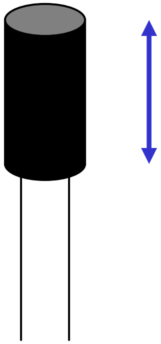
Electricity and Magnetism

- Today
  - Electric current
  - Resitivity/Resistance
  - Ohm’s Law

Dielectrics

In your toolbox:

- Parallel Plate Capacitor:
  - $C = \varepsilon_0 A/d$
  - Ex. $A = 1m^2, d=0.1mm$ $C \sim 0.1\mu F$

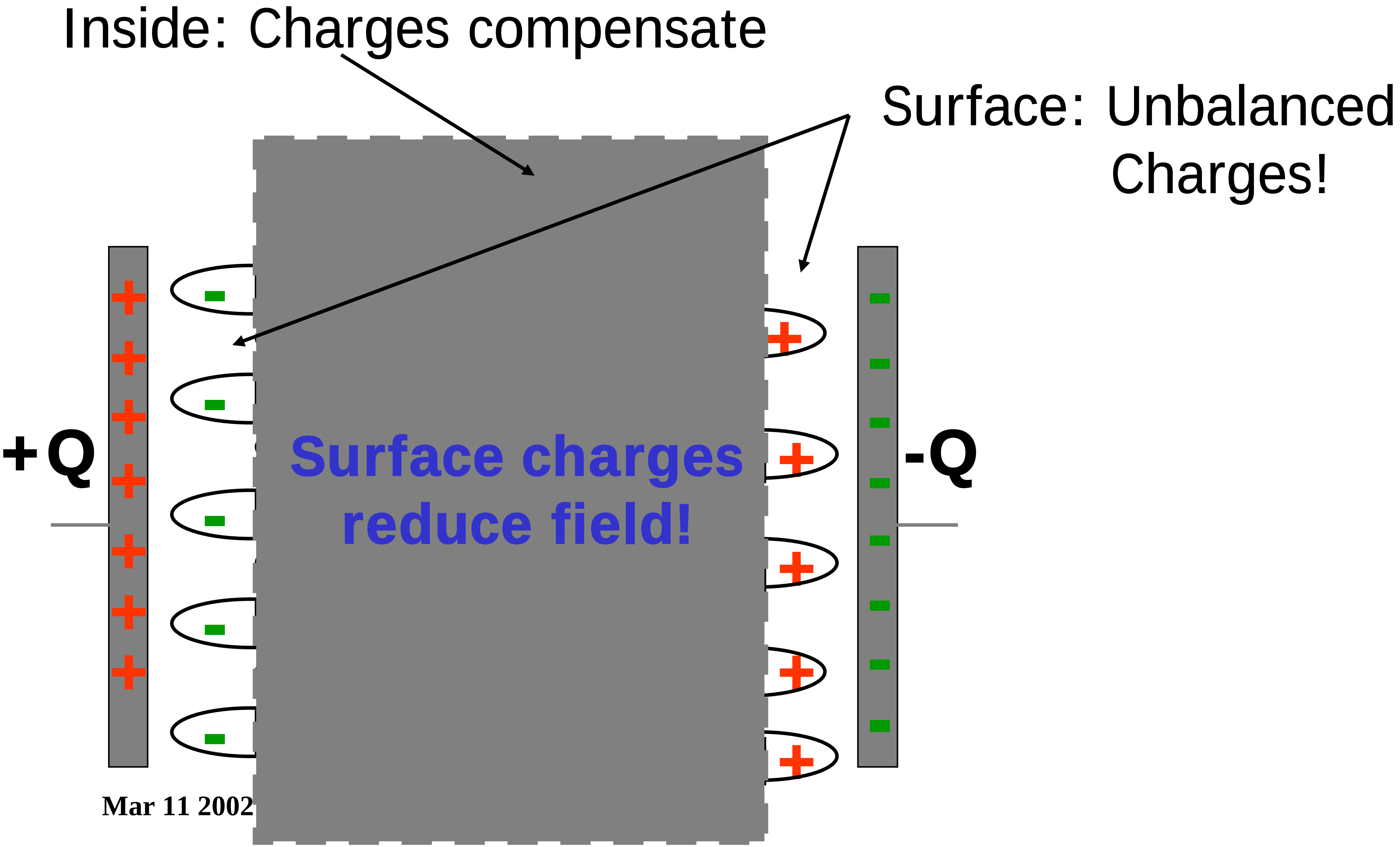


2 cm

C = 1000μF

- C can be increased with Dielectric
  - $C = K \varepsilon_0 A/d$
  - **K: Dielectric Constant**

Microscopic view



Dielectric Constant

- Examples

Material	K
Vacuum	1
Air	1.0006
Plexiglass	3.4
Water	80.4
Ethanol	23
Ceramics	~5000
Glass	5-10

Similar to vacuum

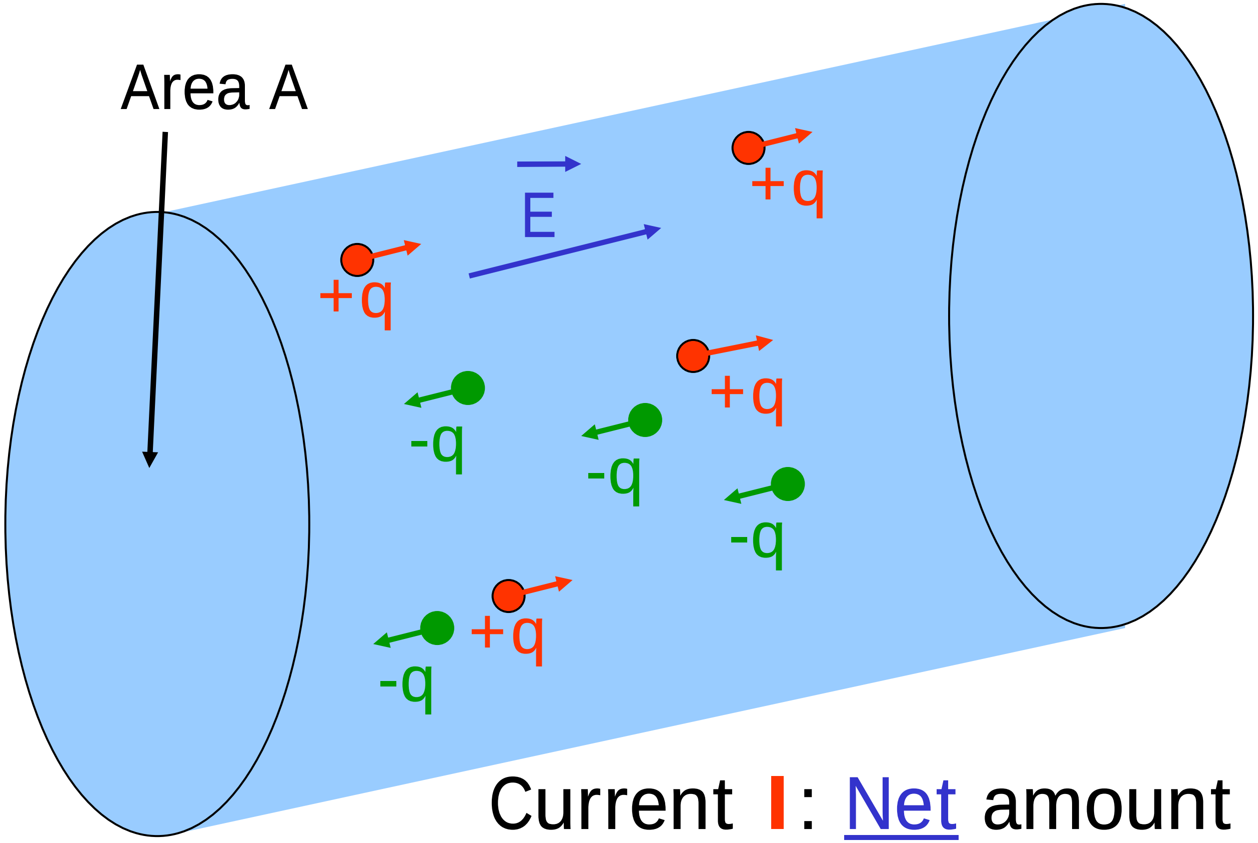
Large!

C in HVPS

Electric Current

- So far: Electrostatics
  - Look at situation where charges don't move
  - **E = 0** in conductors
    - otherwise mobile charges would move!
- Now we look at moving charges!
  - **Electric currents**
  - **|E| > 0** possible in conductor

Electric Current



Current **I**: Net amount of charge passing through conductor per unit time

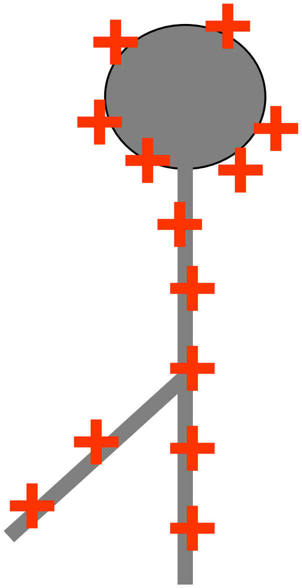
Electric Current

- Electric Current I:
  - **$I = dQ/dt$**
- Units:
  - **$[I] = C/s = A$ (Ampere)**
- 1 A = 1.6 10^{19} q_e/s
- I is a scalar, connected to given conductor

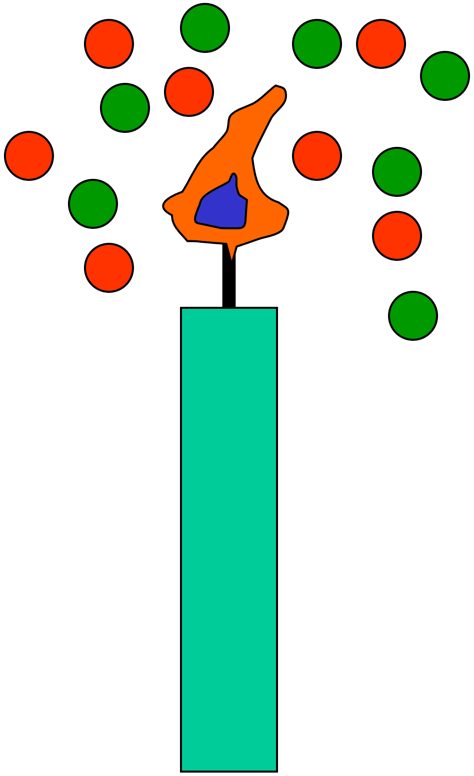
Electric Current

- Current **$I = dQ/dt$** has a direction
  - Convention: Direction of flow of positive charges
  - In our circuits, I carried by electrons
- To get a current:
  - Need mobile charges
  - Need **$|E| > 0$** (Potential difference)

In-Class Demo I

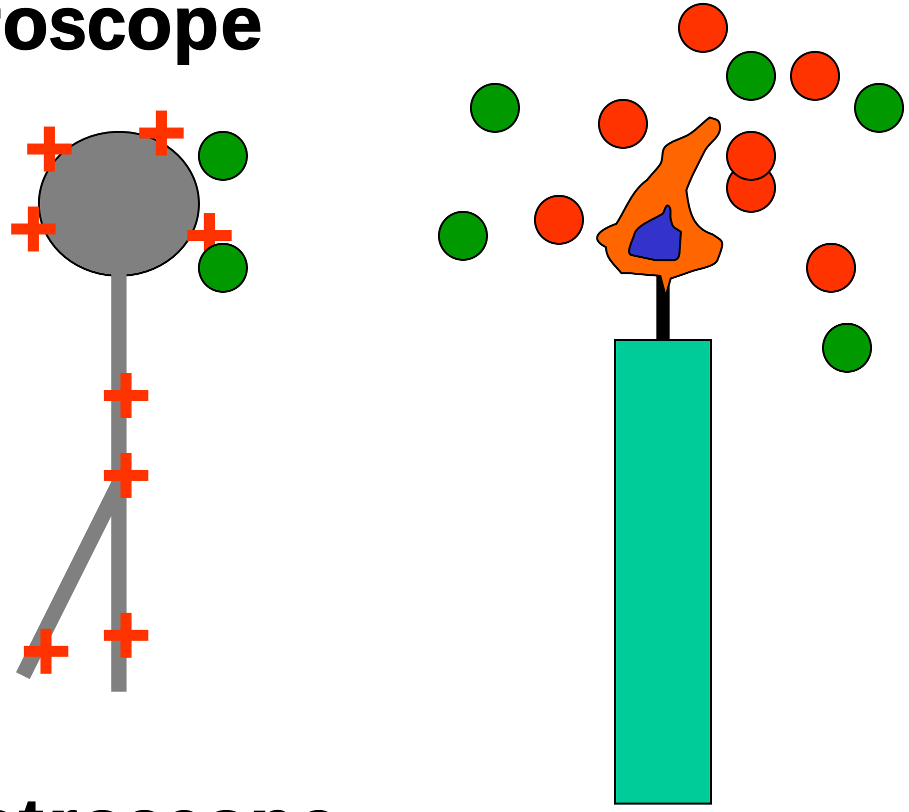


Electroscope



Charged Ions

In-Class Demo I



Ions discharge Electroscope

Charged Ions

Electroscope

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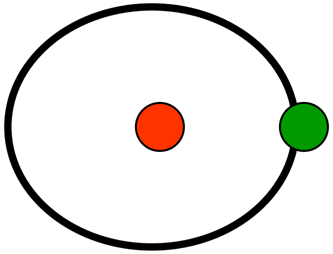
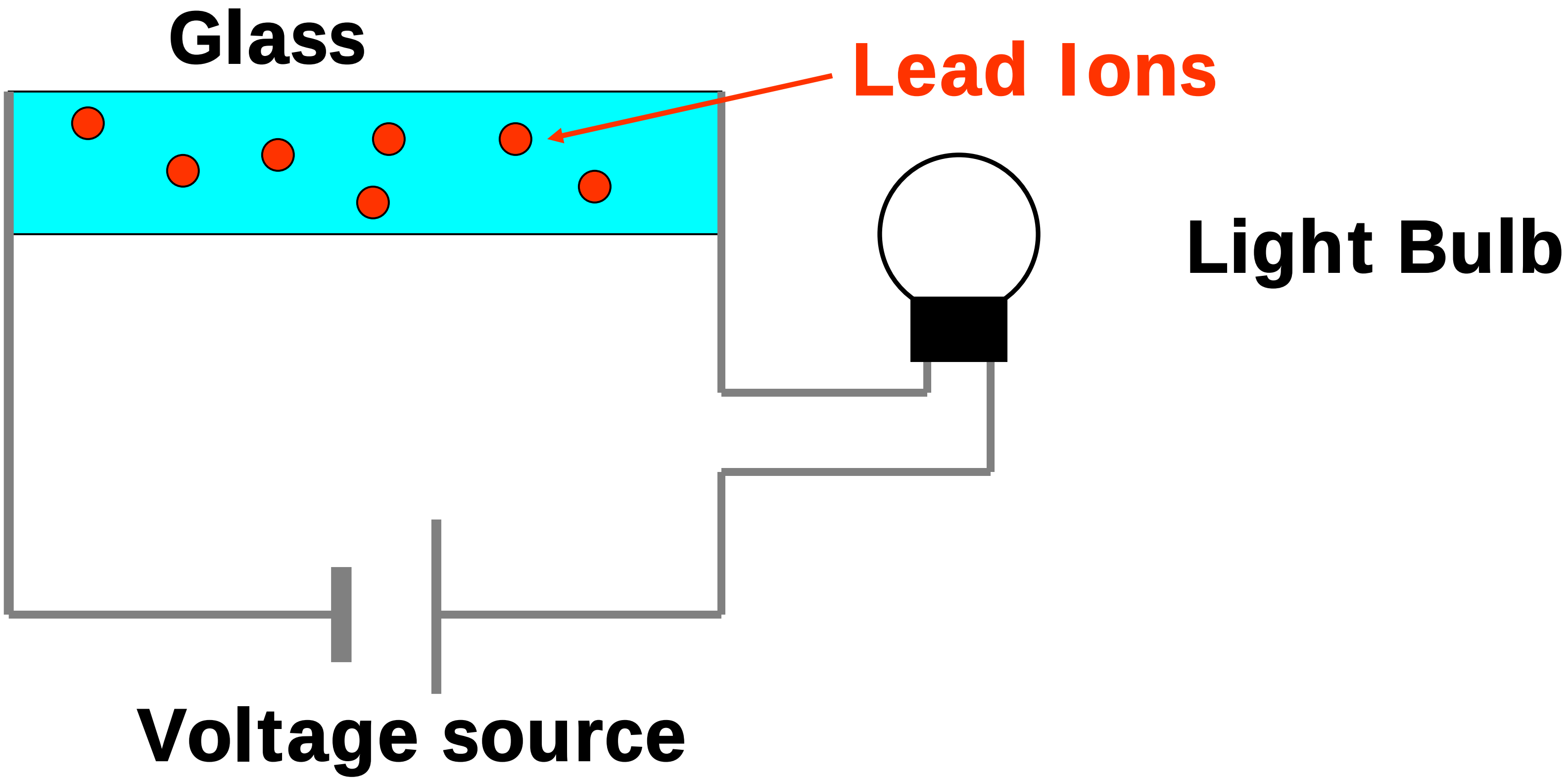
In-Class Demo I



Neutral molecules:
Pos. and neg. charges move together -> No current!

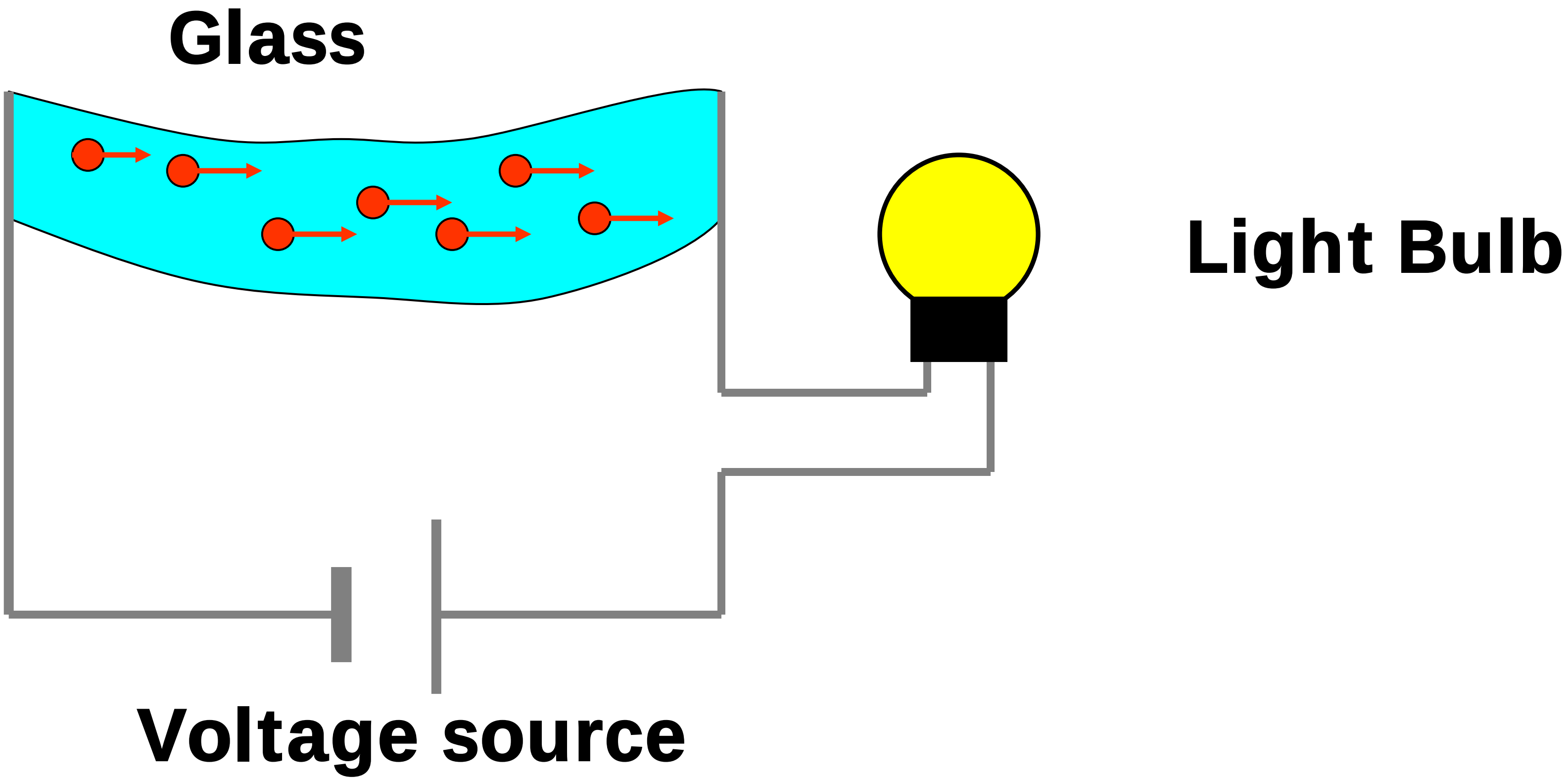
Ions:
Pos. and neg. charges move separately -> Current $|I| > 0$!

In-Class Demo II



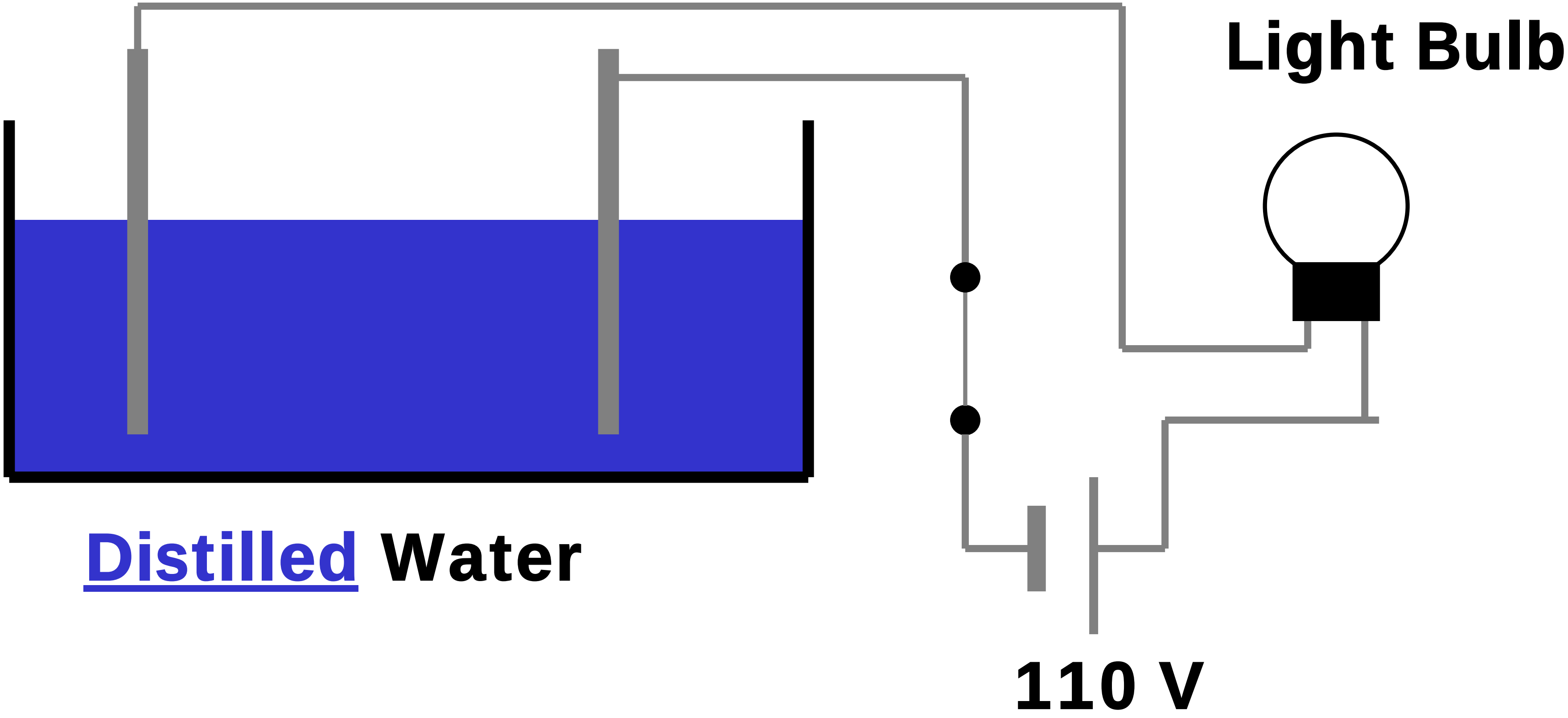
Solid glass: Potential charge carriers are stuck!

In-Class Demo II



Molten glass: Charge carriers become mobile -> Current flows -> Bulb lights up!

In-Class Demo III



Will the bulb light up? NO

No light -> No current -> No mobile charges!

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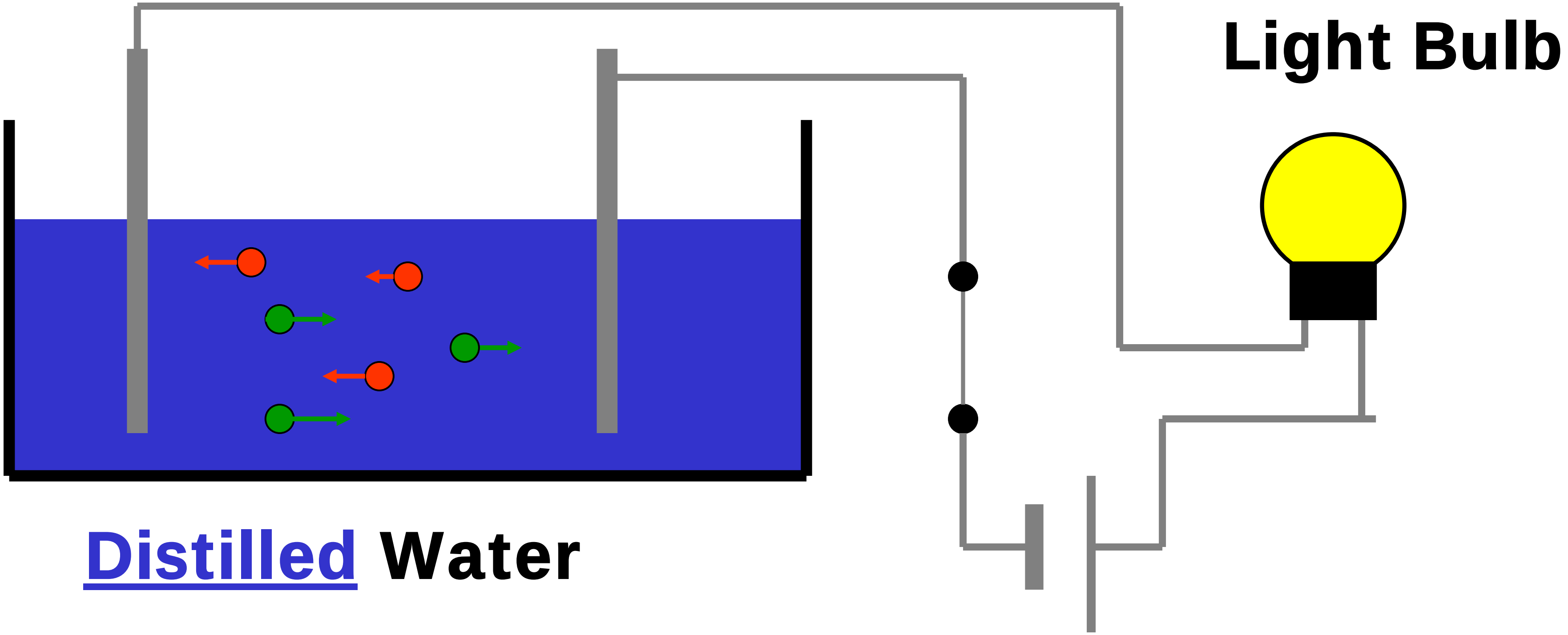
In-Class Demo III



Distilled Water

Add NaCl: Dissociates into Na^+ and Cl^-

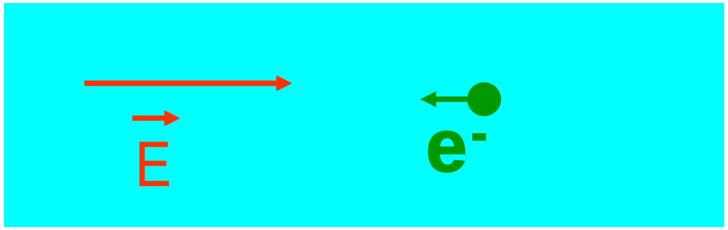
Charge carriers are available -> Current flows -> Bulb lights up

Resistivity

- To get a current, we needed
  - – mobile charge carriers
  - – a Potential difference
- What determines magnitude of I ?
- -> Microscopic analysis

Resistivity

- Consider an electron in a conductor:

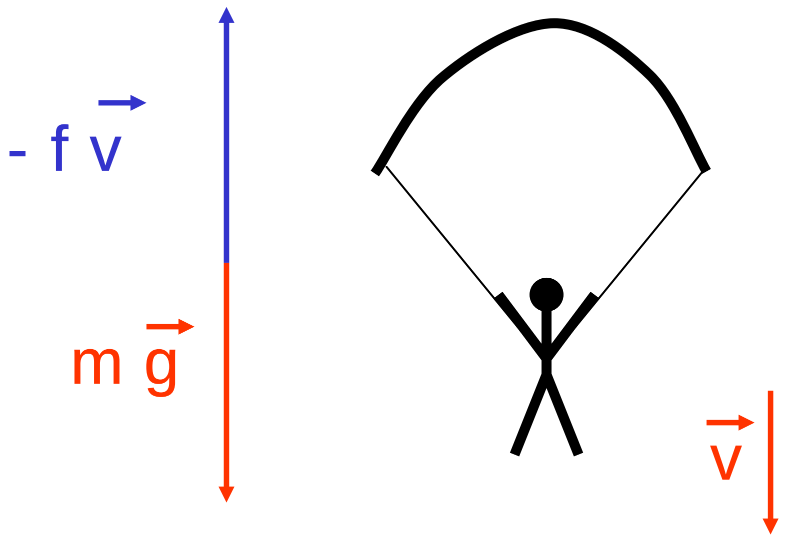


Equation of motion: $m_e\, d\vec{v}/dt = q_e\, \vec{E}$

For $|E| > 0$, $|v|$ increases all the time ($|v| \to$ infinity)

Consider analogy with Gravity

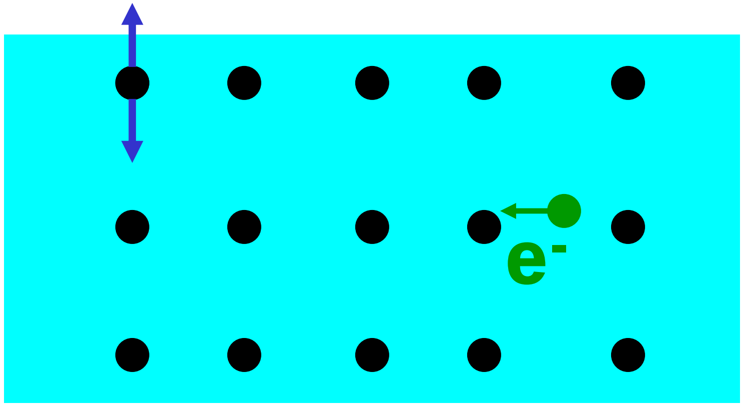
Reminder: Gravity



Equation of motion: $m\,d\vec{v}/dt = m\,\vec{g} - f\,\vec{v}$

Friction grows with $|v|$, limits maximal velocity $v<v_{max}$
$(\ m\,g = \ f\,v_{max}\ )$

Resistivity



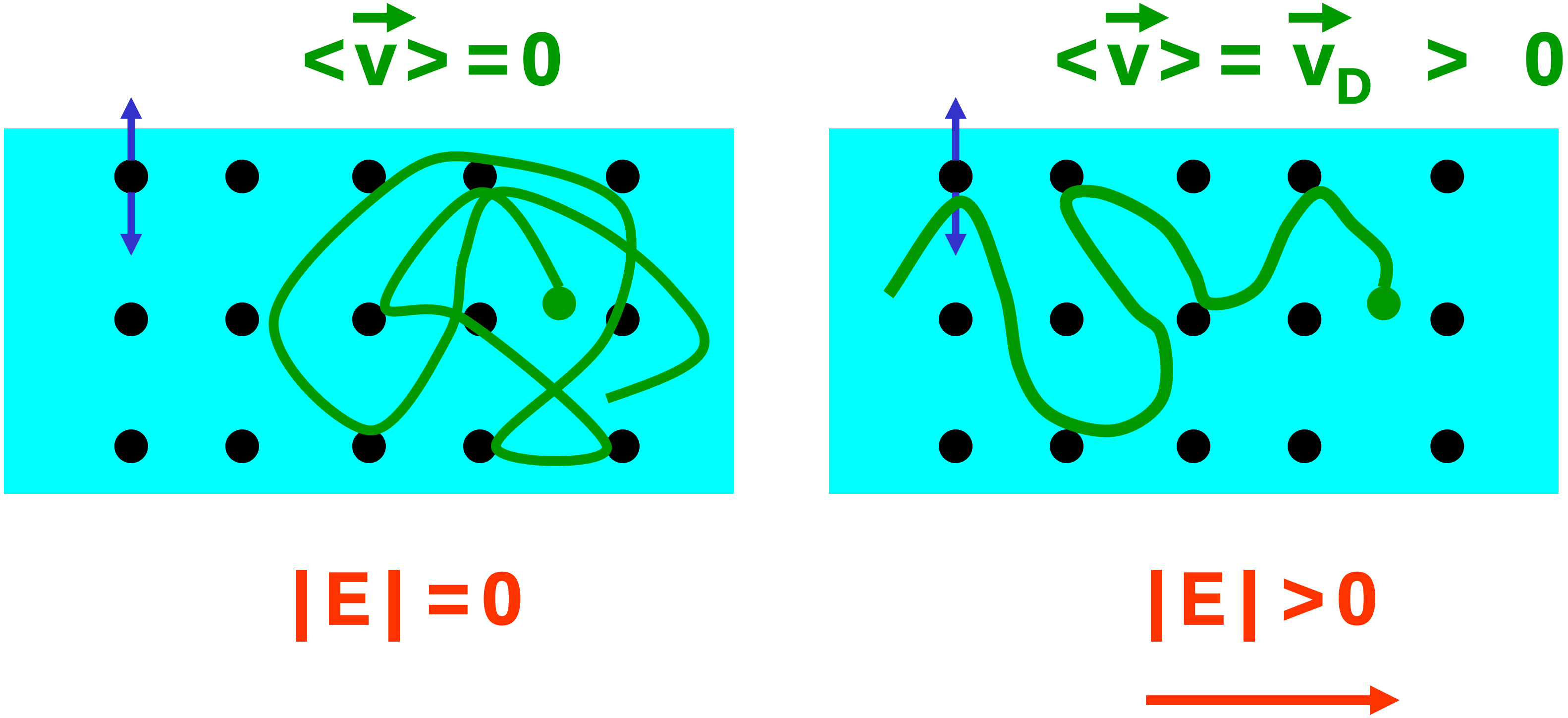
Where does friction come from?

<u>Metal:</u> Electrons move through lattice of atoms

Lattice: Thermal vibrations, average position fixed

Electrons: Light! Bounce around...

Resistivity



$\langle \vec{v} \rangle = 0$

$\langle \vec{v} \rangle = \vec{v}_D > 0$

$|E| = 0$

$|E| > 0$

If $|E| > 0$: Electron accelerated between scatterings

On average: Electron moves in –E direction

Resistivity

- Interplay of scattering and acceleration gives an average velocity $\vec{v}_D$
- v_D is called 'Drift velocity'
- Similar to terminal velocity for parachuting:

Equation of motion: $m_e \, dv/dt = q_e E - f v$

$$\boxed{\text{For } dv/dt = 0 \ \rightarrow \ v_D = (q_e E) / f}$$

Steady State

Friction

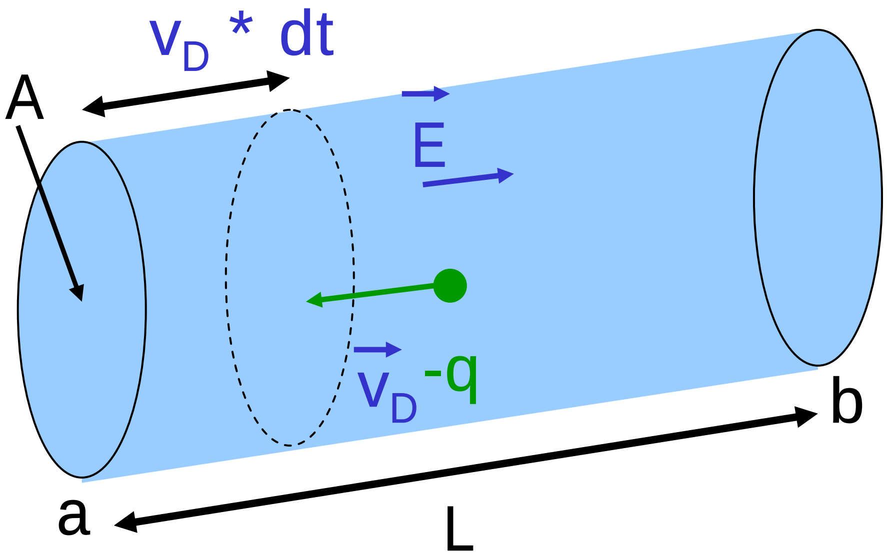
Resistivity

- How fast do the electrons move?
  - Thermal speed is big: $v_{th} \sim 10^6$ m/s
  - Drift velocity is small: $v_D \sim 10^{-3}$ m/s
- How long do I have to wait when switching on the light?
  - $\Delta t = 10\text{m}/v_D = 10^4$ s ~ 3 hours!?!
  - No, $\Delta t = 10\text{m}/c = 3*10^{-7}$ s
- All electrons in conductor start to move, as soon as $E > 0$

Resistivity

- What determines magnitude of current **I**?
- Connect macroscopic description (I) with microscopic description

Resistivity

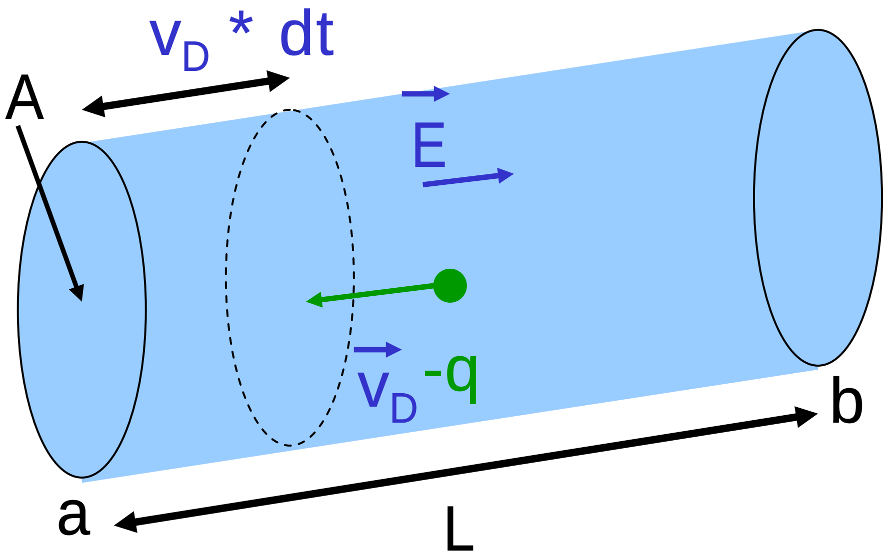


<u>How big is the current I?</u>

$$I = dQ/dt = 1/dt * \underbrace{(\text{Charge going through A})}$$

$$= 1/dt * \overbrace{q * \underbrace{n_q}_{\text{Charges/Volume}} * \underbrace{(A\ v_D\ dt)}_{\text{Volume}}} = q\ n_q\ A\ v_D = \boxed{q\ n_q\ v_D\ A}$$

Resistivity



$$\boxed{I = q\, n_q\, v_D\, A}$$

Velocity

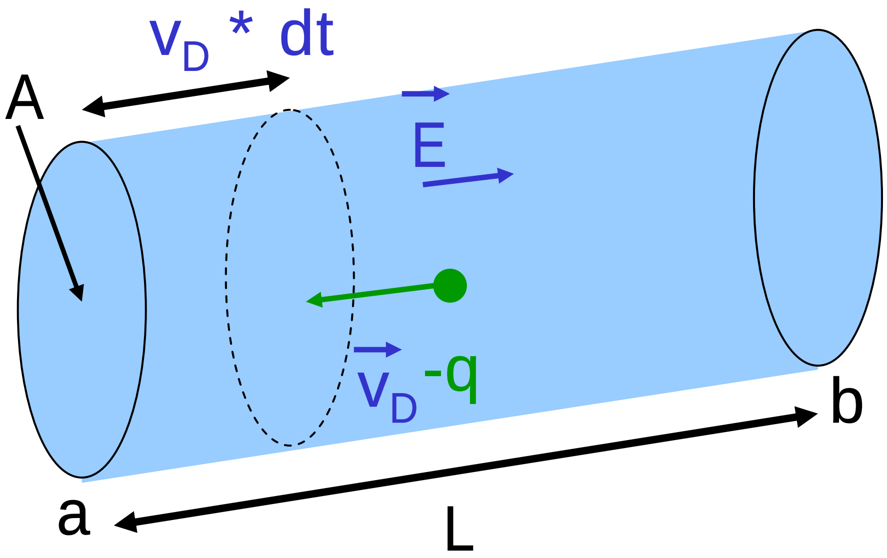
Area

Velocity * Area: Flux! (like flow of water)

Def: $\vec{J} = q\, n_q\, \vec{v}_D$ Current Density ([J] = A/m^2)

Then $I = \vec{J}\, \vec{A}$

Resistivity



How does $\vec{J} = q\ n_q\ v_D$ depend on the Field E ?

Remember: $\vec{v}_D = (q_e\ \vec{E})\ /\ f \;\; -> \;\; \vec{J} = \underbrace{q^2\ n_q/f}_{\text{constant}}\ \vec{E}$

$$\boxed{\vec{J} = 1/\rho\ \vec{E};\ \rho = f\ /(q^2\ n_q)}$$

Resistivity

Ohm

$\rho = f/(q^2 n_q)$: Resistivity

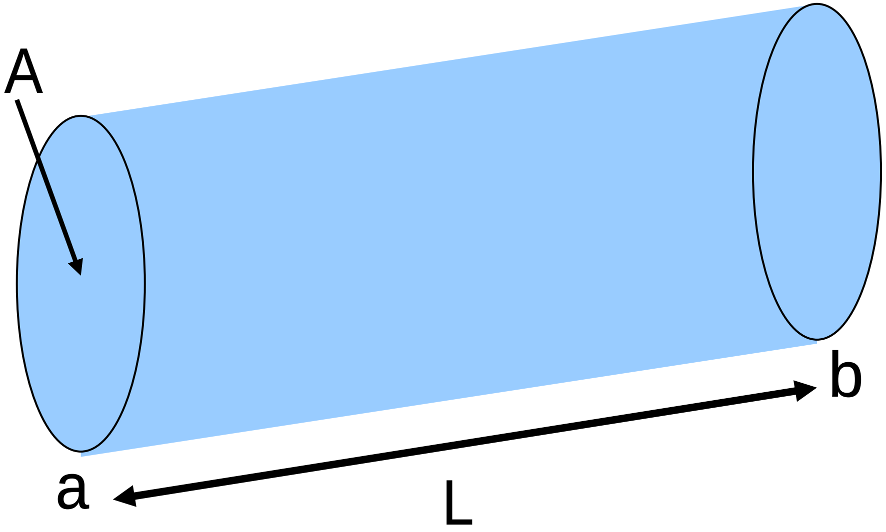
Units: $[\rho] = V/m\ m^2/A = m\ V/A = m\ \Omega$

Material	ρ [m Ω]	
Glass	$<10^{10}$	Insulator
Pure Water	$2*10^5$	
Carbon	$3.5*10^5$	Semiconductor
Silicon	2300	
Sea Water	0.2	Conductor
Gold	$2.4*10^{-8}$	
Copper	$1.7*10^{-8}$	

Energy conservation

- We spend energy to apply potential difference
- But velocity of charges doesn’t increase
- Where does the energy go?
- ‘Friction’ of electrons moving through conductor causes heat

Resistance



$V = V_a - V_b = E\,L$

Define $R = V/I$: Resistance

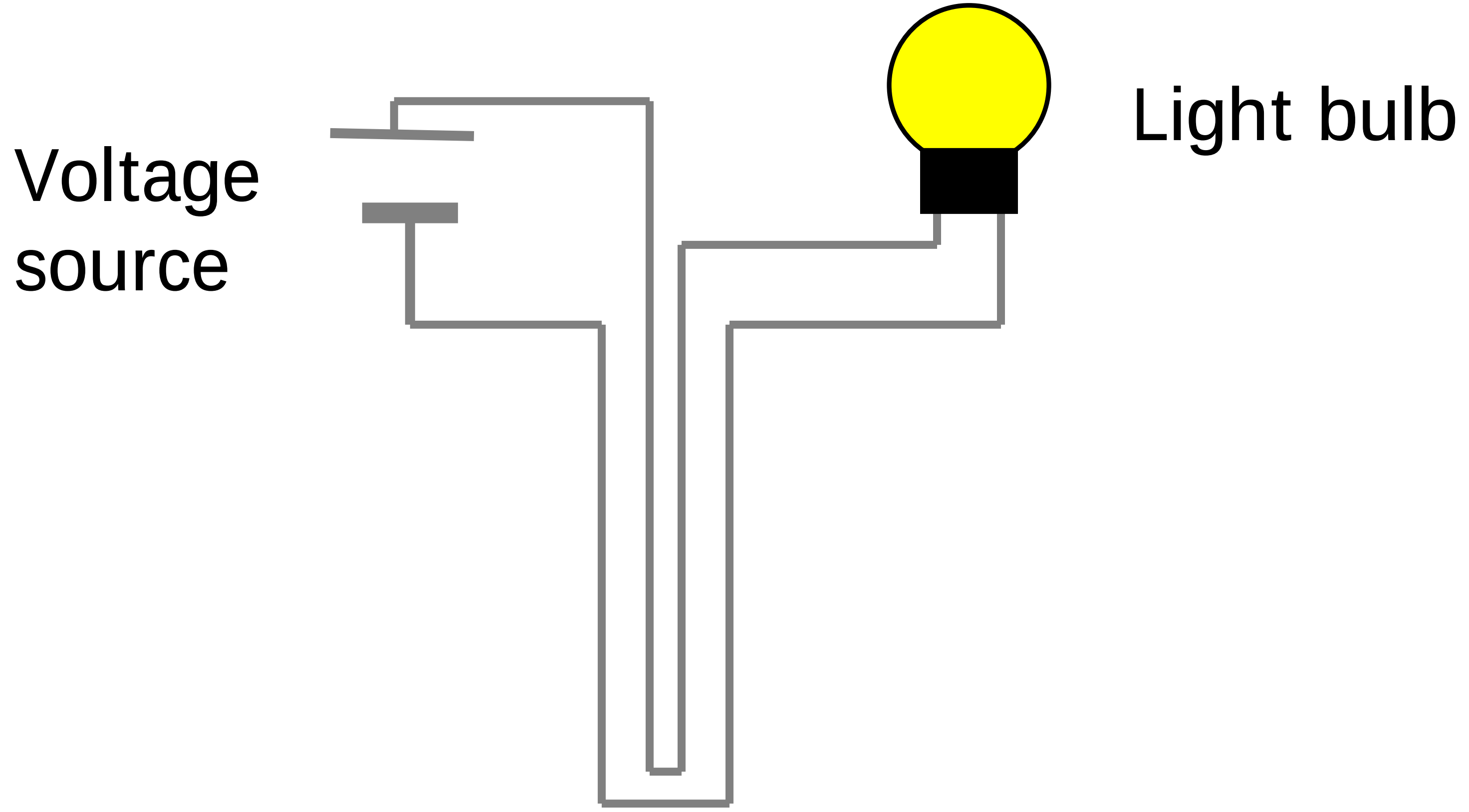
$J = 1/\rho\, E$ and $I = J\,A$ -> $I = R\,V$ for $\boxed{R = \rho\, L\,/A}$

Resistance

$$R = \rho L / A = f/(n_q q^2) L/A$$

- How can we change Resistance?
  - generally, want low resistance (lose less energy to heat)
  - Make Area A big
  - Make length L short
  - Make f small!
- Demo....

In-Class Demo



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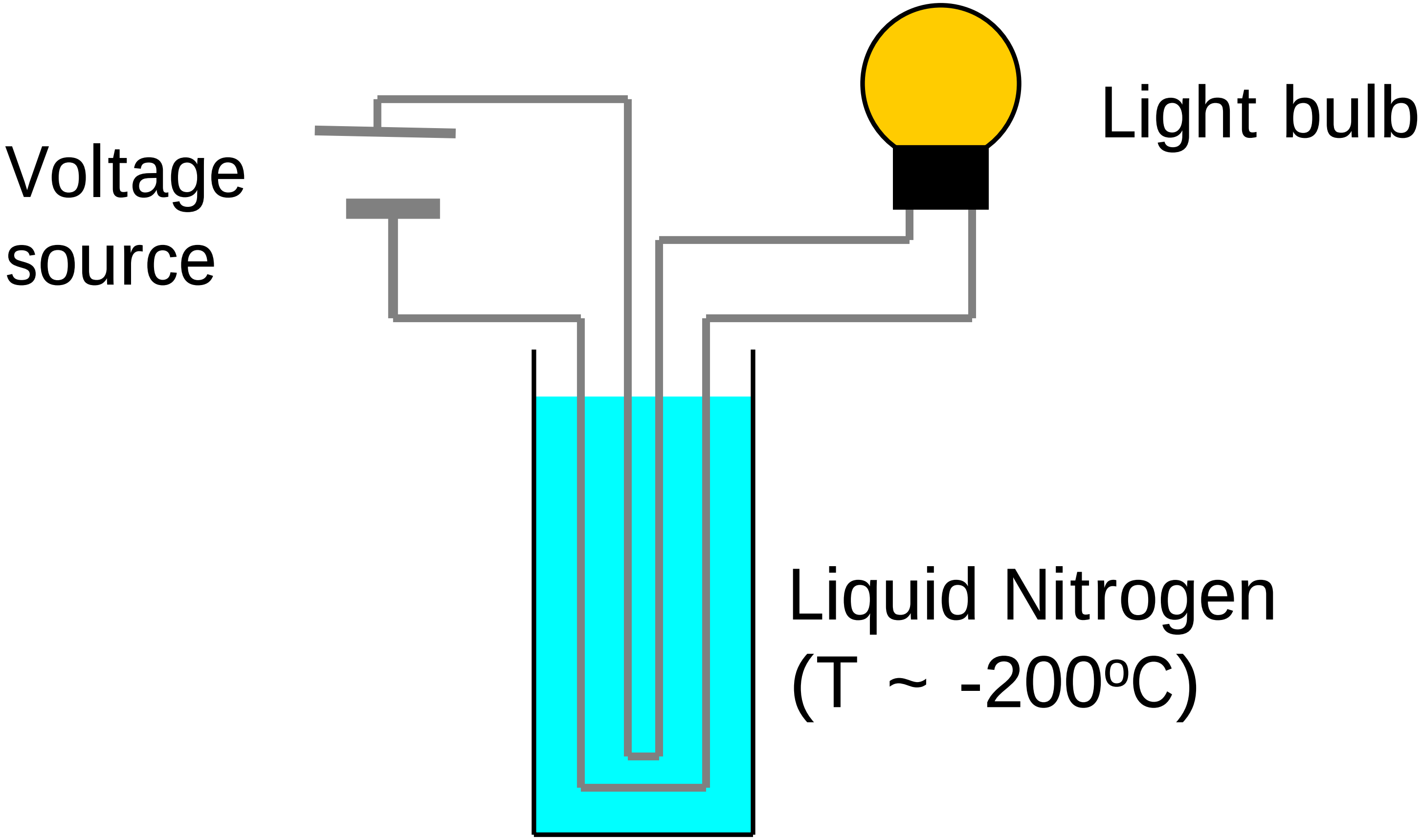
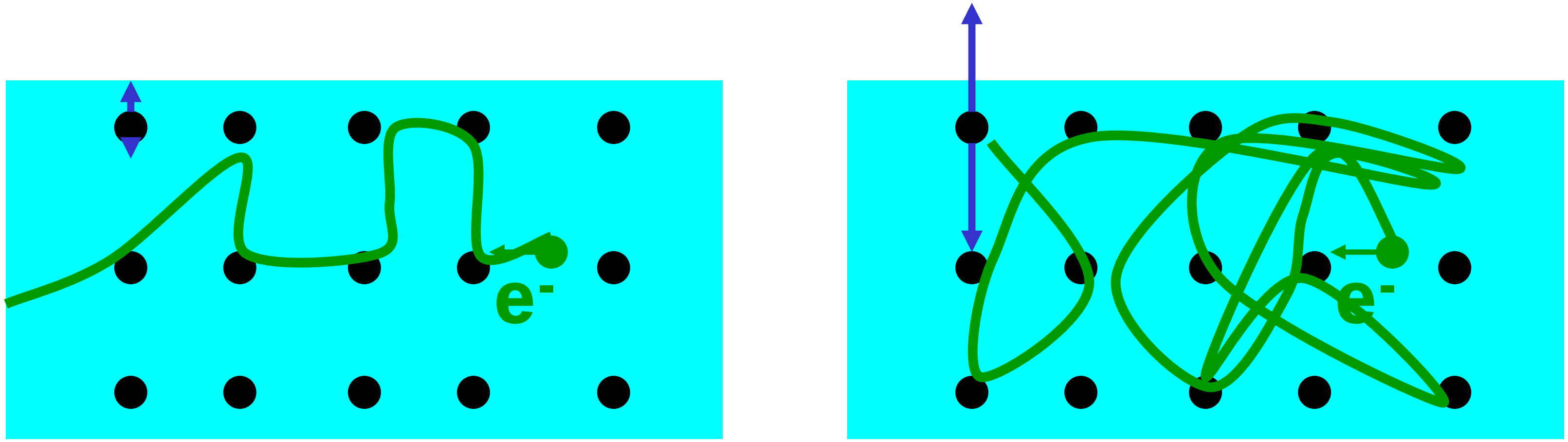
In-Class Demo



Wire cold -> less resistance -> more current -> bulb burns brighter

In-Class Demo



- T low
- Less vibration
- Electrons move through lattice easily

- T high
- Big vibration
- Electrons bounced around