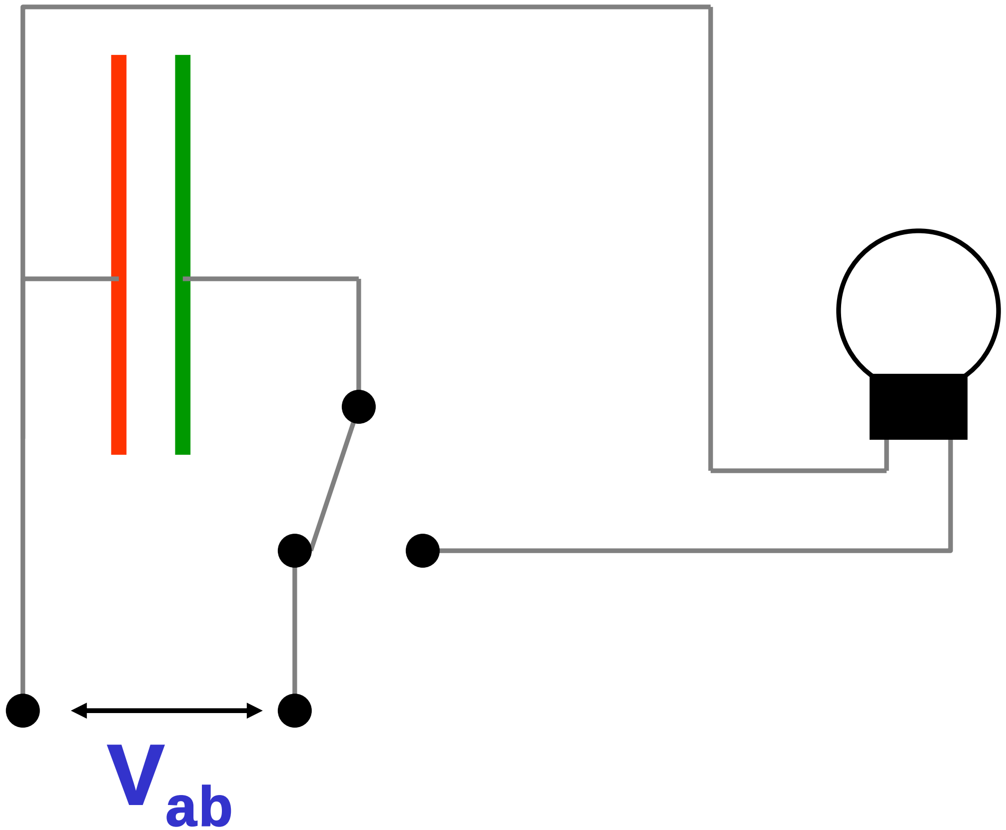
- Can store more energy, if
  - **C** bigger
  - **V** bigger

$$W_{tot} = \frac{1}{2} C V^2$$

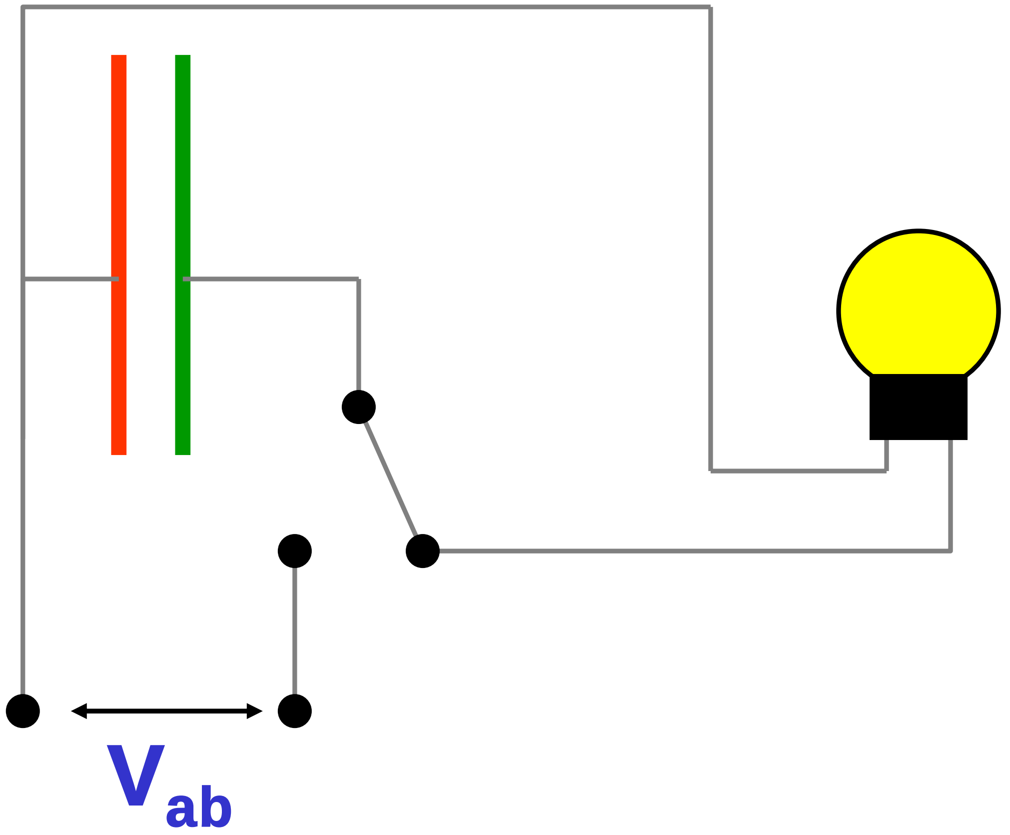
In-Class Demo



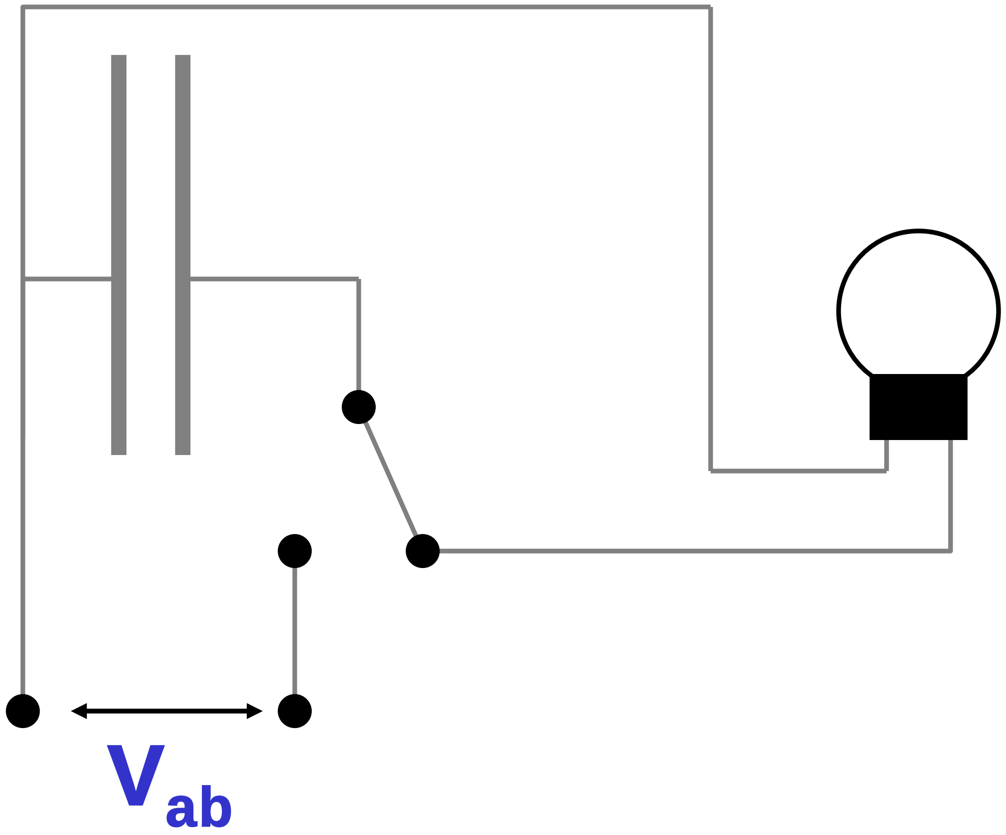
In-Class Demo



In-Class Demo



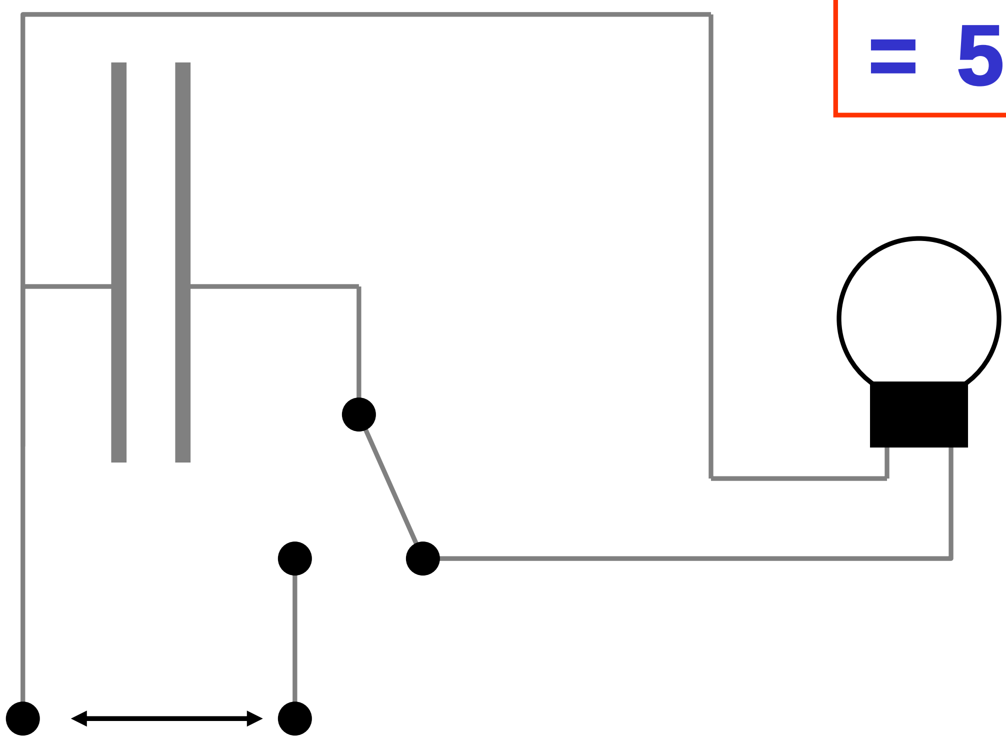
In-Class Demo



In-Class Demo

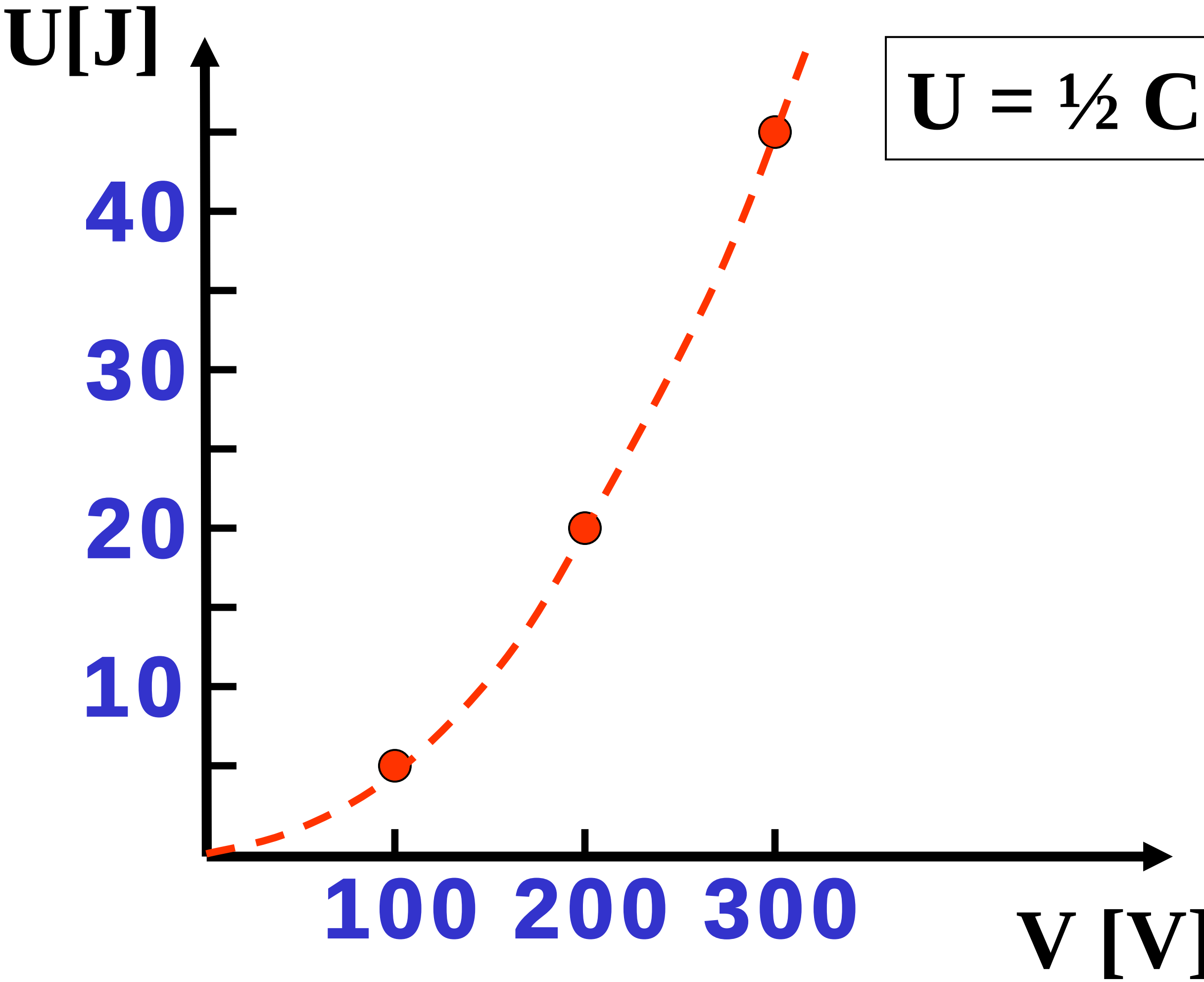
$C = 1000\mu F$

$U = 0.5 * 10^{-3} * 100^2$
$= 5$ J



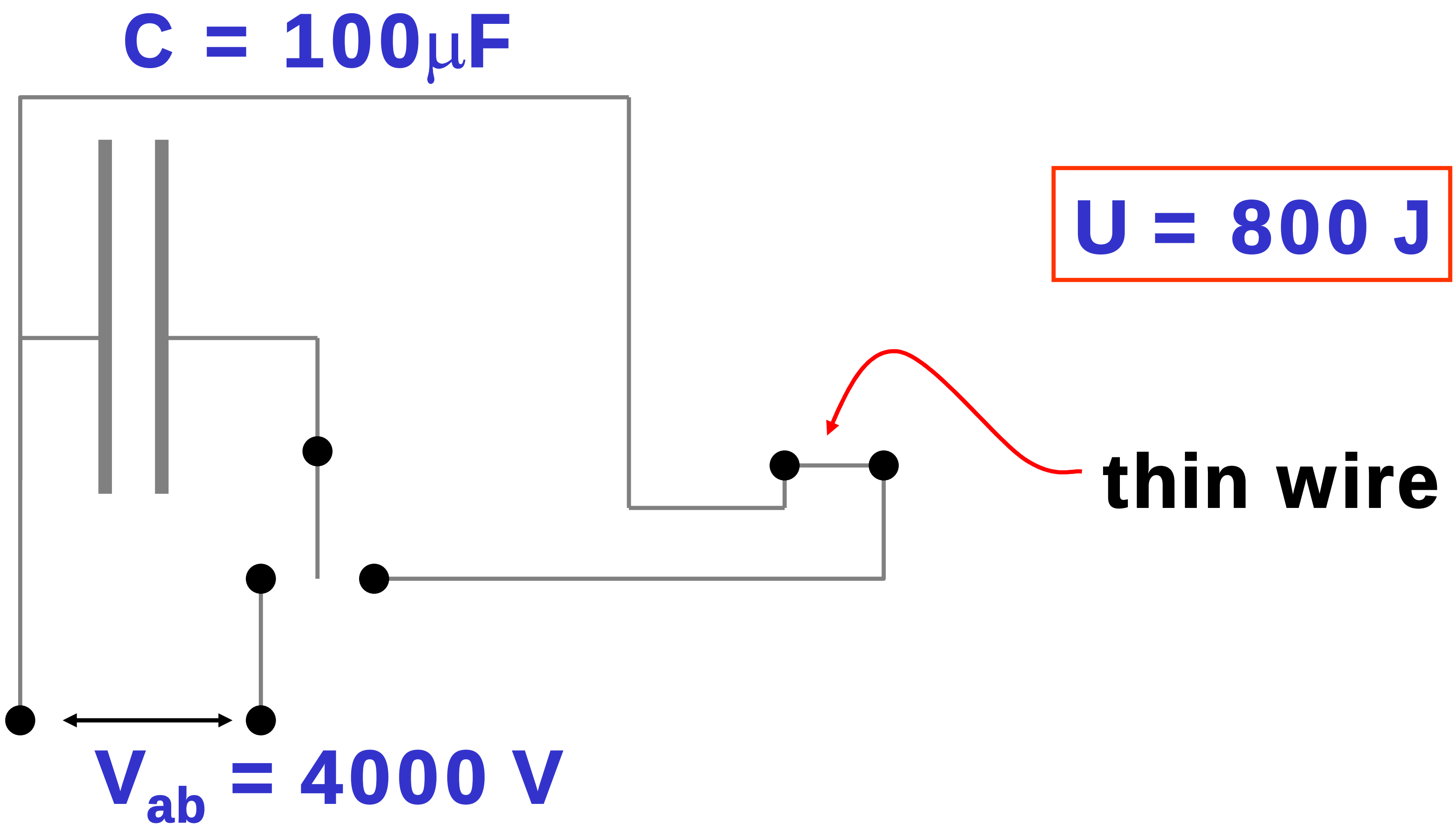
$V_{ab} = 100$ V

Energy vs V



$U = \frac{1}{2} C V^2$

In-Class Demo (same, but more)

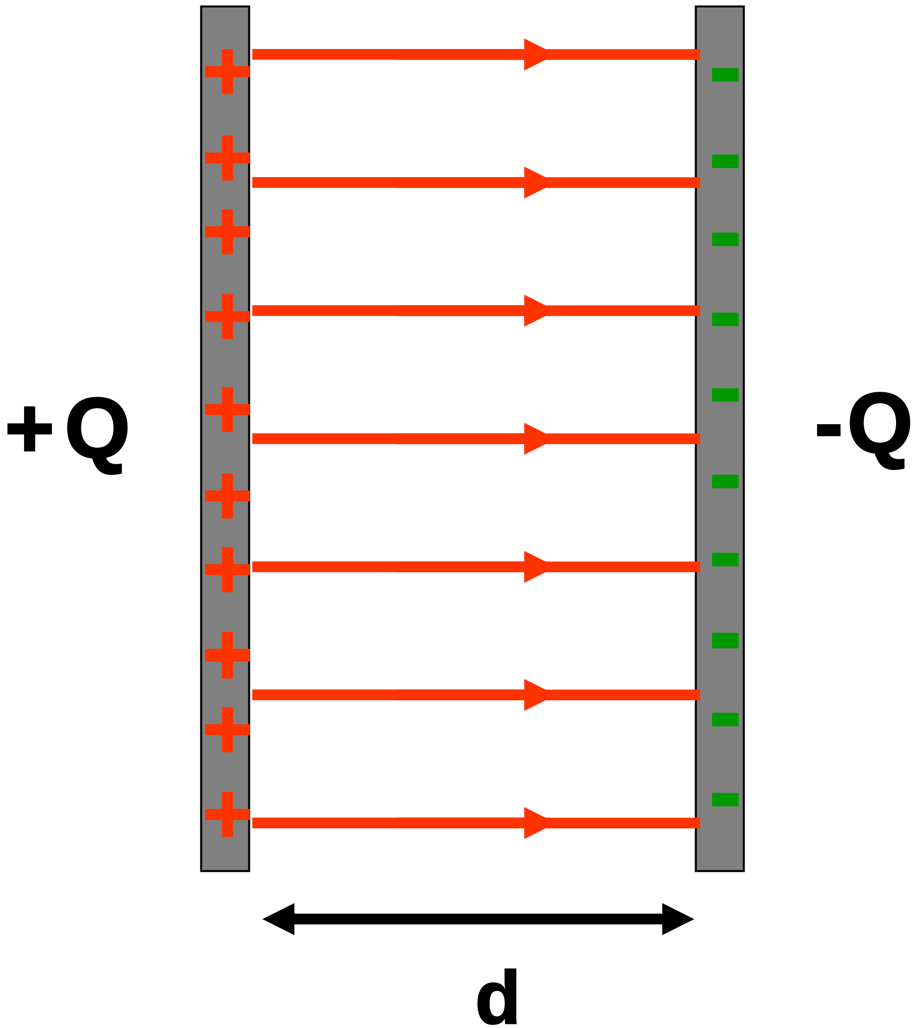


How much Energy is 800J ?

- Cyclist
  - Power p ~200 W => 800 J = 200 W * 4 sec
- Food
  - 500 ‘calories’ => 500*4kJ = $2*10^6$J
  - should keep you going for $2*10^6$J/200W = 10000 sec = 3h
  - sounds about right…

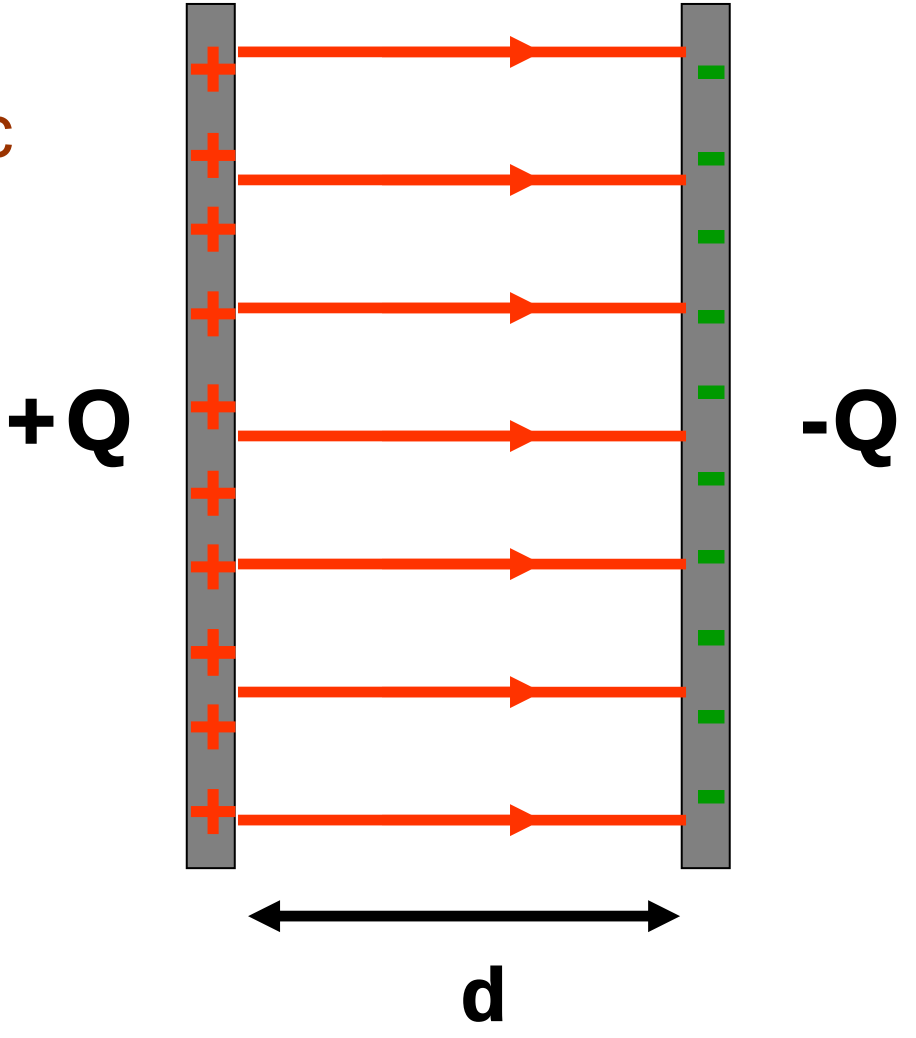
Where is the energy stored?

$$U_{stored} = \frac{1}{2}CV^2 = \frac{1}{2}(\epsilon_0 \frac{A}{d})(E\ d)^2$$
$$= \frac{1}{2}\epsilon_0 E^2\ Volume$$

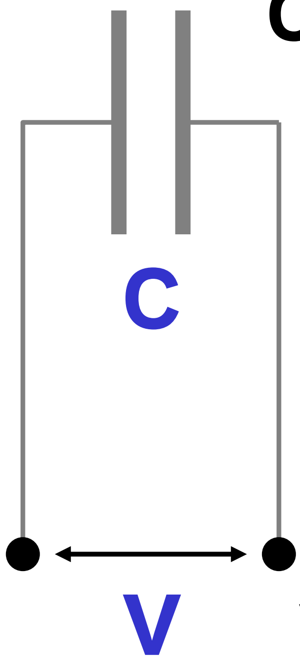


Where is the energy stored?

- Energy is stored in Electric Field
- **$U = ½\ e_0\ E^2$*Volume**
- **E^2** gives Energy Density:
- **$U/Volume = ½\ e_0\ E^2$**



Electric Circuits

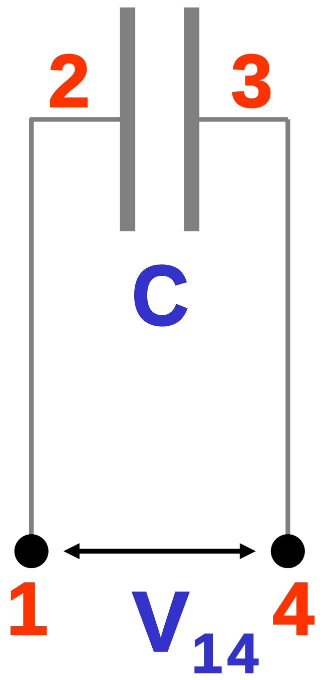


Capacitor

Wire

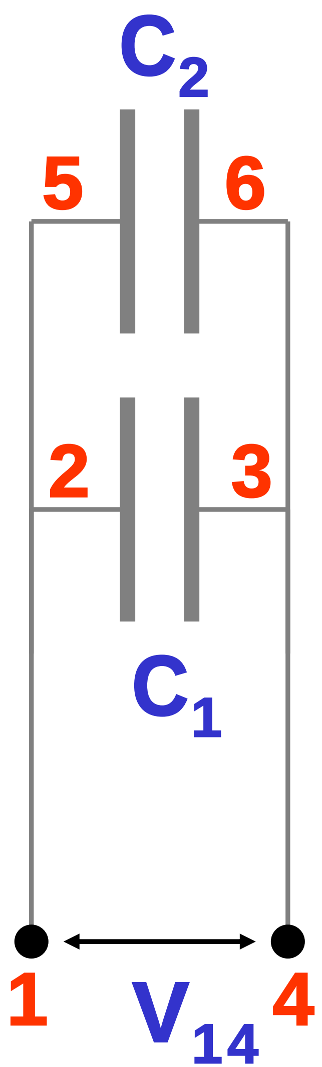
Voltage source (like LVPS)

Electric Circuits



- **V_{14}** provided by some source (like LVPS)
- **V_{12} = 0,** because wire is conductor -> V constant
- **V_{34} = 0**
- **V_{23} = V_{14}** (after capacitor is charged)

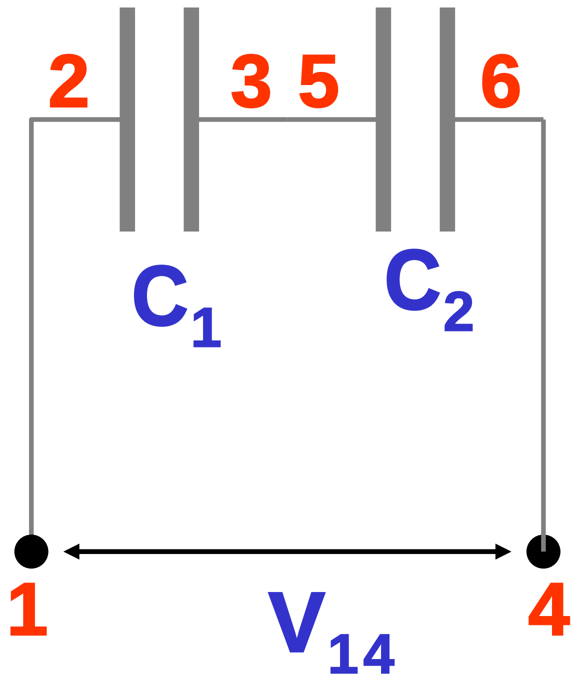
Electric Circuits



- Two capacitors in **parallel**
- $V_{56} = V_{23} = V_{14}$ (after capacitor is charged)
- $Q_1 / C_1 = Q_2 / C_2 = V_{14}$
- $Q_{tot} = Q_1 + Q_2$
- $C_{tot} = (Q_1 + Q_2) / V_{14} = C_1 + C_2$

- Capacitors in **parallel -> Capacitances add!**

Electric Circuits



- Two capacitors in **series**
- $V_{14} = V_{23} + V_{56}$
- $Q = Q_1 = Q_2$
- $V_{tot} = Q_1/C_1 + Q_2/C_2 = Q/(C_1+C_2)$
- $1/C_{tot} = 1/C_1 + 1/C_2$
- **Inverse Capacitances add!**