Electricity and Magnetism

- Today
  - DC Circuits
  - Kirchoff's Rules
  - RC Circuits

Ohm's law

$$V = R\ I$$

- Def. $R = V/I$ for any conductor
- Ohm's Law says that for some conductors, current and voltage are proportional
  - Ohmic conductors (e.g. Resistors)
- For real conductors, that's an approximation (e.g. $R = R(T)$ and $T = T(I)$)

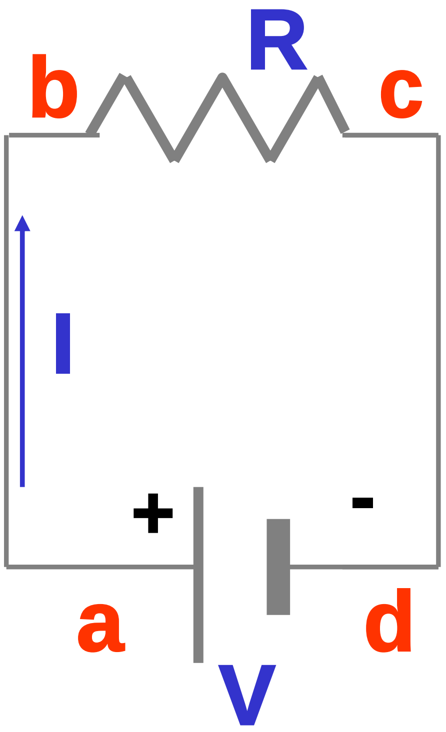
Electric Power

- Fundamental application of Electricity
  - Deliver Electric Power
  - Converted to
    - Mechanical power
    - Heat
    - Light

Power = Energy/time =

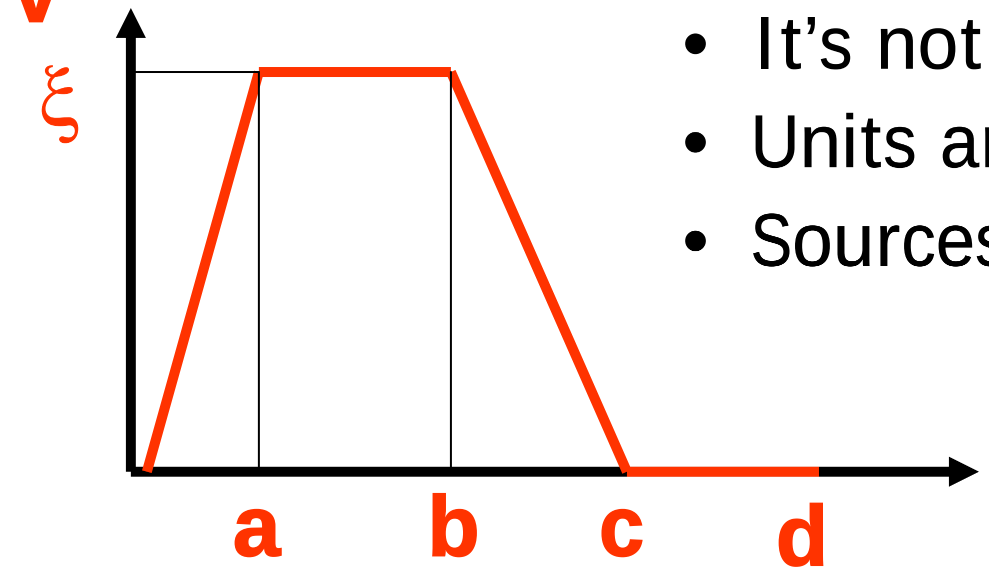
dW/dt = (dq V)/dt =

dq/dt V = $\underline{I\ V}$ = I^2R = V^2/R



- To keep charge moving
  - work $W = q\,V$ to get from d to a

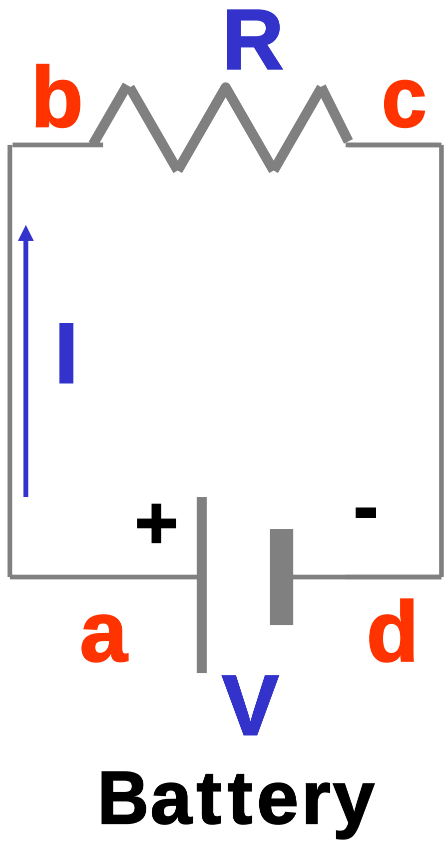
- Def: $\xi =$ Work/unit charge
- ξ is 'Electromotive Force' (EMF)



- It's not a Force!
- Units are [V]
- Sources of EMF: Battery, LVPS

Electric Circuits

Resistor

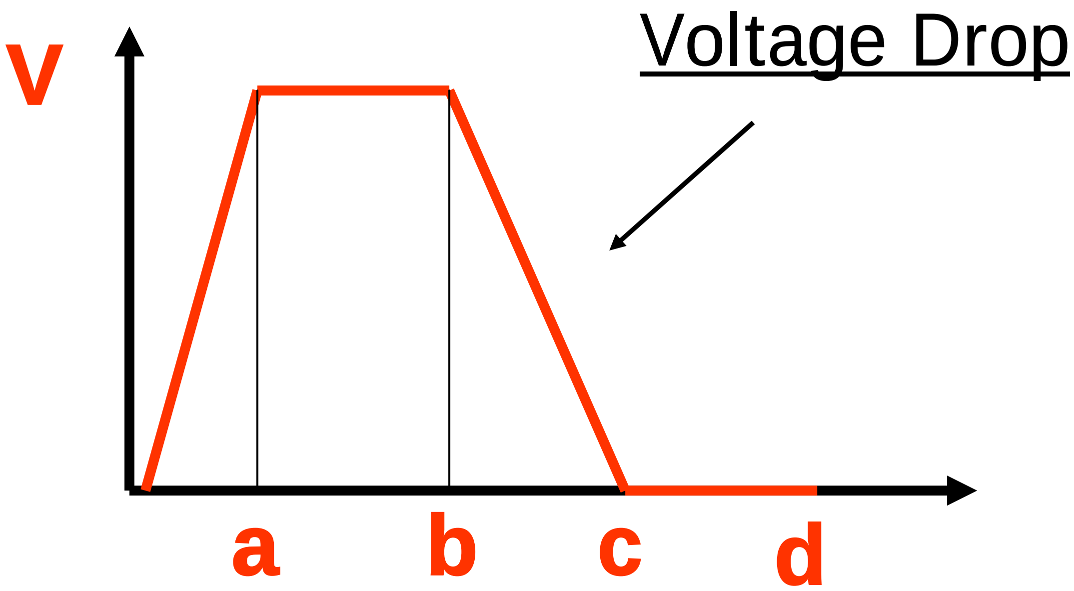


$V_{ad} = V$

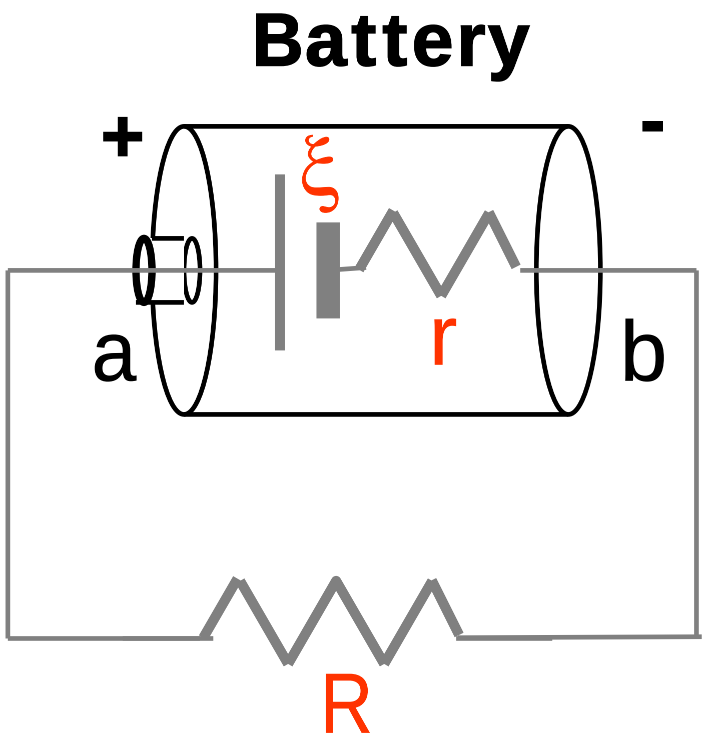
$V_{ab} = 0$

$V_{cd} = 0$

$\rightarrow V_{bc} = V_{ad} = \underline{I\ R}$



Internal Resistance



$$V_{ab} = \xi - I\,r$$

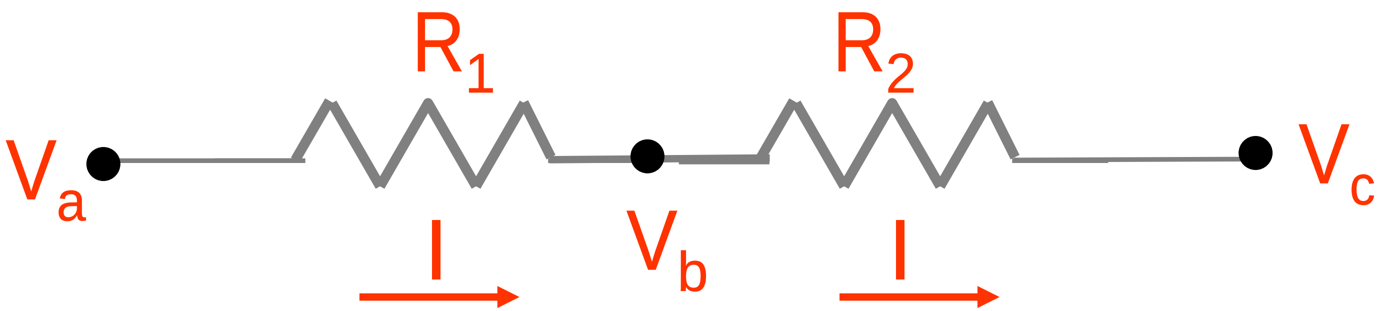
$$V_{ab} = I\,R$$

-> $\xi - I\,r = IR$

-> $I = \xi\,/(r+R)$

DC Circuits

Resistors in <u>series</u>

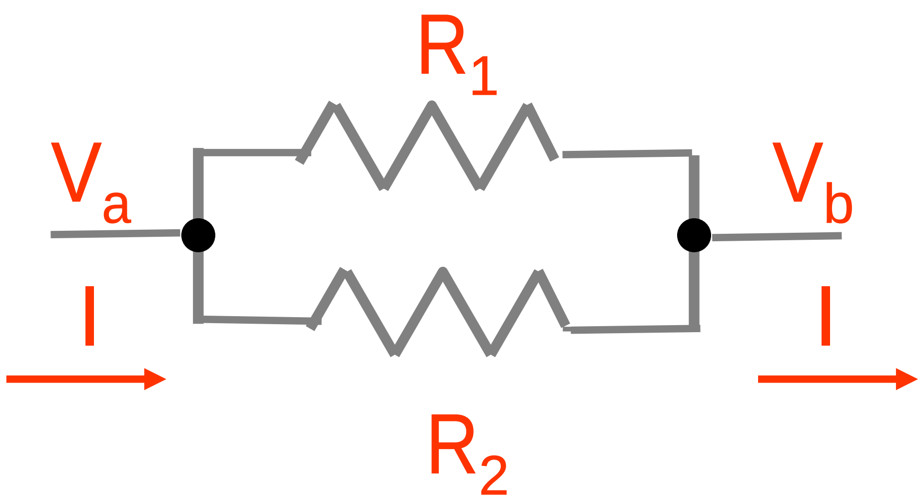


$$V_{ac} = V_{ab} + V_{ac} = I\,R_1 + I\,R_2 = I\,(R_1 + R_2)$$

$$= I\,R_{eq}$$ for $$\boxed{R_{eq} = (R_1 + R_2)}$$

DC Circuits

Resistors in parallel



$$I = I_1 + I_2 = V_{ab}/R_1 + V_{ab}/R_2 = V_{ab}/R_{eq}$$

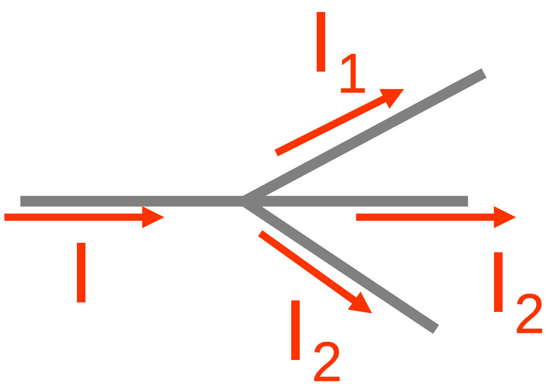
$$\rightarrow \boxed{1/R_{eq} = 1/R_1 + 1/R_2}$$

Kirchoff's Rules

- Junction rule

At junctions:

$$\sum I_{i_{in}} = \sum I_{j_{out}}$$



Charge conservation

- Loop rule

Around closed loops:

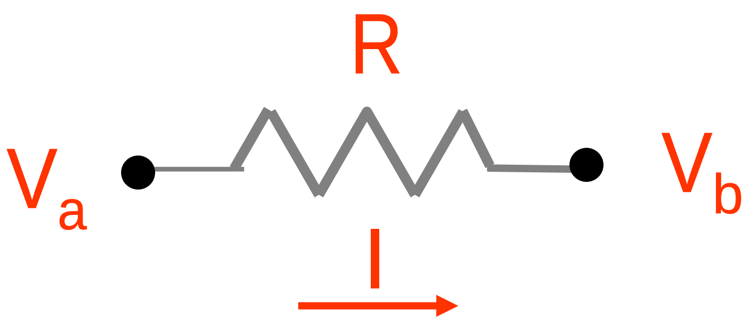
$$\sum \Delta V_j = 0$$

ΔV for both EMFs and Voltage drops

Energy conservation

Kirchoff’s Rules

- Kirchoff’s rules allow us to calculate currents for complicated DC circuits
- Main difficulty: Signs!
- Rule for resistors:



$\Delta V = V_b - V_a = -\, I\, R$, if we go in the direction of I (voltage drop!)

Kirchoff's Rules

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- Main difficulty: Signs!
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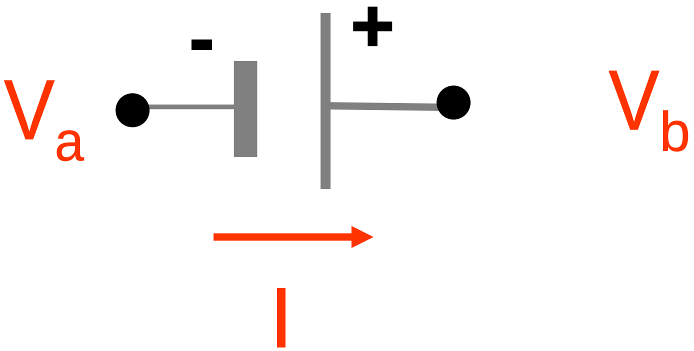
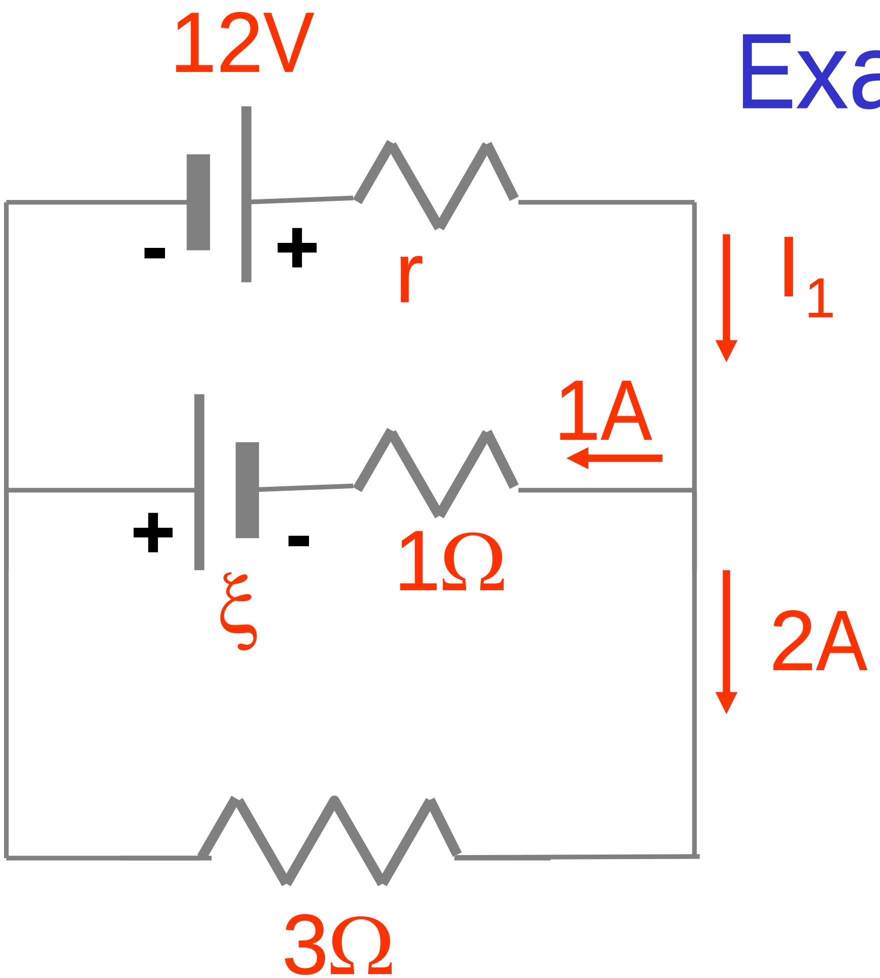


$\Delta V = V_b - V_a = \xi$, if we go in the direction of I

Example



r, ξ, I_1 ? 3 unknowns

- Pick signs for I_1, ξ
- Junction rule

  $I_1 = 1A + 2A = 3A$
- Loop rule (1)

  $12V - 6V - 3A\ r = 0$

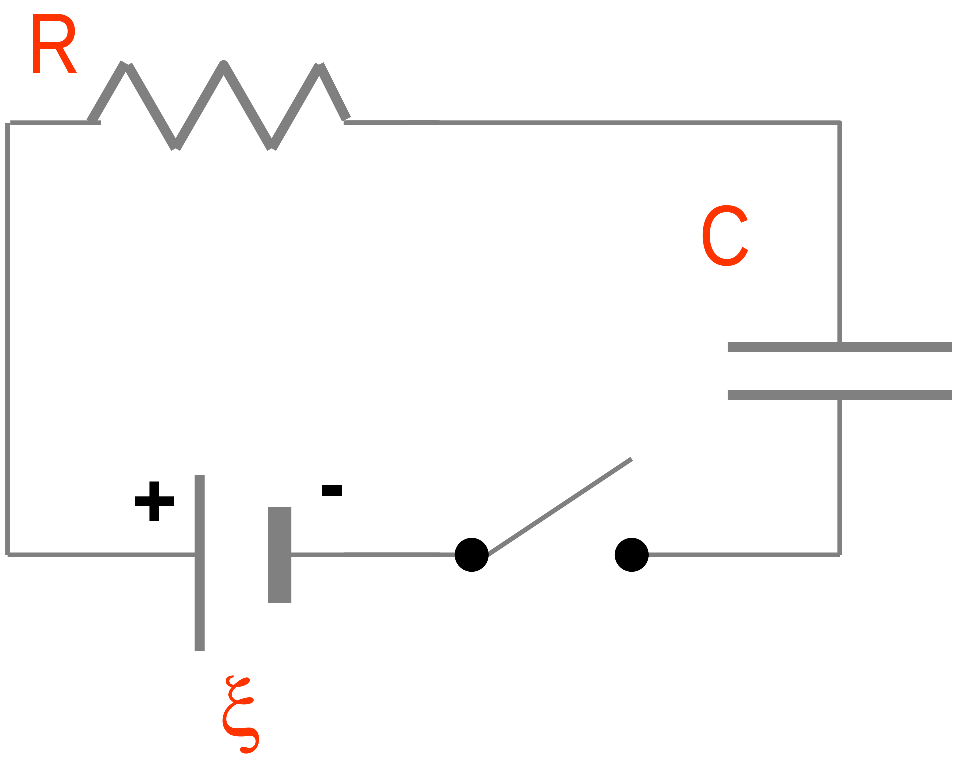
  $\rightarrow r = 6/3\ \Omega = 2\ \Omega$
- Loop rule (2)

  $12V - 6V - 1V + \xi = 0$

  $\rightarrow \xi = -5V$

RC Circuits

- Currents change with time
- Example: Charging a capacitor



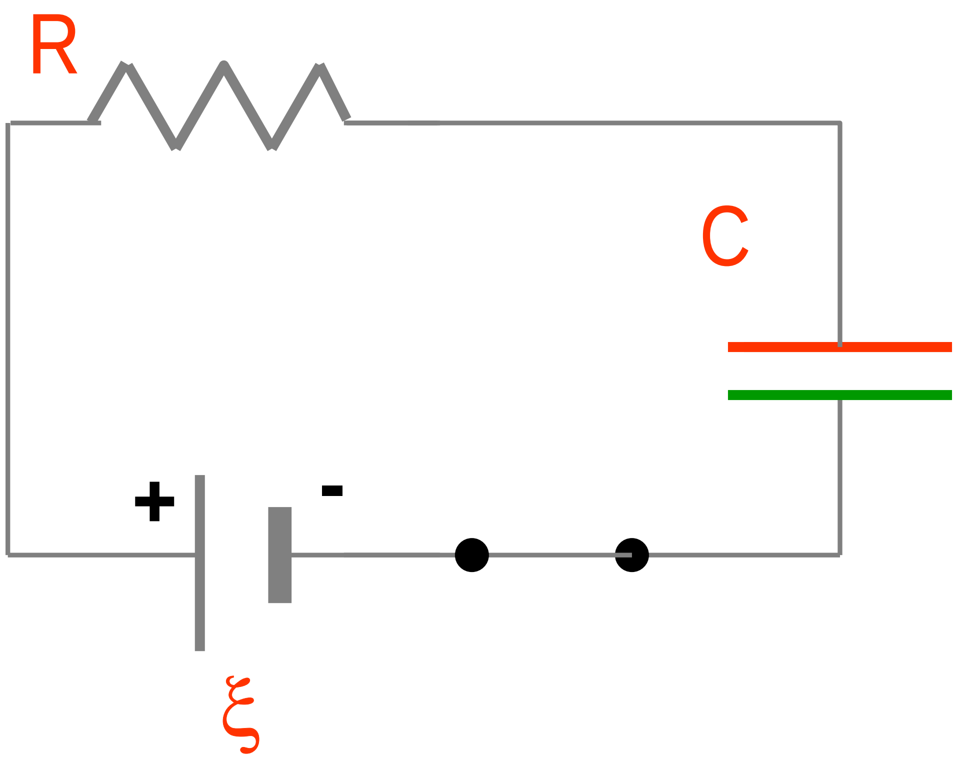
$t = 0$

$q = 0$

$V_c = q/C$

RC Circuits

- Currents change with time
- Example: Charging a capacitor



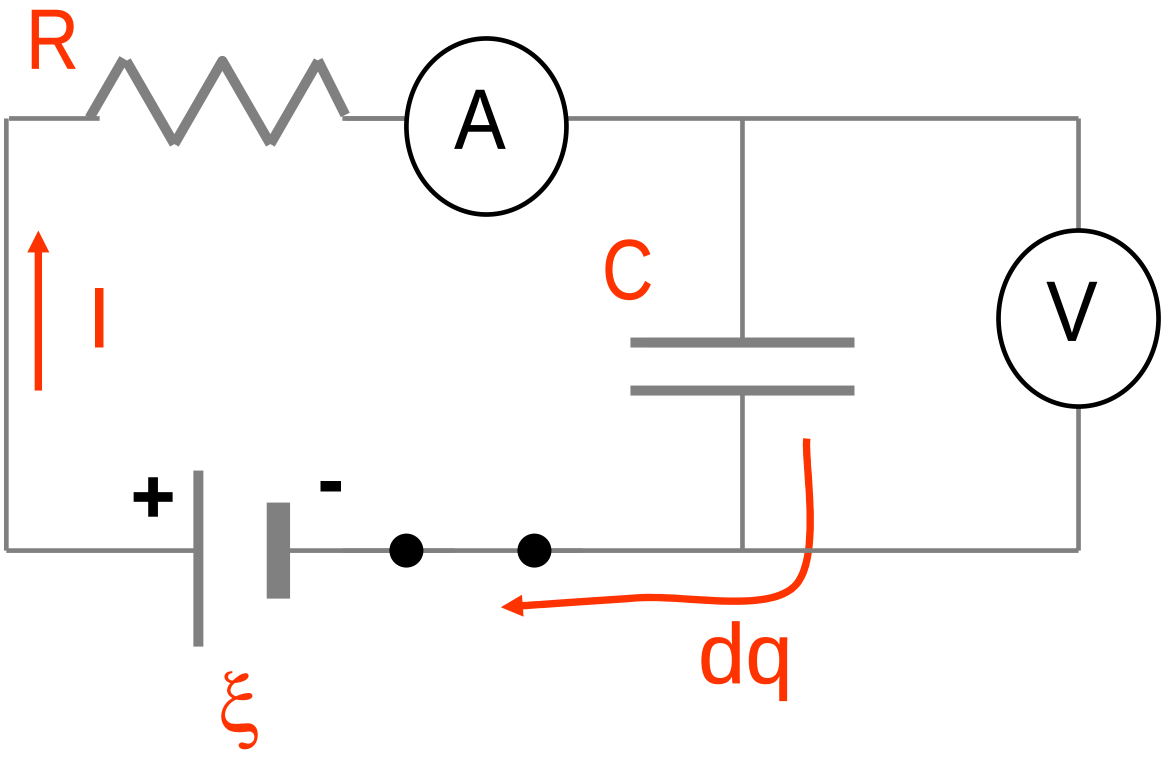
t = infinity

$q = C\,\xi$

$V_c = q/C = \xi$

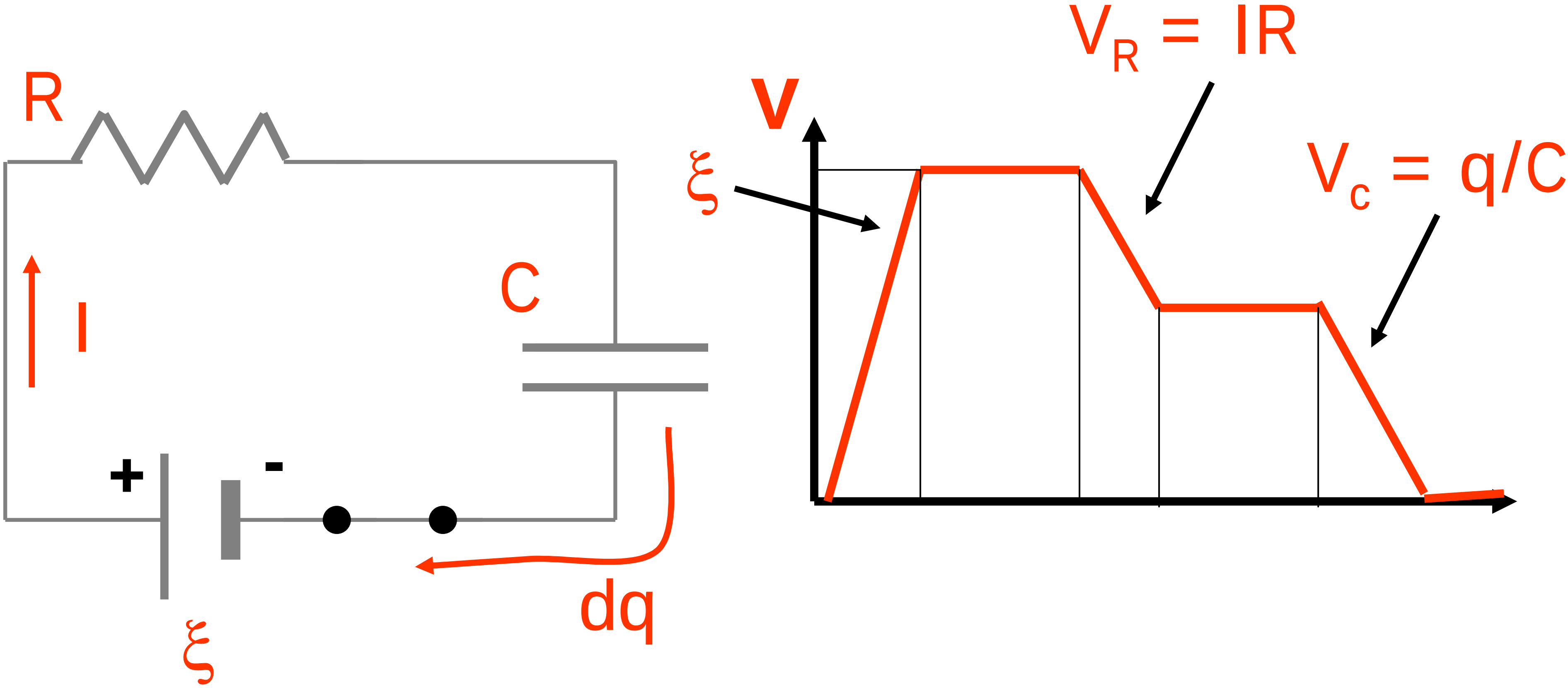
RC Circuits

- What happens between t=0 and infinity?



RC Circuits

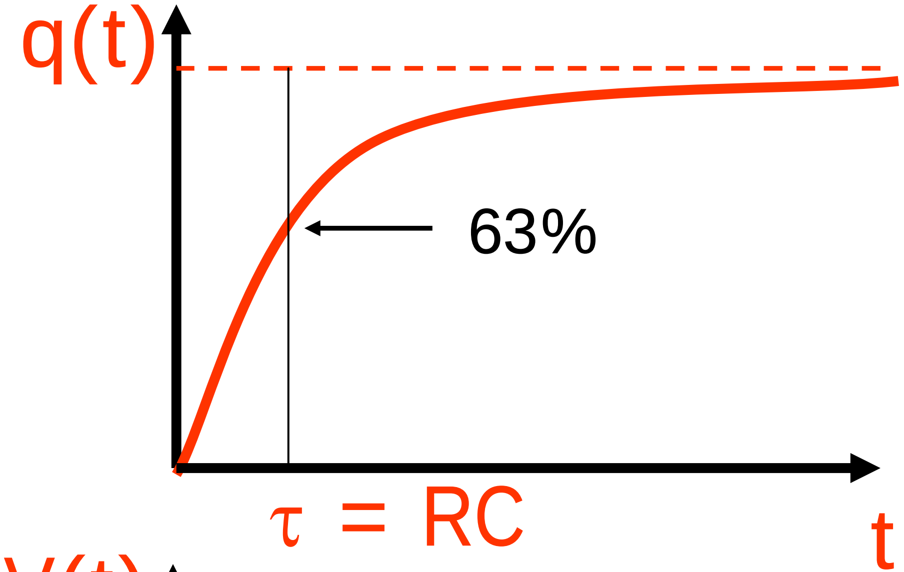
- What happens between t=0 and infinity?

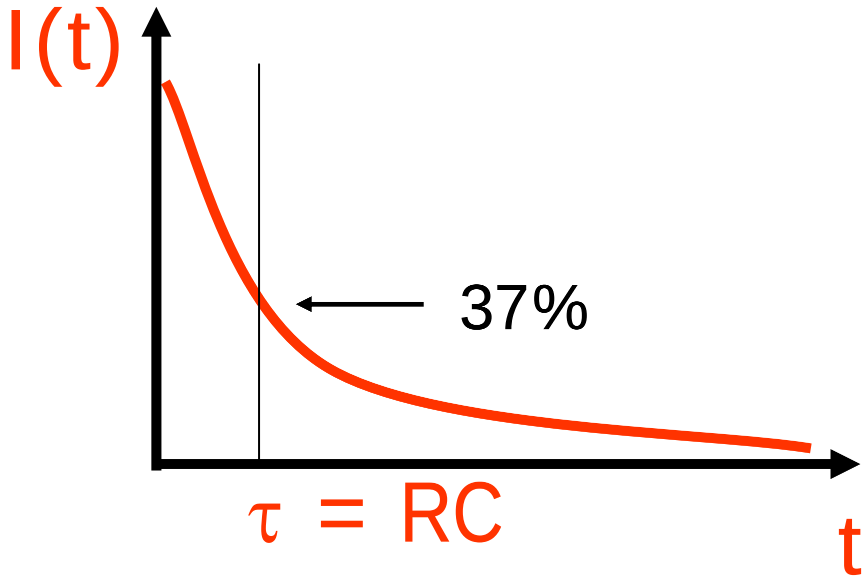


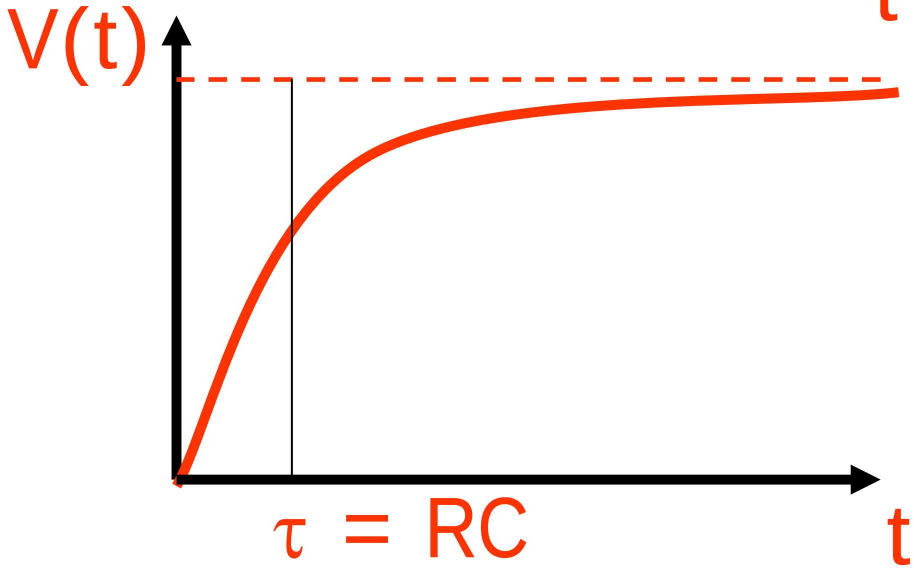
Charging capacitor

$$\begin{aligned}
\xi - \frac{q}{C} - IR &= 0 \quad \text{Loop Rule} \\
\xi - \frac{q}{C} - \frac{dq}{dt}R &= 0 \quad \text{note } q(t=0)=0, \quad I(t=0) = \frac{\xi}{R} \\
\Rightarrow \frac{dq}{dt} &= -\frac{q}{RC} + \frac{\xi}{R} \quad \text{separate variables} \\
\Rightarrow \frac{dq}{q-\xi C} &= -\frac{dt}{RC} \quad \text{integrate} \\
\Rightarrow \int_0^Q \frac{1}{q-\xi C} dq &= \int_0^t -\frac{dt}{RC} \\
\Rightarrow \ln\left(\frac{q-\xi C}{-\xi C}\right) &= -\frac{t}{RC} \quad \text{exponentiate} \\
\Rightarrow \frac{q-\xi C}{-\xi C} &= \exp\left(-\frac{t}{RC}\right) \\
\Rightarrow q(t) &= \xi C\left[1-\exp\left(-\frac{t}{RC}\right)\right] \\
\Rightarrow I(t) = \frac{dq}{dt} &= -\xi C \exp\left(-\frac{t}{RC}\right) \times \left(-\frac{1}{RC}\right) \\
&= \frac{\xi}{R} \exp\left(-\frac{t}{RC}\right) \\
\Rightarrow V(t) = \frac{q(t)}{C} &= \xi\left[1-\exp\left(-\frac{t}{RC}\right)\right]
\end{aligned}$$

Charging C







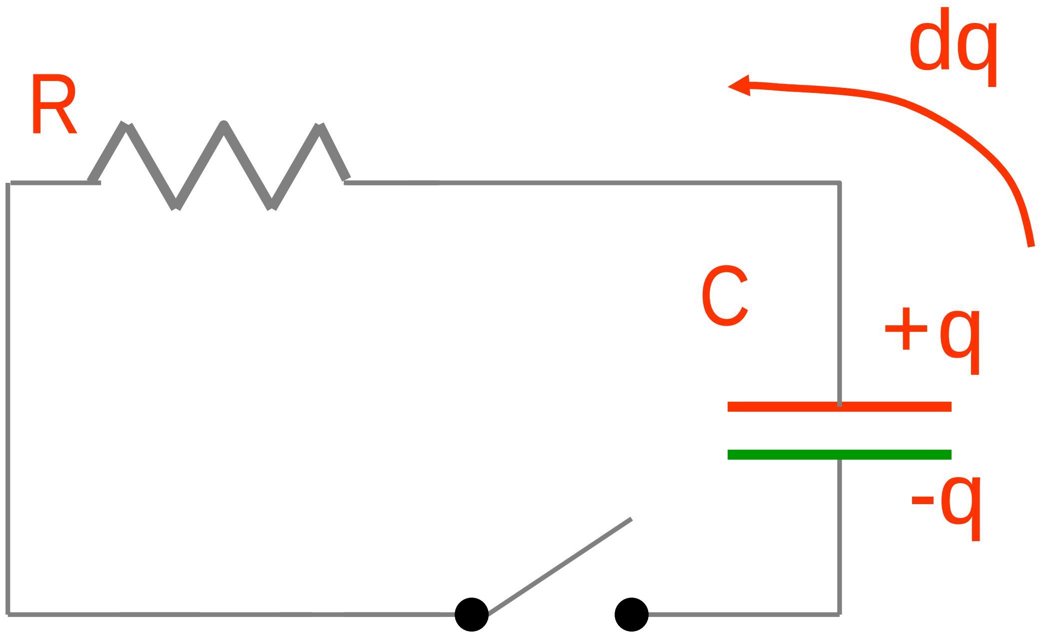
At t = t,

$q(t) = (1-1/e)\, q_{max}$

$= \underline{63\ \%\ q_{max}}$

In-Class Demo

Discharging a capacitor



Discharging Capacitor

$$
\begin{aligned}
-\frac{q}{C} - IR &= 0 \quad \text{Loop Rule for } \xi = 0, q(t=0) = Q_{final} = \xi C \\
-\frac{q}{C} - \frac{dq}{dt}R &= 0 \\
-\frac{dq}{q} = \frac{dt}{RC} & \\
\Rightarrow \int_{Q_{final}}^{q} \frac{1}{q} dq &= -\int_0^t \frac{dt}{RC} \\
\Rightarrow \ln\left(\frac{q}{Q_{final}}\right) &= -\frac{t}{RC} \\
\Rightarrow \boxed{q(t) = Q_{final} \exp\left(-\frac{t}{RC}\right)} & \\
&= \xi C \exp\left(-\frac{t}{RC}\right) \\
\Rightarrow I(t) = \frac{dq}{dt} &= \xi C \exp\left(-\frac{t}{RC}\right) \times \left(-\frac{1}{RC}\right) \\
&= -\frac{\xi}{R} \exp\left(-\frac{t}{RC}\right) \quad \text{note sign!} \\
V(t) = \frac{q(t)}{C} &= \xi \exp\left(-\frac{t}{RC}\right)
\end{aligned}
$$

Discharging C