Electricity and Magnetism

- Today
  - Ohm's Law
  - Electric Power
  - Electromotive Force
  - Circuits
  - Kirchoff's Rules

Electric Current

- We left Electrostatics
  - Now: Charges can move in steady state
- Electric Current **I**:
  - **$I = dQ/dt$**
  - Net amount of charge moving through conductor per unit time
- Units:
  - **$[I] = C/s = A$ (Ampere)**

Electric Current

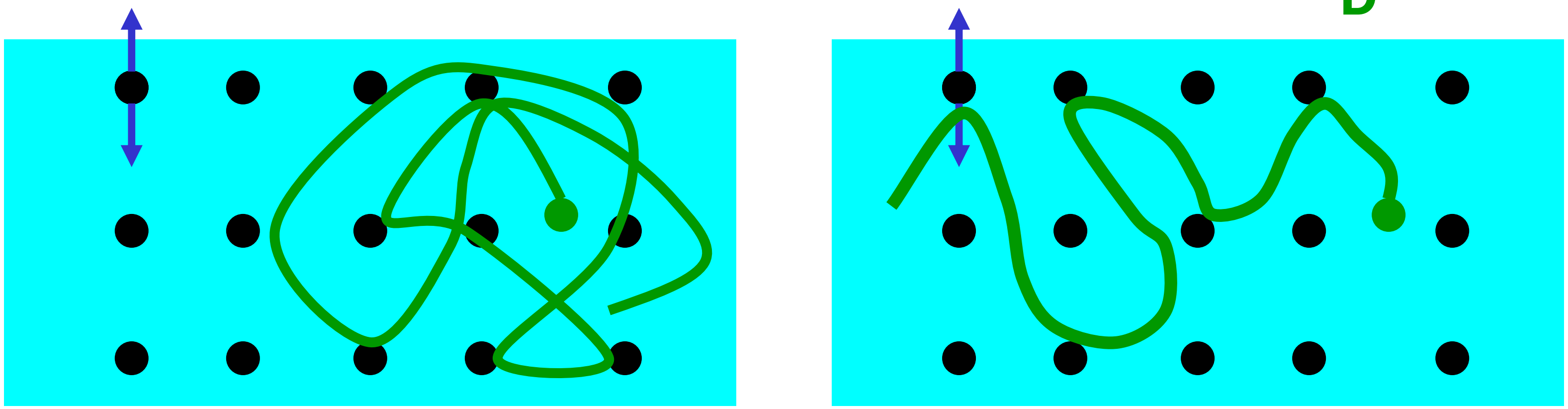
- Current **$I = dQ/dt$** has a direction
  - Convention: Direction of flow of positive charges
  - In our circuits, I carried by electrons
- To get a current:
  - Need mobile charges
  - Need **$|E| > 0$** (Potential difference)

Resistivity

- Suppose we have
  - mobile charge carriers
  - a Potential difference
- What determines magnitude of **I** ?
  - limited by <u>friction</u>
- Charge carriers lose energy in collisions with e.g. metal lattice

Resistivity

$\langle \vec{v} \rangle = 0$ $\qquad$ $\langle \vec{v} \rangle = \vec{v}_D > 0$



$|E| = 0$ $\qquad$ $|E| > 0$

If $|E| > 0$: Electron accelerated between scatterings

<u>On average</u>: Electron moves in $-E$ direction

Resistivity

- Interplay of scattering and acceleration gives an average velocity v_D
- v_D is called 'Drift velocity'
- How fast do the electrons move?
  - Thermal speed is big: $v_{th} \sim 10^6$ m/s
  - Drift velocity is small: $v_D \sim 10^{-3}$ m/s
- All electrons in conductor start to move, as soon as $E > 0$

Resistivity

Ohm

$\rho = f/(q^2 n_q)$: Resistivity

Units: $[\rho] = V/m\ m^2/A = m\ V/A = m\ \Omega$

Material	ρ [m Ω]
Glass	$>10^{10}$
Pure Water	$2*10^5$
Carbon	$3.5*10^5$
Silicon	2300
Sea Water	0.2
Gold	$2.4*10^{-8}$
Copper	$1.7*10^{-8}$

Insulator

Semiconductor

Conductor

Resistance

- Define $R = V/I$: Resistance
- $R = \rho L/A = f/(n_q q^2) L/A$
  - for constant cross section A
- R is measured in Ohm [W] = [V/A]

- Resistivity ρ is property of material (e.g. glass)

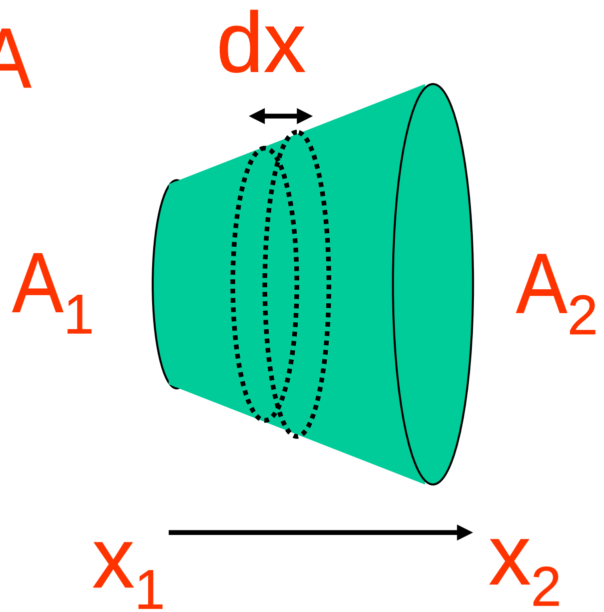
- Resistance R is property of specific conductor, depending on material (ρ) and geometry

Resistance

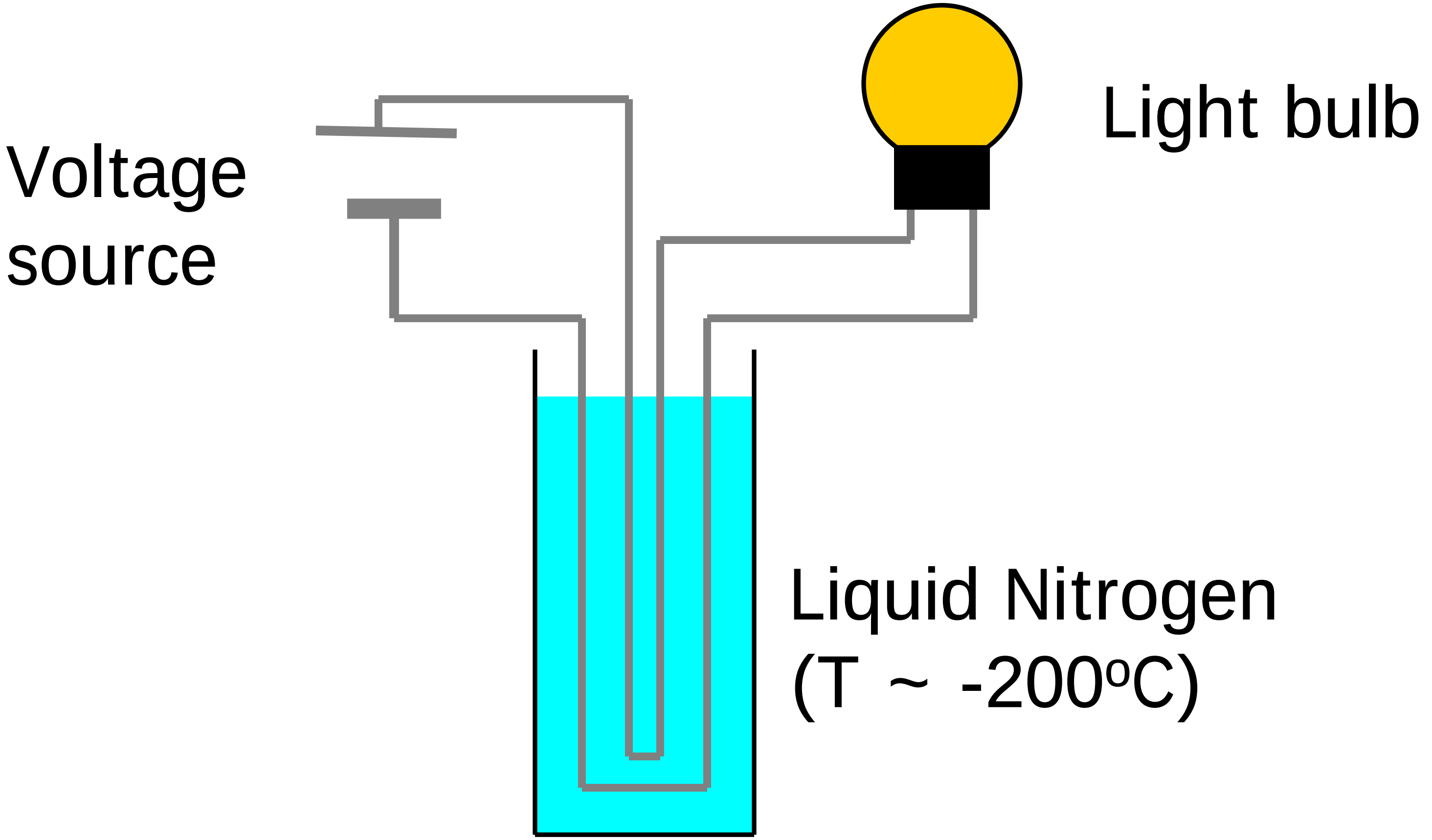
- $R = \rho L/A = f/(n_q q^2) L/A$
  - assuming constant cross-section A
- What if $A = A(x)$?
- Slice into pieces dx with constant A

$$dR = \rho \; dx/A(x)$$

- Integrate



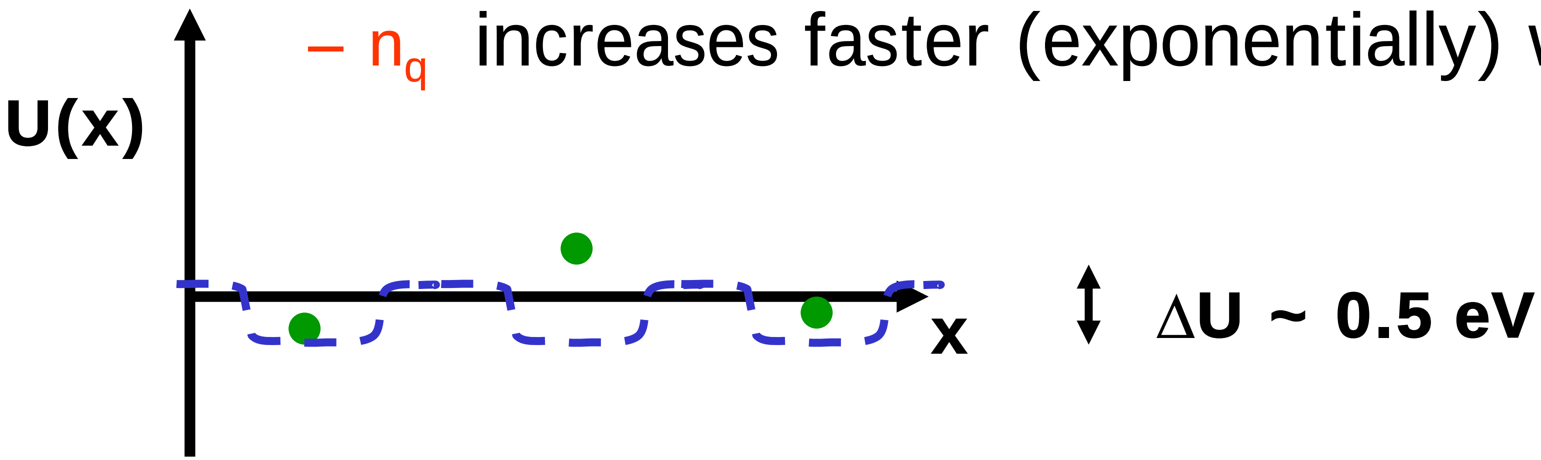
In-Class Demo



Resistivity depends on Temperature

Resistivity vs Temperature

- Resistivity $\rho = f/(q^2 n_q)$
- For Conductors: f increases with T
  - more scattering from vibrating lattice
- For Semiconductors: Different behaviour
  - f increases with T
  - n_q increases faster (exponentially) with T



Ohm's law

$$V = R\ I$$

- Isn't that just the definition of R?
- Not quite
  - Def. $R = V/I$ for any conductor
  - Ohm's Law says that for some conductors, current and voltage are linear
  - For real conductors, that's an approximation (e.g. $R = R(T)$ and $T = T(I)$)

Electric Power

- Fundamental application of Electricity
  - Deliver Electric Power
  - Converted to
    - Mechanical power
    - Heat
    - Light

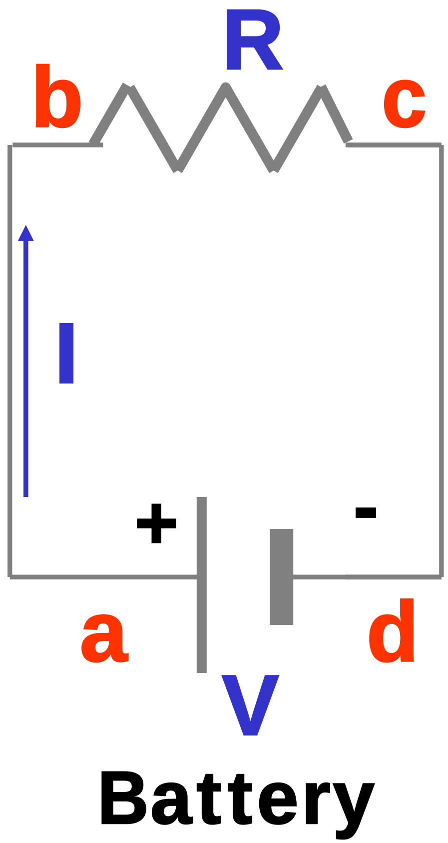
Power = Energy/time =

dW/dt = (dq V)/dt =

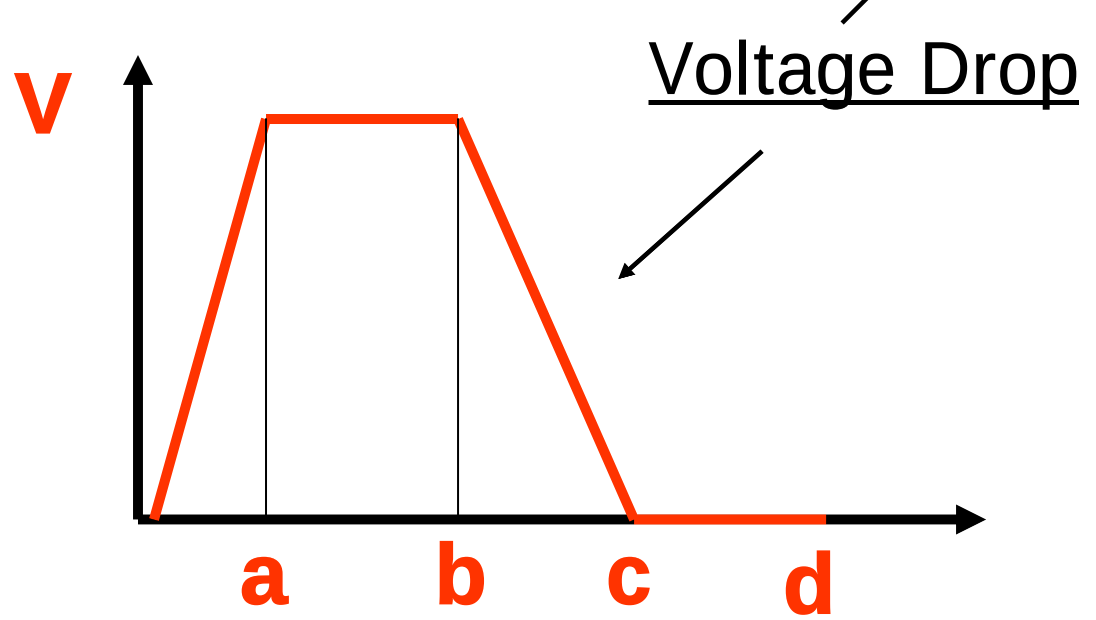
dq/dt V = <u>I V</u> = $I^2R = V^2/R$

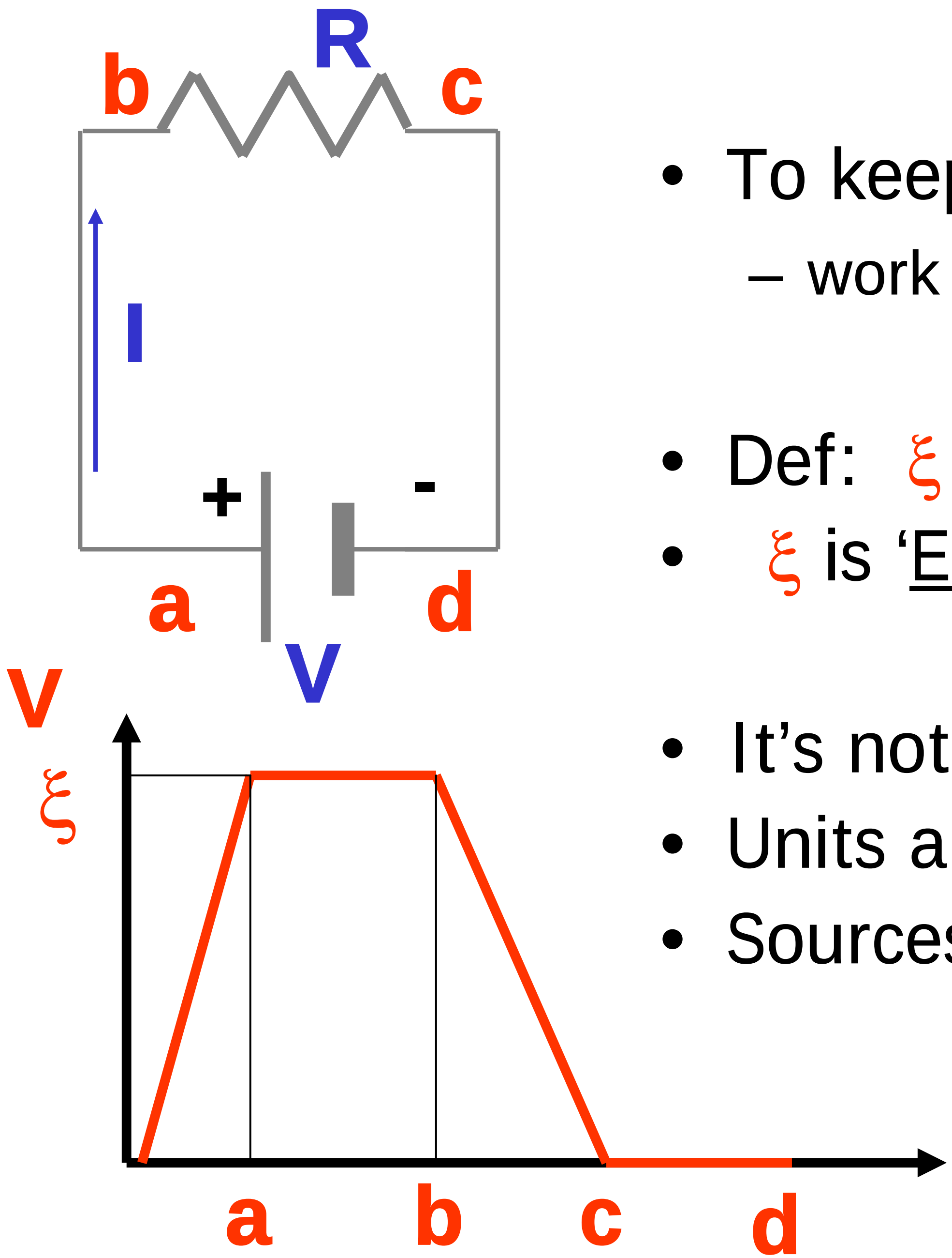
Electric Circuits

Resistor



$$V_{ad} = V$$
$$V_{ab} = 0$$
$$V_{cd} = 0$$
$$\Rightarrow V_{bc} = V_{ad} = \underline{I\,R}$$





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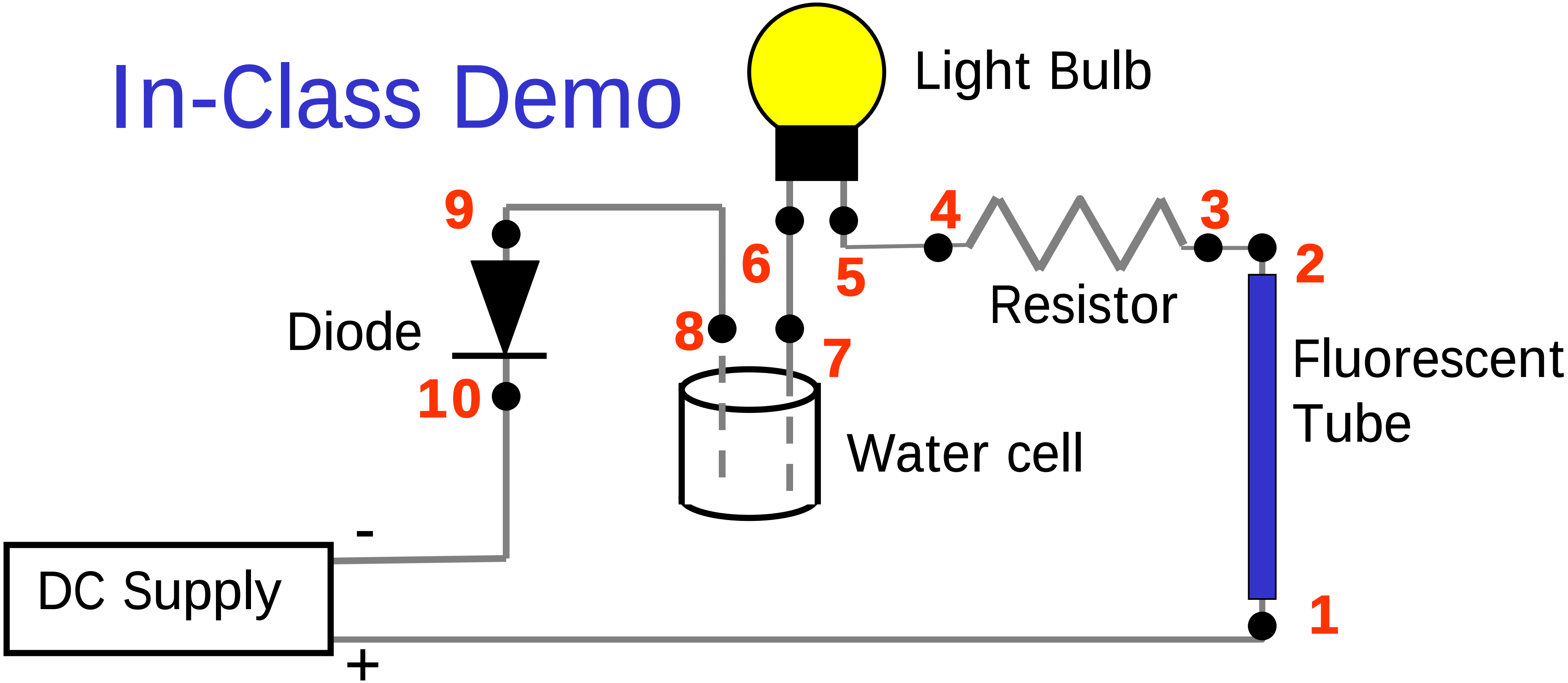
- To keep charge moving
  - work $W = q V$ to get from d to a

- Def: ξ = Work/unit charge
- ξ is ‘Electromotive Force’ (EMF)

- It’s not a Force!
- Units are [V]
- Sources of EMF: Battery, LVPS

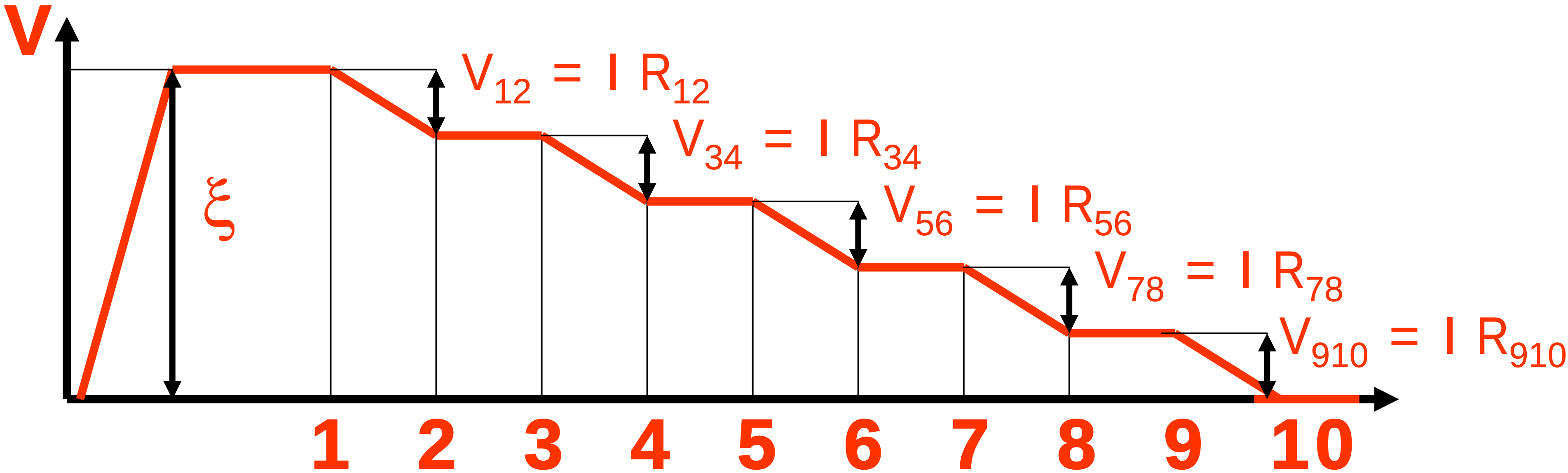
In-Class Demo



?

$$\text{EMF } \xi = V_{12} + V_{34} + V_{56} + V_{78} + V_{910}$$

Electromotive Force

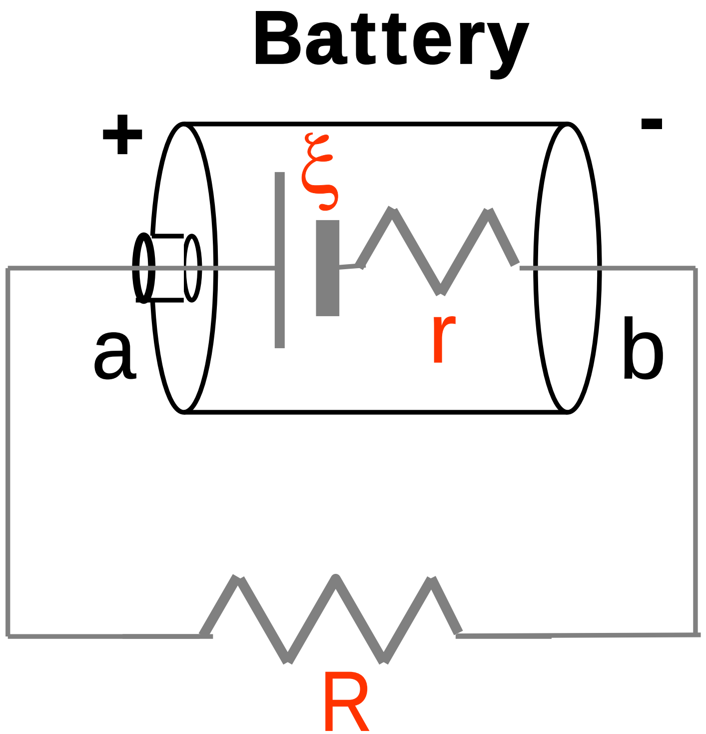


In general, $\Sigma\ \xi_i = \Sigma\ V_j$ <u>Loop Rule</u>

Electromotive Force

- Ideal source of EMF: $\xi = V = \text{constant}$
  - independent of I
- Problem: Power $P = IV = V^2/R$
  - for $R \to 0$, power gets infinite
- Real source of EMF: $V = \xi - I\,r$
  - r is internal resistance

Electromotive Force



$$V_{ab} = \xi - I\ r$$

$$V_{ab} = I\ R$$

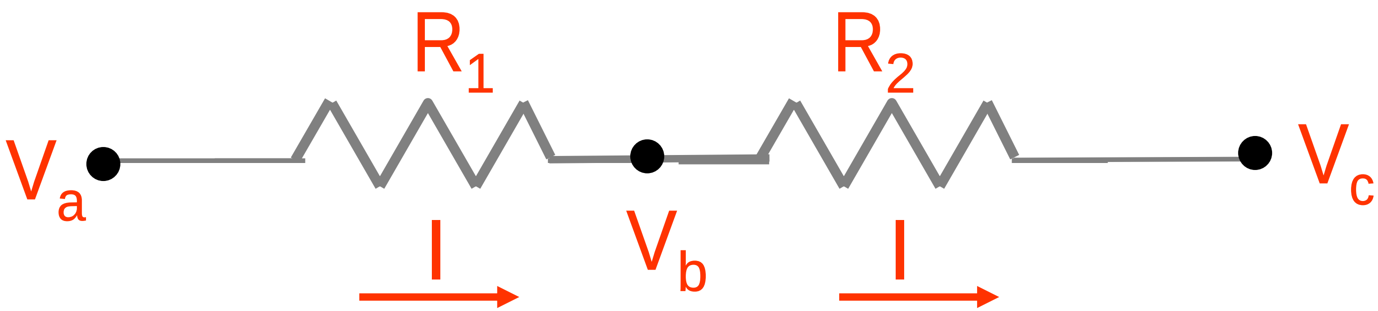
$$\text{->}\ \xi - I\ r = IR$$

$$\text{->}\qquad I = \xi\ /(r+R)$$

Old batteries: r gets larger

DC Circuits

- DC: Direction of current doesn't change
- Example: Resistors in <u>series</u>

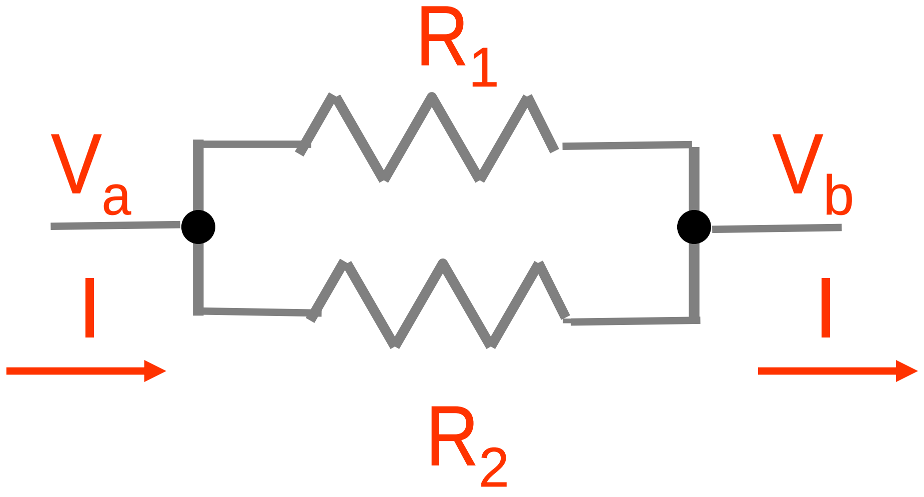


$$V_{ac} = V_{ab} + V_{ac} = I\,R_1 + I\,R_2 = I\,(R_1 + R_2)$$

$= I\,R_{eq}$ for $R_{eq} = (R_1 + R_2)$

DC Circuits

- Next example: Resistors in parallel



$I = I_1 + I_2 = V_{ab}/R_1 + V_{ab}/R_2 = V_{ab}/R_{eq}$

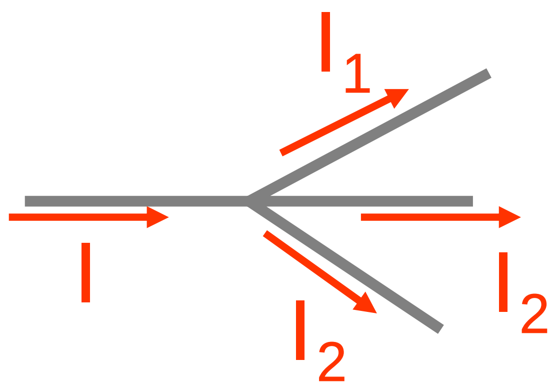
$\rightarrow 1/R_{eq} = 1/R_1 + 1/R_2$

Kirchoff's Rules

- Junction rule

At junctions:

$$\Sigma\ I_{i_{in}} = \Sigma\ I_{j_{out}}$$



Charge conservation

- Loop rule

Around closed loops:

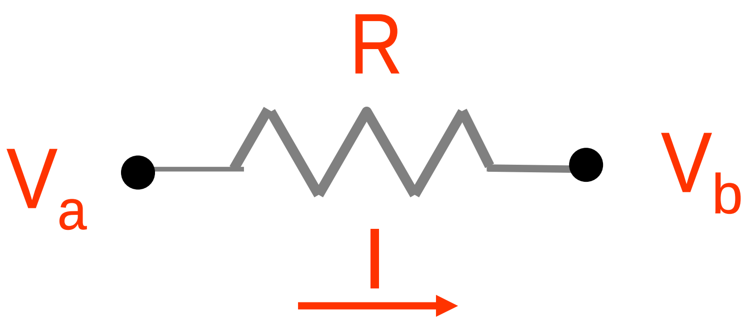
$$\Sigma\ \Delta\ V_j = 0$$

$\Delta\ V$ for both EMFs and Voltage drops

Energy conservation

Kirchoff's Rules

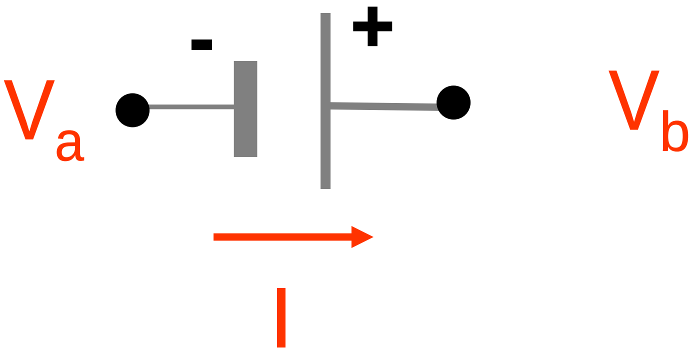
- Kirchoff's rules allow us to calculate currents for complicated DC circuits
- Main difficulty: Signs!
- Rule for resistors:



$\Delta V = V_b - V_a = -IR$, if we go in the direction of I (voltage drop!)

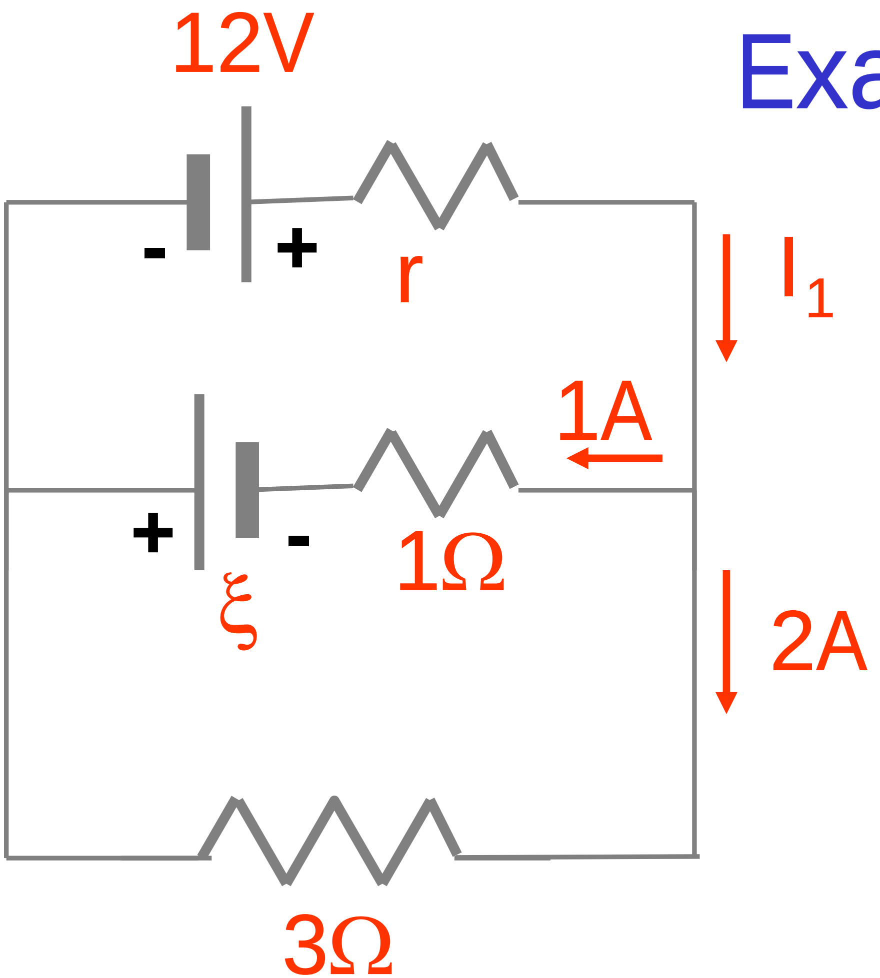
Kirchoff's Rules

- Kirchoff's rules allow us to calculate currents for complicated DC circuits
- Main difficulty: Signs!
- Rule for EMFs:



$\Delta V = V_b - V_a = \xi$, if we go in the direction of I

Example



r, ξ, I_1 ? 3 unknowns

- Pick signs for I_1, ξ
- Junction rule

  I_1 = 1A + 2A = 3A
- Loop rule (1)

  12V – 6V – 3A r = 0

  -> r = 6/3 Ω = 2 Ω
- Loop rule (2)

  12V – 6V – 1V + ξ = 0

  -> ξ = 5V