Electricity and Magnetism

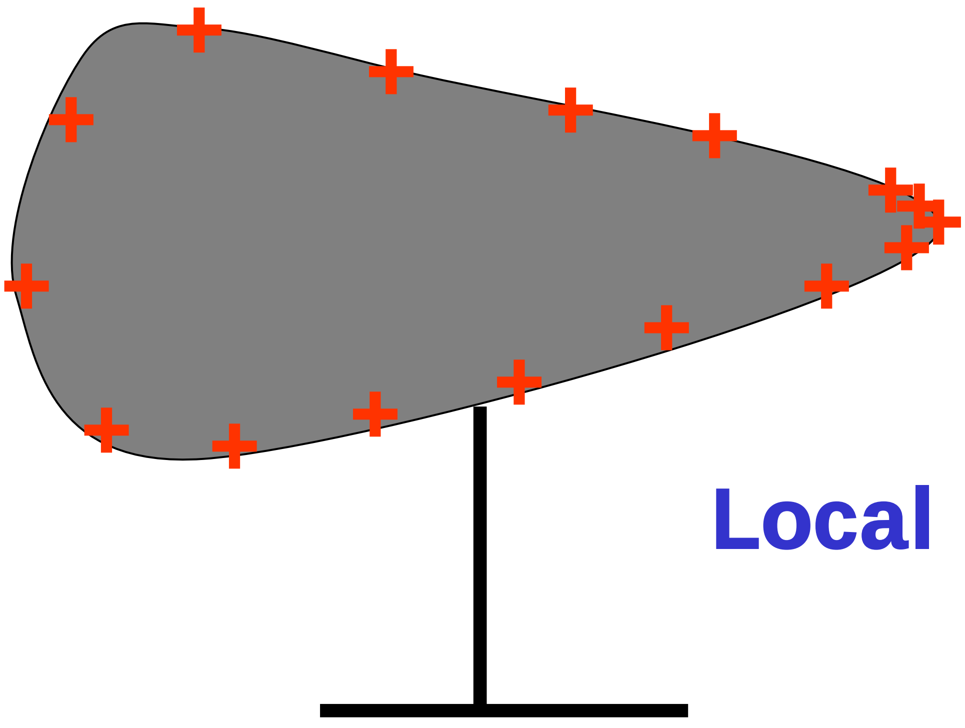
- Today: Review for Quiz #2
  - Conductors in E-Field/Potential
  - Capacitance/Capacitors
  - Dielectrics/Polarization
  - Current/Current Density
  - Resistance/Resistivity
  - DC circuits
  - Electric Power
  - Kirchoff's Rules
  - RC Circuits

Conductors in Electrostatics

- $E = 0$ inside (in Electro*statics*)
  - otherwise charges would move
- No charges inside
  - Gauss
- $E = \sigma/\varepsilon_0$ perpendicular to surface
  - otherwise charges on surface would move
- Potential is constant on conductor

Charge Density

In-Class Demo: **Application: Lightning rod - Biggest E near pointy tip!**



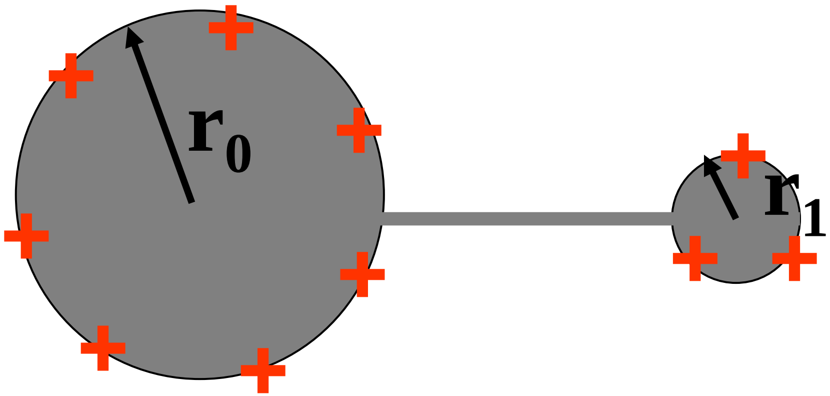
$$\Rightarrow \quad E \propto \frac{1}{r} \propto \sigma$$

Local radius of curvature

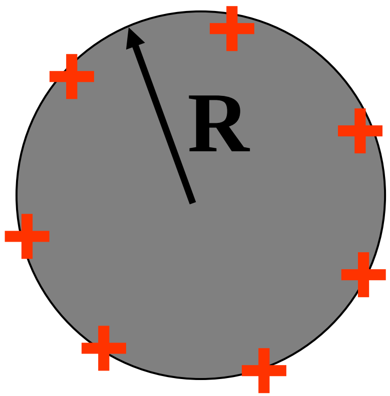
Charge distribution and Geometry

$$V_0 = \frac{Q_0}{4\pi\epsilon_0}\frac{1}{r_0} =$$
$$V_1 = \frac{Q_1}{4\pi\epsilon_0}\frac{1}{r_1}$$
$$\Rightarrow Q_0/r_0 = Q_1/r_1 = 4\pi\epsilon_0 V = \text{ const.}$$

$$E_i = \frac{\sigma_i}{2\epsilon_0} = \frac{Q_i}{4\pi\epsilon_0 r_i^2}$$
$$\Rightarrow E \propto \frac{1}{r} \propto \sigma$$



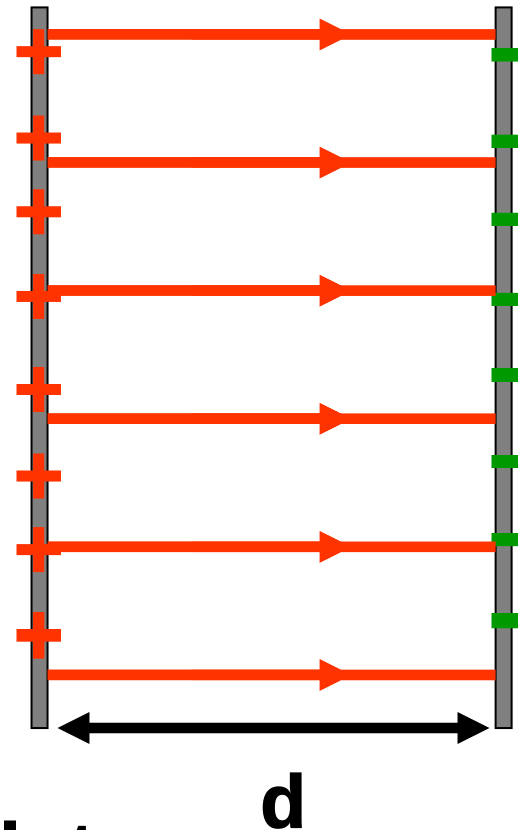
Charge and Potential



Charged Sphere

$$V=1/(4\ \pi\ \varepsilon_0\ R)\ Q$$

Geometry!



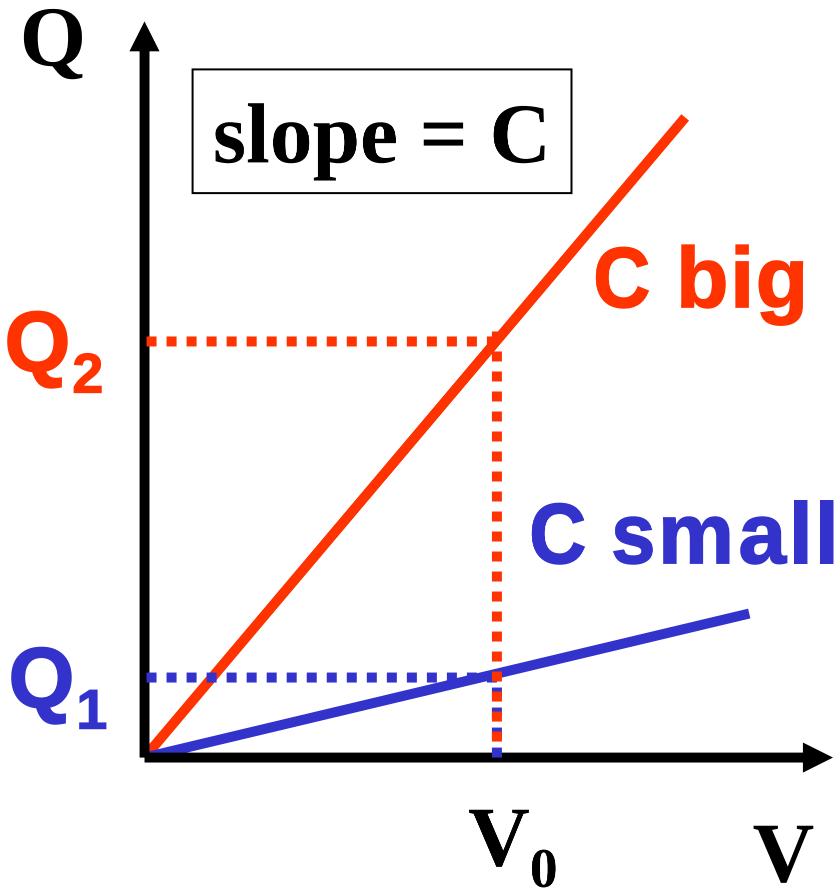
Parallel Plate Capacitor

$$V= d/(A\ \varepsilon_0)\ Q$$

Charge and Potential

- For given geometry, Potential and Charge are proportional
- Define
  - **$Q = C V$ -> C is Capacitance**
- Measured in [F] = [C/V] : Farad
- C tells us, how easy it is to store charge on it ($V = Q/C$)

Capacitance

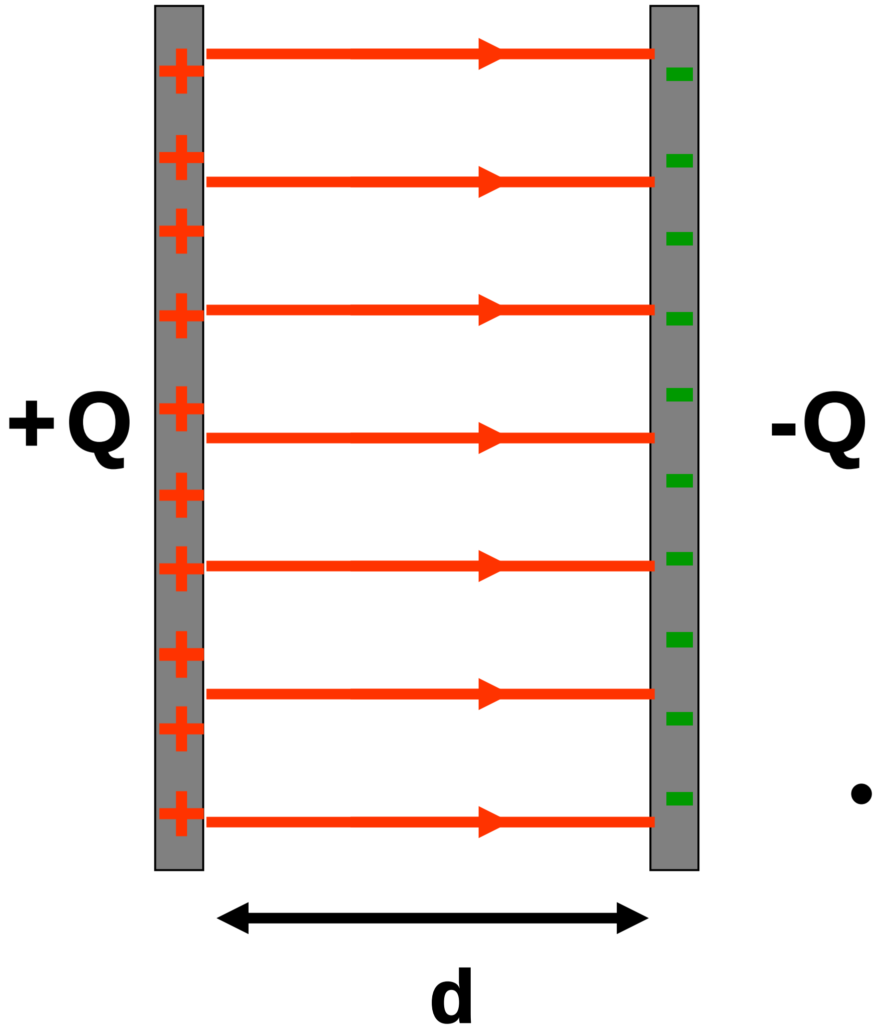


C bigger -> Can store more Charge!

Capacitor

- Def: Two conductors separated by insulator
- Charging capacitor:
  - take charge from one of the conductors and put on the other
  - separate + and - charges

Parallel Plate Capacitor



$$C = \frac{Q}{V(a) - V(b)} = \frac{Q}{E\,d}$$

$$= \frac{Q}{\frac{\sigma}{\epsilon_0}\,d} = \frac{Q}{\frac{Q}{A\epsilon_0}\,d} =$$

$$= \epsilon_0 \frac{A}{d}$$

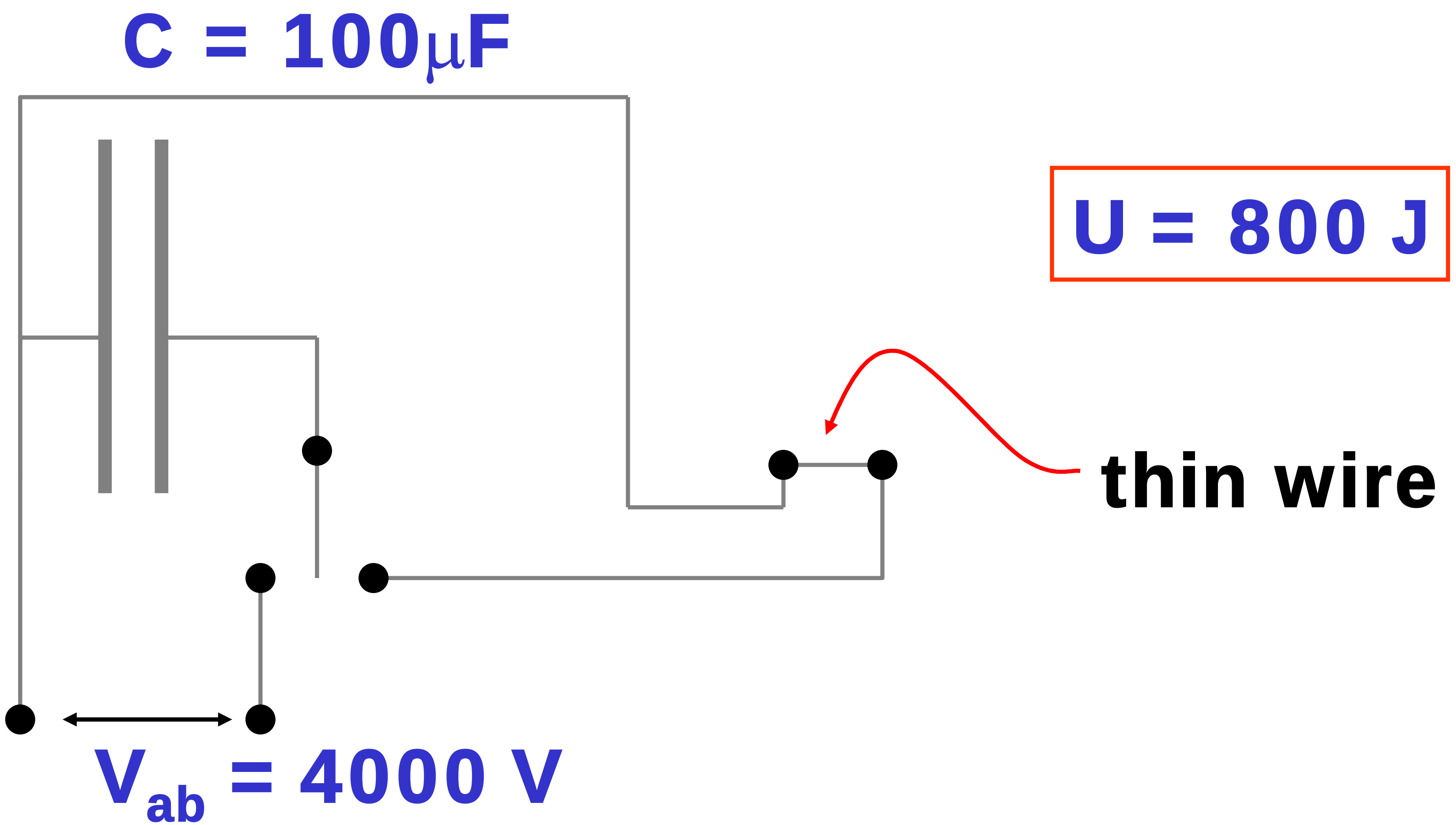
- To store lots of charge
  - make A big
  - make d small

Energy stored in Capacitor

$$
\begin{aligned}
W_{tot} &= \int_{Q_{initial}}^{Q_{final}} V \; dq = \int_0^Q V \; dq \\
&= \int_0^Q q/C \; dq = \frac{1}{C}\int_0^Q q \, dq \\
&= \frac{1}{C}\frac{Q^2}{2}
\end{aligned}
$$

- Work $W = ½\, Q^2/C = ½\, C\, V^2$ needed to charge capacitor
- Energy conserved
- But power can be amplified
  - Charge slowly
  - Discharge very quickly

In-Class Demo



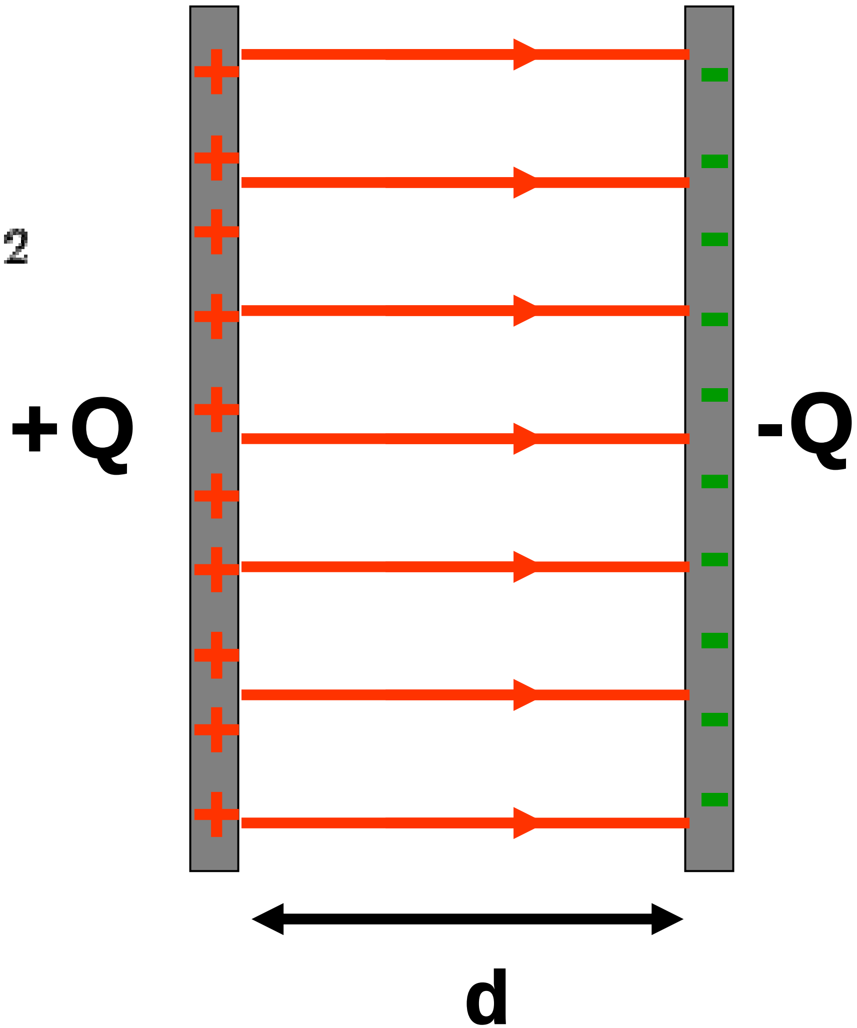
Where is the energy stored?

- Energy is stored in Electric Field

$$U_{stored} = \frac{1}{2}CV^2 = \frac{1}{2}(\epsilon_0\frac{A}{d})(E\ d)^2$$

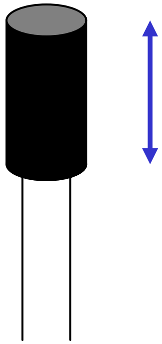
$$= \boxed{\frac{1}{2}\epsilon_0 E^2\ Volume}$$

- **E^2** gives Energy Density:
- **U/Volume = ½ e_0 E^2**



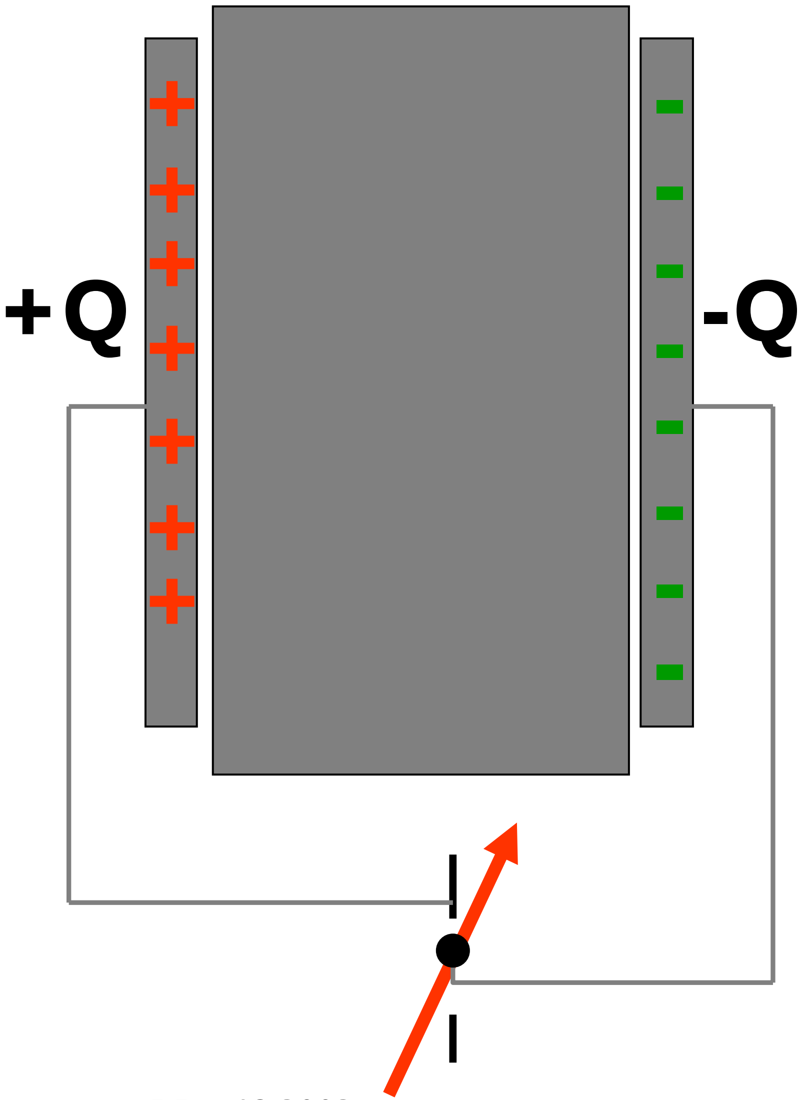
Dielectrics

In your toolbox:



- Parallel Plate Capacitor:
  - **$C = \varepsilon_0 A/d$**
  - Ex. **$A = 1m^2$, $d=0.1mm$ -> $C \sim 0.1\mu F$**
- How can one get small capacitors with big capacity?

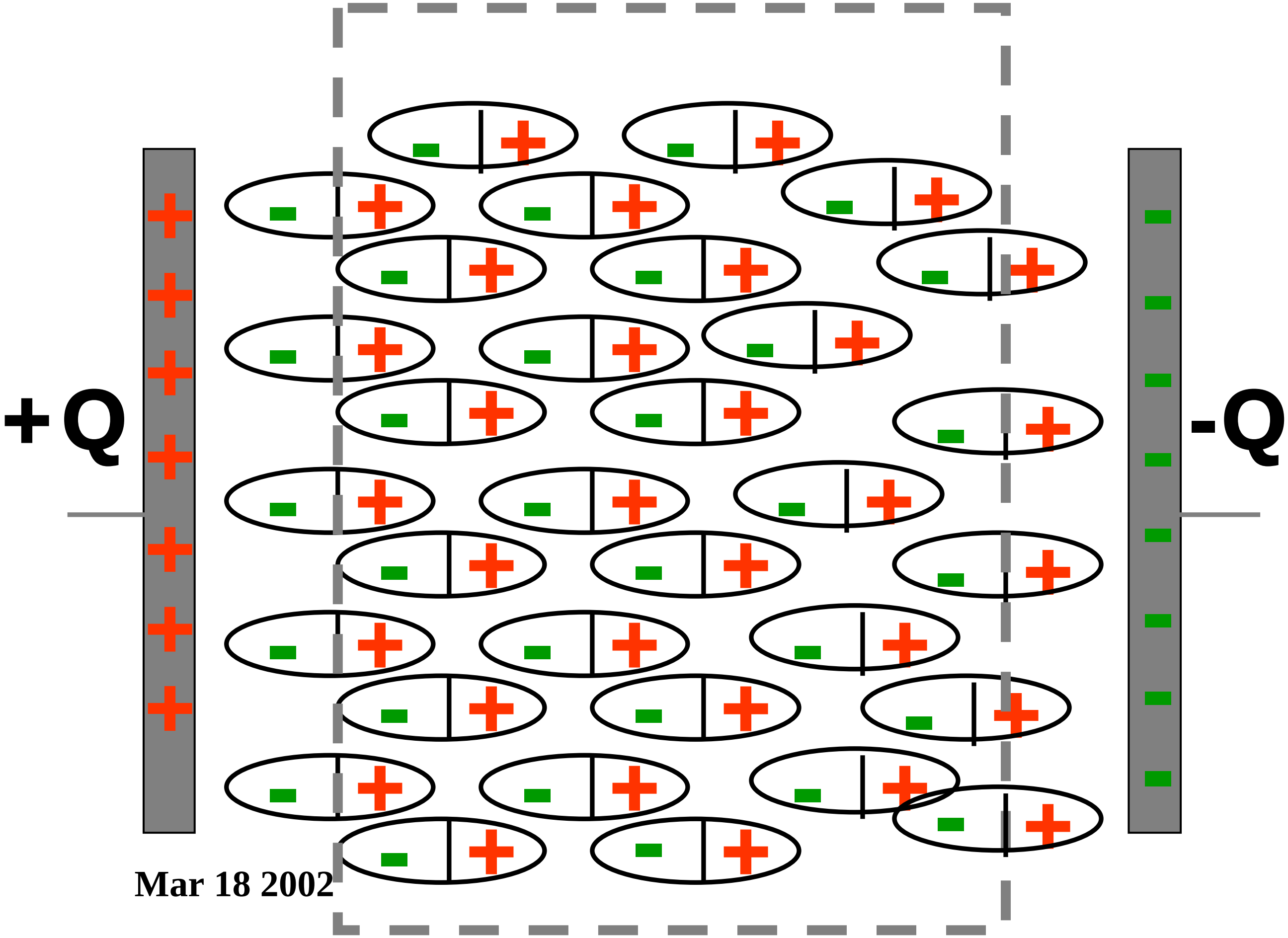
Dielectric Demo



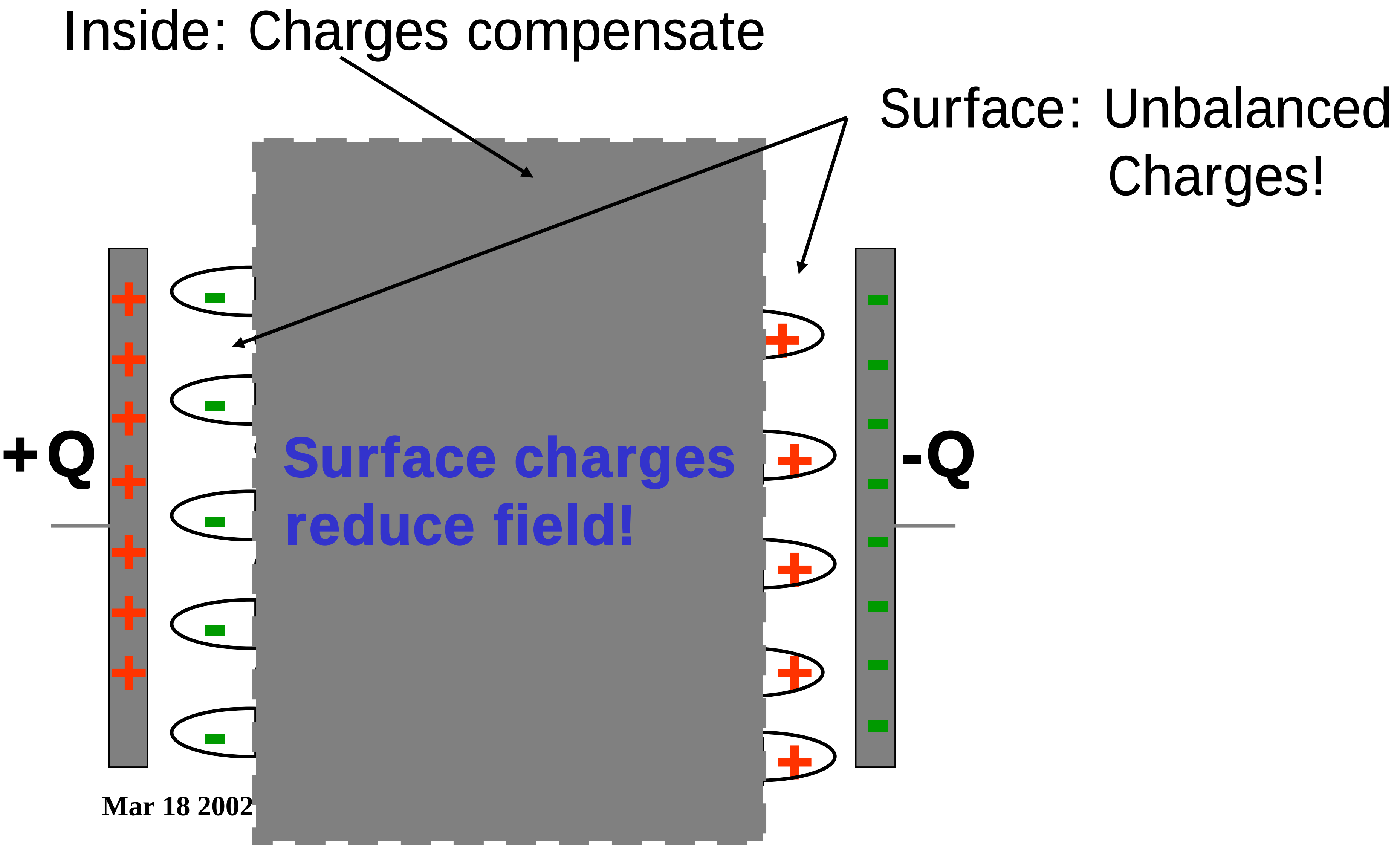
- Start w/ charged capacitor
- **d** big -> **C** small -> **V** large
- Insert Glass plate
- Now **V** much smaller
- **C** bigger
- But **A** and **d** unchanged !
- Glass is a **Dielectric**

Microscopic view

Polarization $\vec{P}$ = const. $\vec{E} = \varepsilon_0 \chi \vec{E}$



Microscopic view



Dielectric Constant

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} - \frac{|\sigma_p^+|}{2\epsilon_0} - \frac{|\sigma_p^-|}{2\epsilon_0|}$$

Add contributions to E

$$= \frac{\sigma}{\epsilon_0} - \frac{P}{\epsilon_0}$$

E from plates and E from Dielectric surface charge

$$= E_0 + \chi E$$

$$\rightarrow E = \frac{E}{1+\chi} \equiv \frac{E_0}{K}$$

K: Dielectric Constant

Field w/o Dielectric

Dielectric Constant

- Dielectric reduces field E_0 ($K > 1$)
  - $E = 1/K \; E_0$
- Dielectric increases Capacitance
  - $C = Q/V = Q/(E\,d) = K\,Q/(E_0 d)$
- This is how to make small capacitors with large **C** !

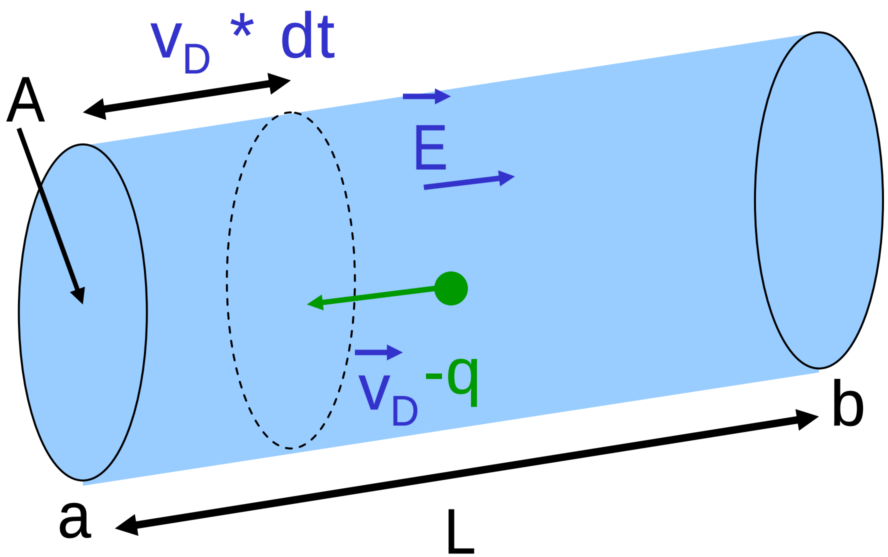
Electric Current

- We left Electrostatics
  - Now: Charges can move in steady state
- Electric Current **I**:
  - **$I = dQ/dt$**
  - Net amount of charge moving through conductor per unit time
- Units:
  - **[I] = C/s = A (Ampere)**

Electric Current

- Current **$I = dQ/dt$** has a direction
  - Convention: Direction of flow of positive charges
  - In our circuits, I carried by electrons
- To get a current:
  - Need mobile charges
  - Need **$|E| > 0$** (Potential difference)

Resistivity



$I = q\, n_q\, v_D\, A$

Velocity   Area

Current Density $\vec{J} = q\, n_q\, v_D$ vs Field E ?

Remember: $\vec{v}_D = (q_e \vec{E}) / f \;\; \text{->} \;\; \vec{J} = \underbrace{q^2 n_q/f}_{\text{constant}}\, \vec{E}$

$\vec{J} = 1/\rho\, \vec{E};\ \rho = f/(q^2 n_q)$

Resistivity

- Interplay of scattering and acceleration gives an average velocity v_D
- v_D is called 'Drift velocity'
- How fast do the electrons move?
  - Thermal speed is big: $v_{th} \sim 10^6$ m/s
  - Drift velocity is small: $v_D \sim 10^{-3}$ m/s
- All electrons in conductor start to move, as soon as $E > 0$

Resistivity

Ohm

$\rho = f/(q^2 n_q)$: <u>Resistivity</u>

Units: $[\rho] = V/m\ m^2/A = m\ V/A = m\ \Omega$

Material	ρ [m Ω]
Glass	$>10^{10}$
Pure Water	$2*10^5$
Carbon	$3.5*10^5$
Silicon	2300
Sea Water	0.2
Gold	$2.4*10^{-8}$
Copper	$1.7*10^{-8}$

Insulator (Glass, Pure Water)

Semiconductor (Carbon, Silicon)

Conductor (Sea Water, Gold, Copper)

Resistance

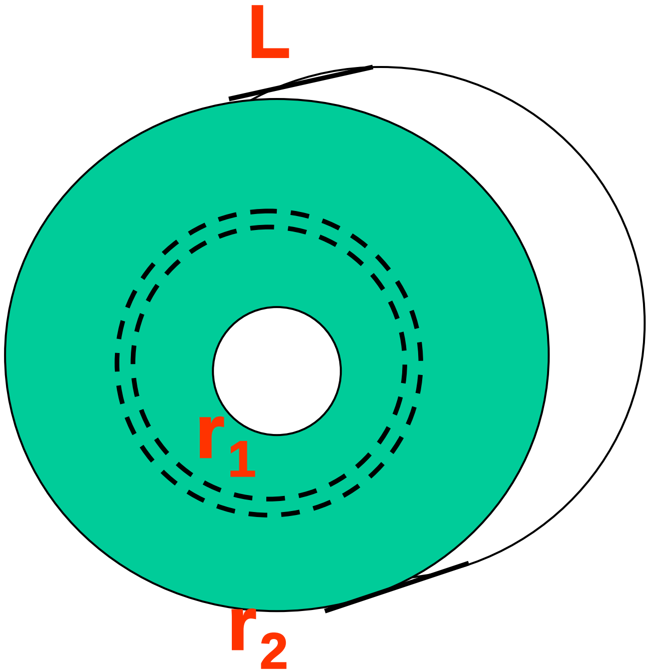
- Define $R = V/I$: Resistance
- $R = \rho\ L/A = f/(n_q\ q^2)\ L/A$
  - for constant cross section A
- R is measured in Ohm [W] = [V/A]

- Resistivity ρ is property of material (e.g. glass)

- Resistance R is property of specific conductor, depending on material (ρ) and geometry

Resistance

- $R = \rho L / A = f/(n_q q^2) L/A$
  - assuming <u>constant cross-section</u> A
- What if $A = A(x)$?
- Ex. hollow cylinder, R between inner and outer surface



- Slice into pieces with constant A

$$dR = \rho \; dr/A(r)$$
$$= \rho \, dr/(2 \pi r L)$$

- Integrate

$$R = \rho/(2 \pi L) \ln(r_2/r_1)$$

Ohm's law

$$V = R\ I$$

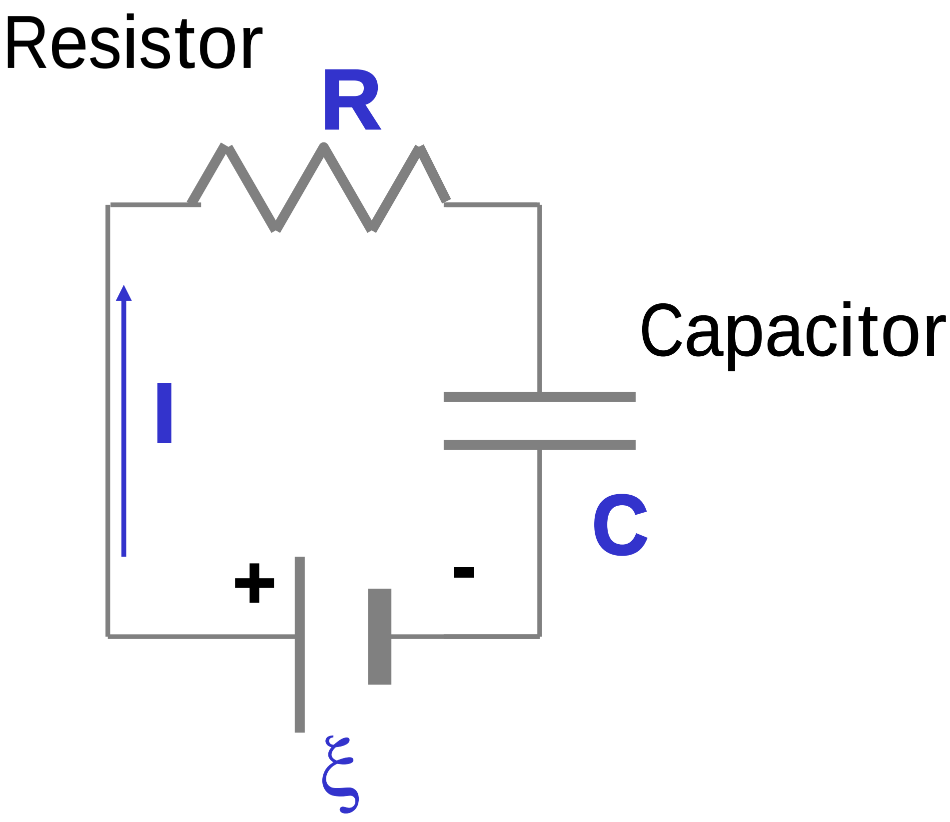
- Conductor is 'Ohmic', if R does not depend on V,I
- For real conductors, that's is only approximately true (e.g. $R = R(T)$ and $T = T(I)$)
- Approximation
  - valid for resistors in circuits
  - not valid for e.g. light bulbs

Electric Power

- Use moving charges to deliver power

$$\text{Power} = \text{Energy/time} =$$
$$dW/dt = (dq\ V)/dt =$$
$$dq/dt\ V = \underline{I\ V} = I^2R = V^2/R$$

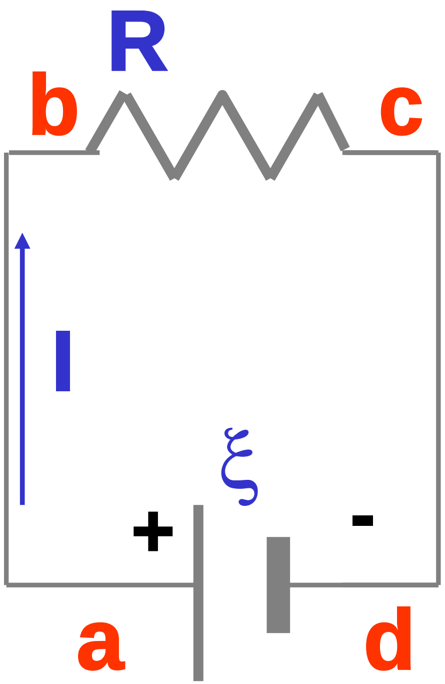
Electric Circuits

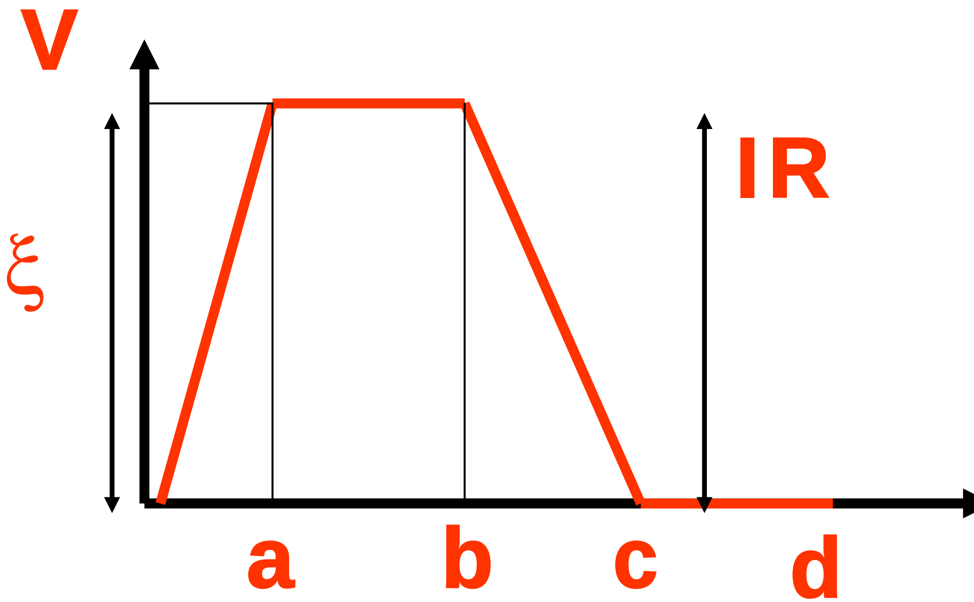


Source of EMF

Electromotive Force EMF

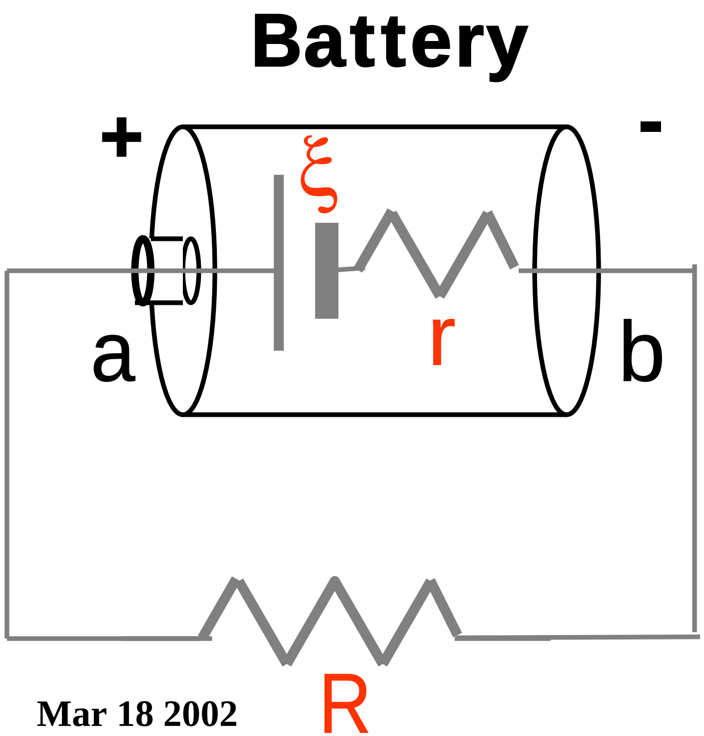
- Def: $\xi =$ Work/unit charge
- ξ is 'Electromotive Force' (EMF)
- Units are [V]





Internal Resistance

- Sources of EMF have internal resistance r
- Can’t suppy infinite power



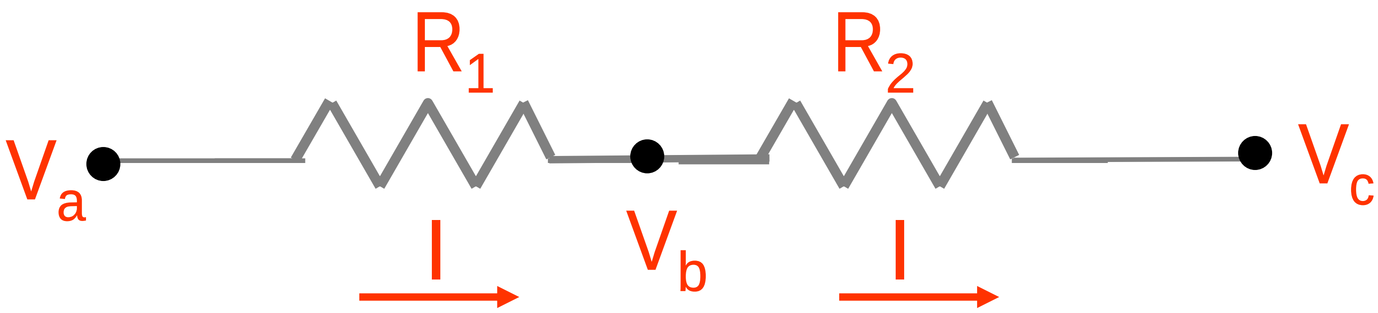
$$V_{ab} = \xi - I r$$

$$= IR$$

-> $I = \xi / (r+R)$

Electric Circuits

Resistors in series

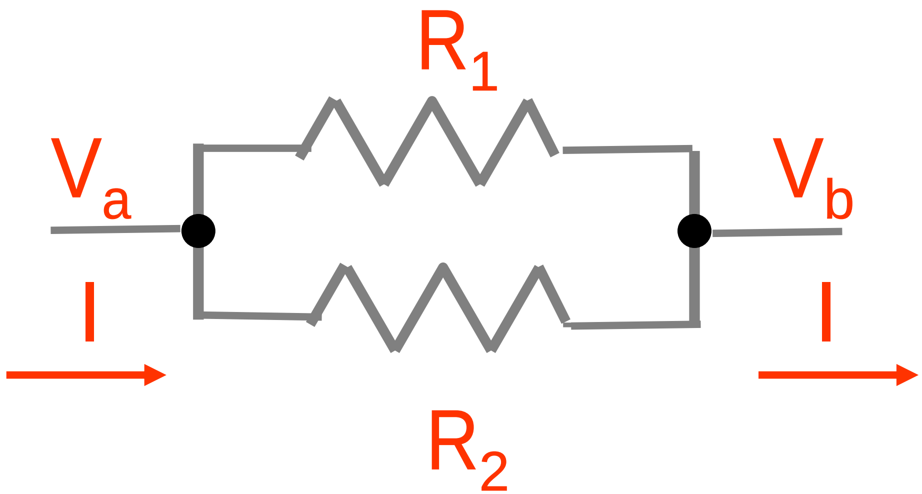


$$V_{ac} = V_{ab} + V_{ac} = I\,R_1 + I\,R_2 = I\,(R_1 + R_2)$$

$$= I\,R_{eq} \quad \text{for} \quad \boxed{R_{eq} = (R_1 + R_2)}$$

Electric Circuits

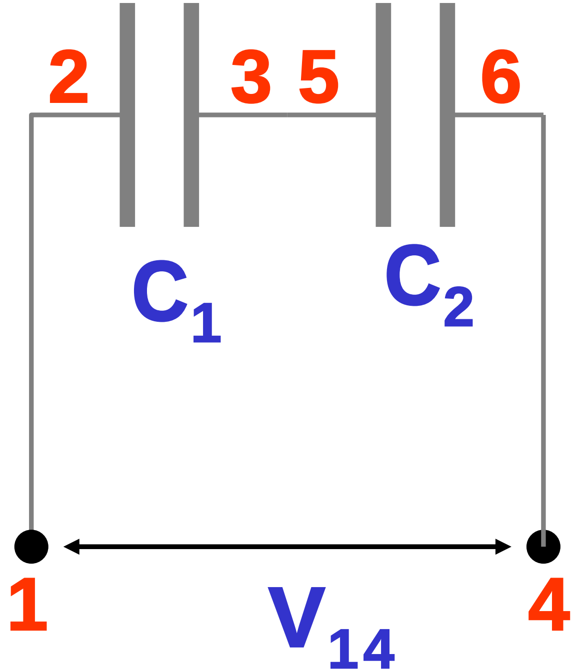
Resistors in parallel



$$I = I_1 + I_2 = V_{ab}/R_1 + V_{ab}/R_2 = V_{ab}/R_{eq}$$

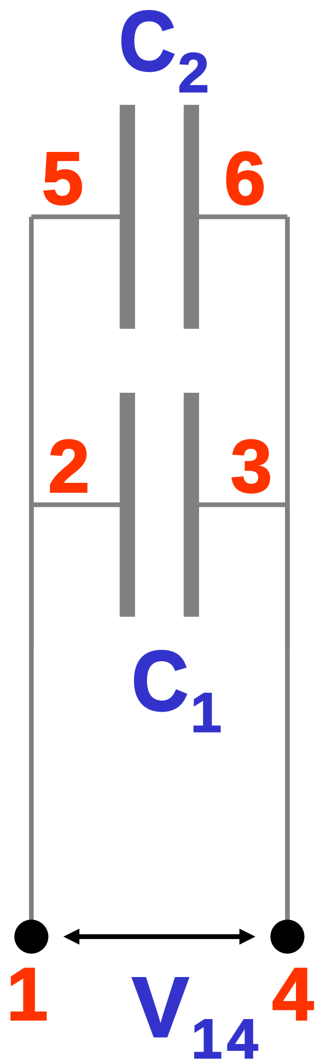
$$\rightarrow \boxed{1/R_{eq} = 1/R_1 + 1/R_2}$$

Electric Circuits



- Two capacitors in **series**
- $V_{14} = V_{23} + V_{56}$
- $Q = Q_1 = Q_2$
- $V_{tot} = Q_1/C_1 + Q_2/C_2 = Q/(C_1+C_2)$
- $1/C_{tot} = 1/C_1 + 1/C_2$
- **Inverse Capacitances add!**

Electric Circuits



- Two capacitors in **parallel**
- $V_{56} = V_{23} = V_{14}$ (after capacitor is charged)
- $Q_1/C_1 = Q_2/C_2 = V_{14}$
- $Q_{tot} = Q_1 + Q_2$
- $C_{tot} = (Q_1 + Q_2)/V_{14} = C_1 + C_2$

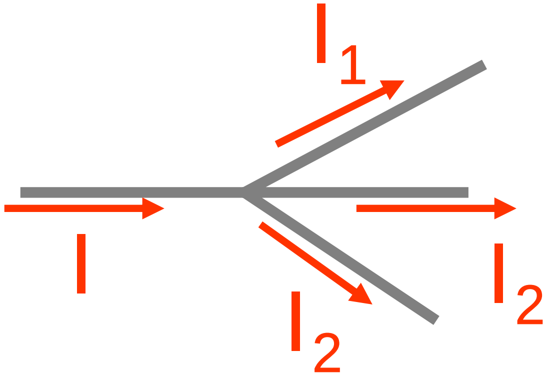
- Capacitors in **parallel -> Capacitances add!**

Kirchoff's Rules

- Junction rule

At junctions:

$$\sum I_{i_{in}} = \sum I_{j_{out}}$$



<u>Charge conservation</u>

- Loop rule

Around closed loops:

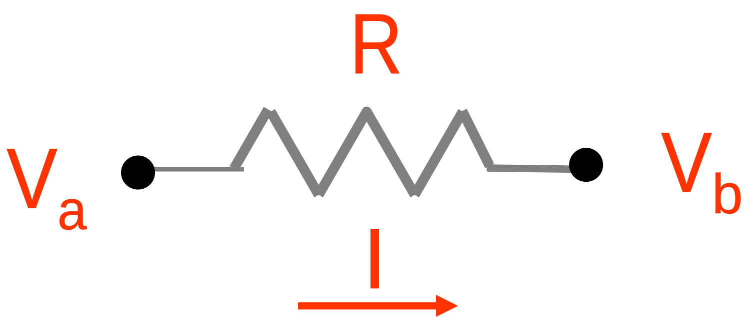
$$\sum \Delta V_j = 0$$

ΔV for both EMFs and Voltage drops

<u>Energy conservation</u>

Kirchoff's Rules

- Kirchoff's rules allow us to calculate currents for complicated DC circuits
- Main difficulty: Signs!
- Rule for resistors:



$\Delta V = V_b - V_a = - I R$, if we go in the direction of I (voltage drop!)

Kirchoff's Rules

- Kirchoff's rules allow us to calculate currents for complicated DC circuits
- Main difficulty: Signs!
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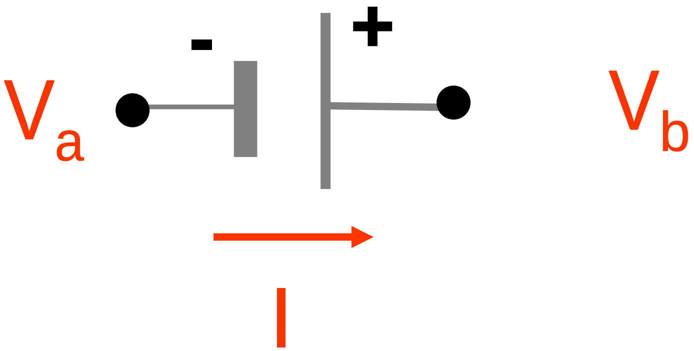
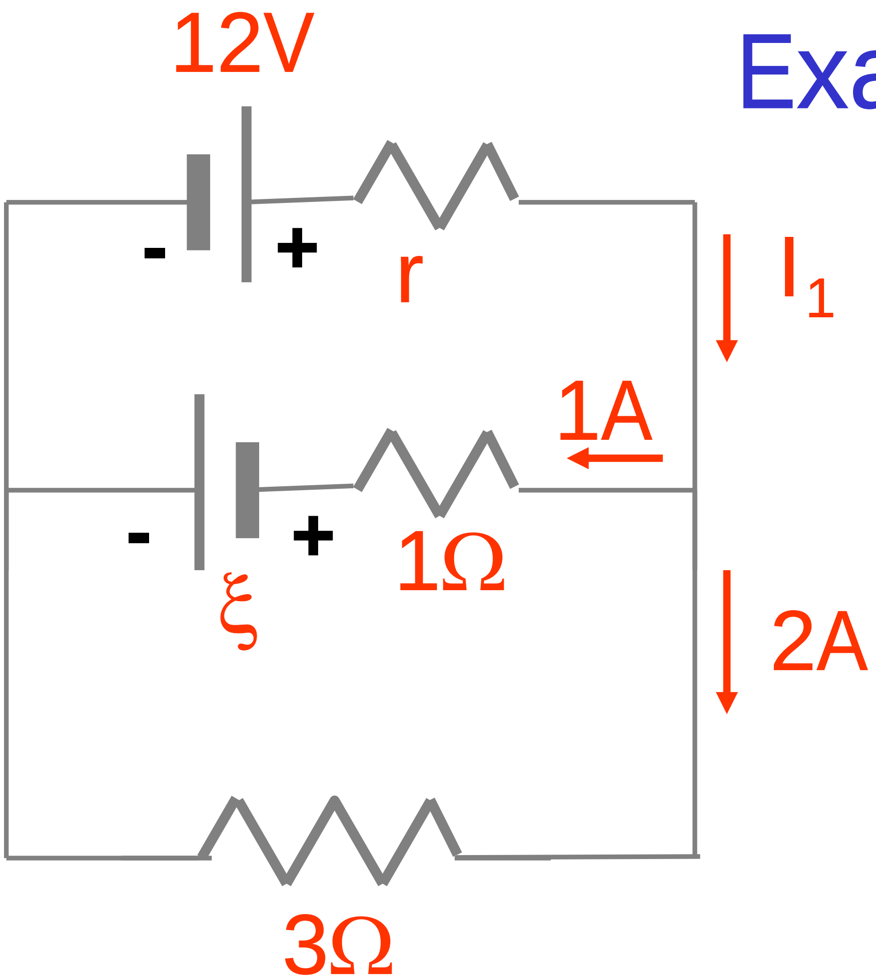


$\Delta V = V_b - V_a = \xi$, if we go in the direction of I

Example



r, ξ, I_1 ? 3 unknowns

- Pick signs for I_1, ξ
- Junction rule

  $I_1 = 1A + 2A = 3A$
- Loop rule (1)

  $12V - 6V - 3A\ r = 0$

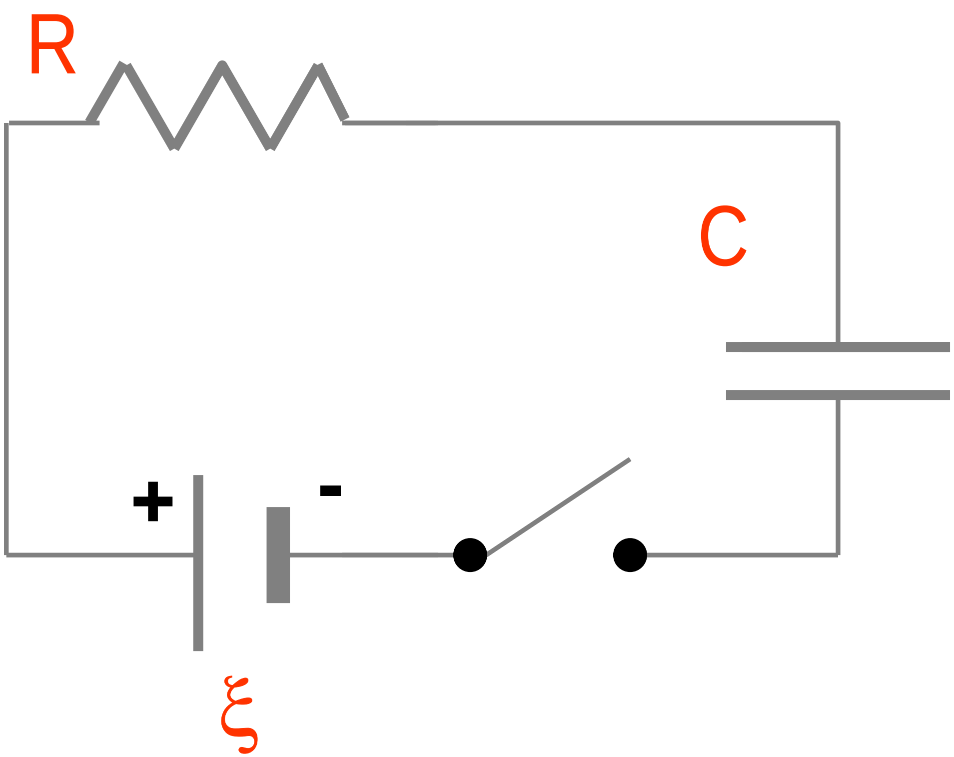
  $\rightarrow r = 6/3\ \Omega = 2\ \Omega$
- Loop rule (2)

  $12V - 6V - 1V - \xi = 0$

  $\rightarrow \xi = 5V$

RC Circuits

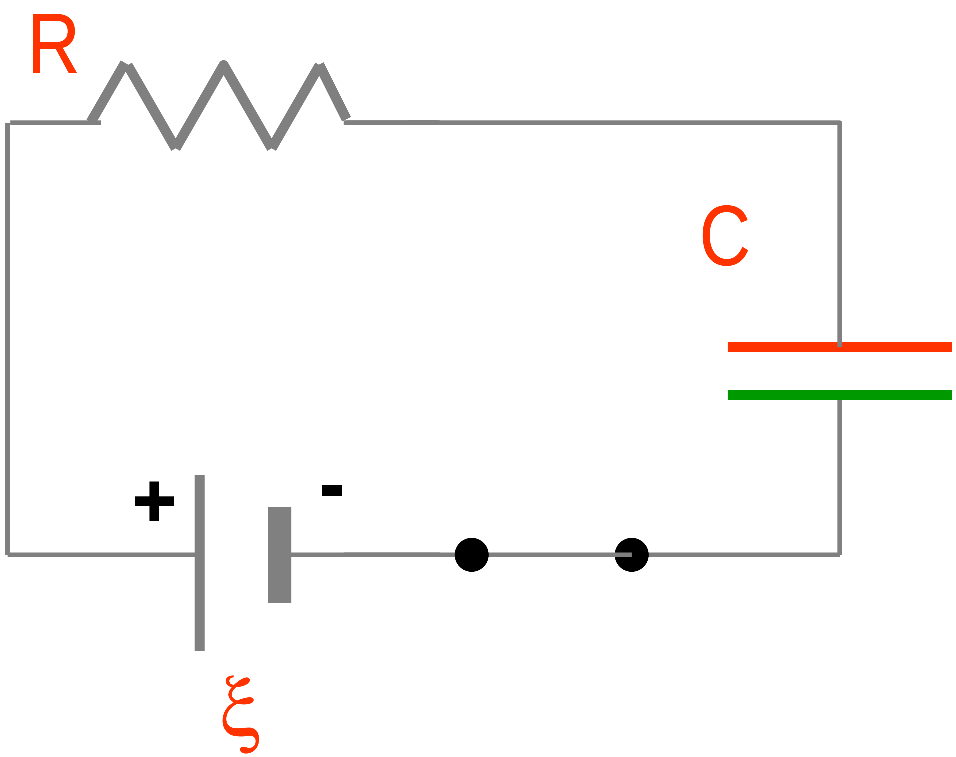
- Currents change with time
- Example: Charging a capacitor



$$t = 0$$
$$q = 0$$
$$V_c = q/C$$

RC Circuits

- Currents change with time
- Example: Charging a capacitor



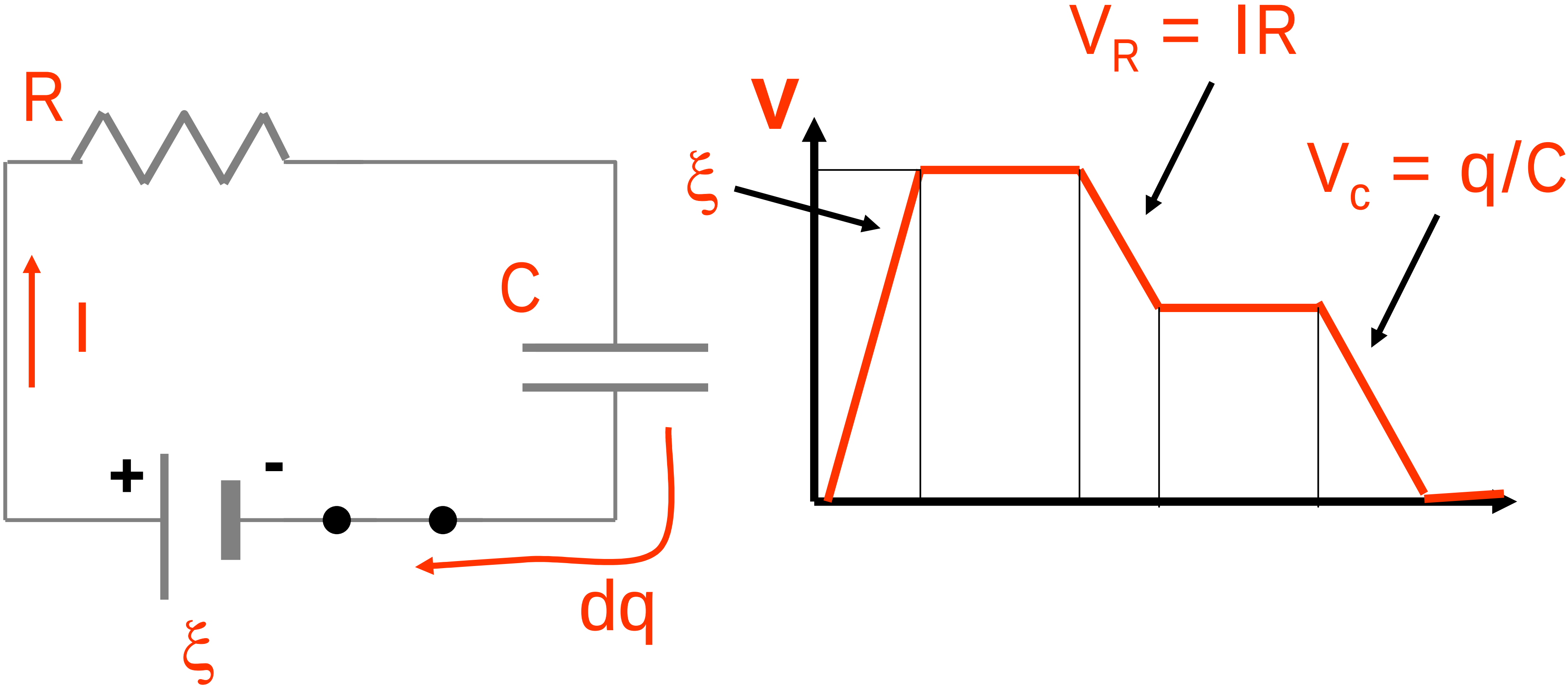
t = infinity

$q = C\,\xi$

$V_c = q/C = \xi$

RC Circuits

- What happens between t=0 and infinity?



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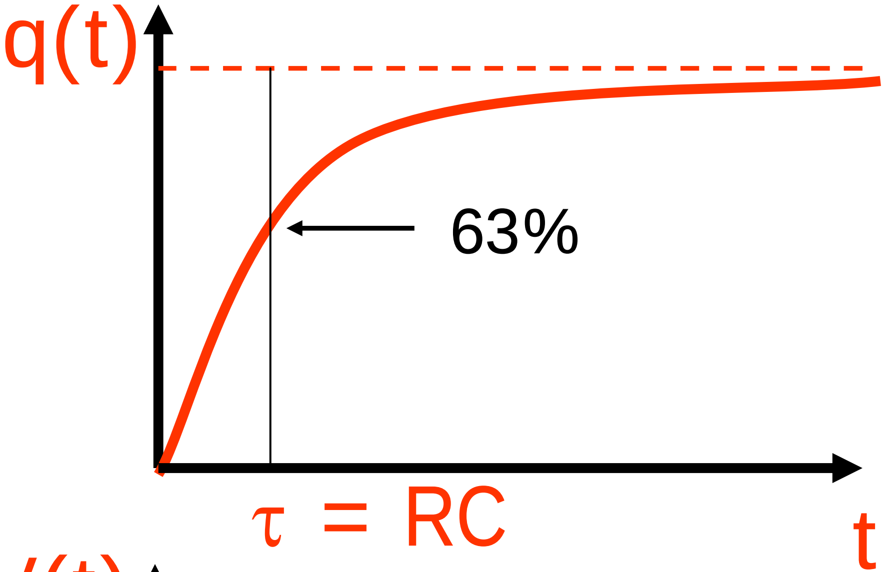
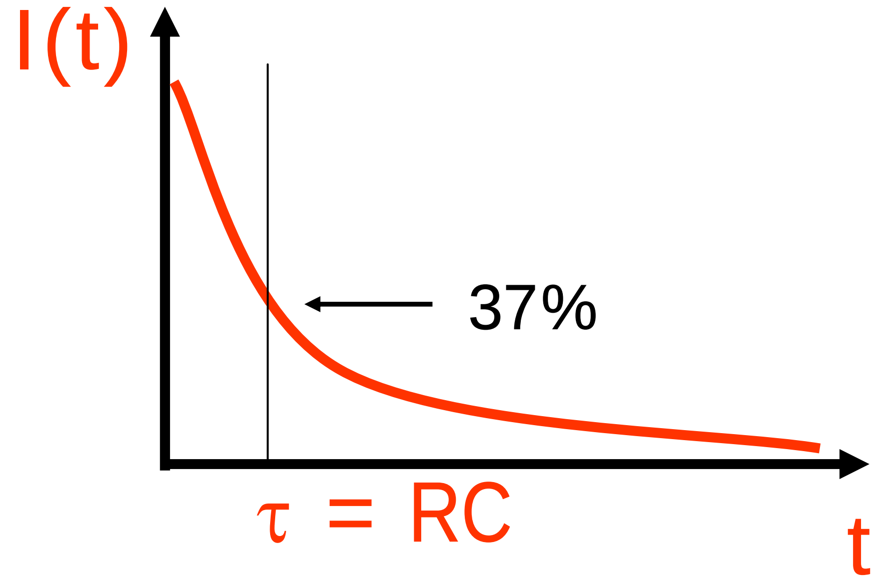
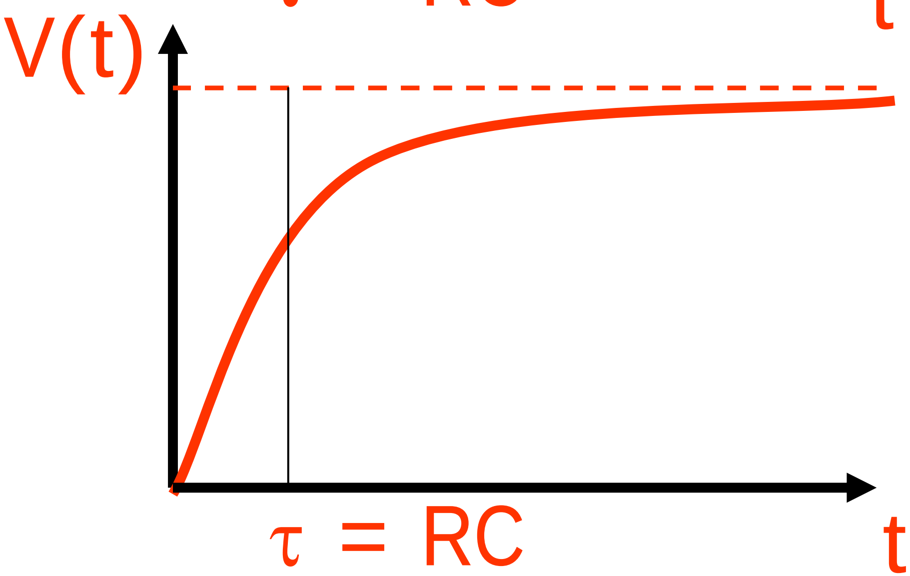
Charging C





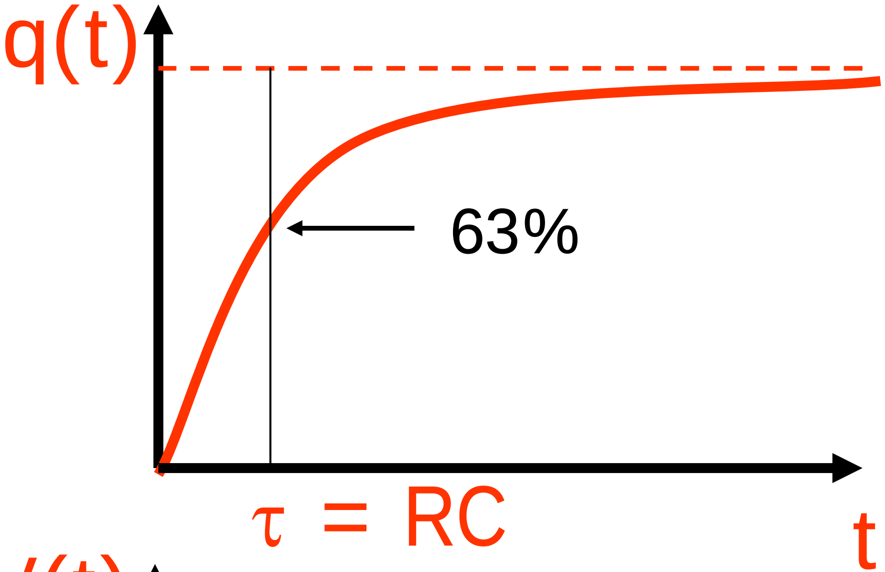


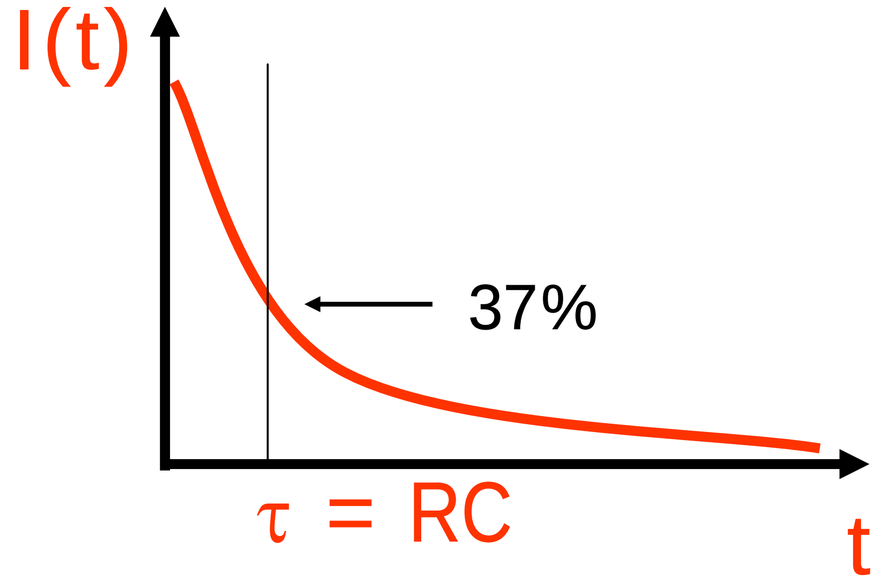
$$q(t) = \xi C[1-\exp(-t/\tau)]$$

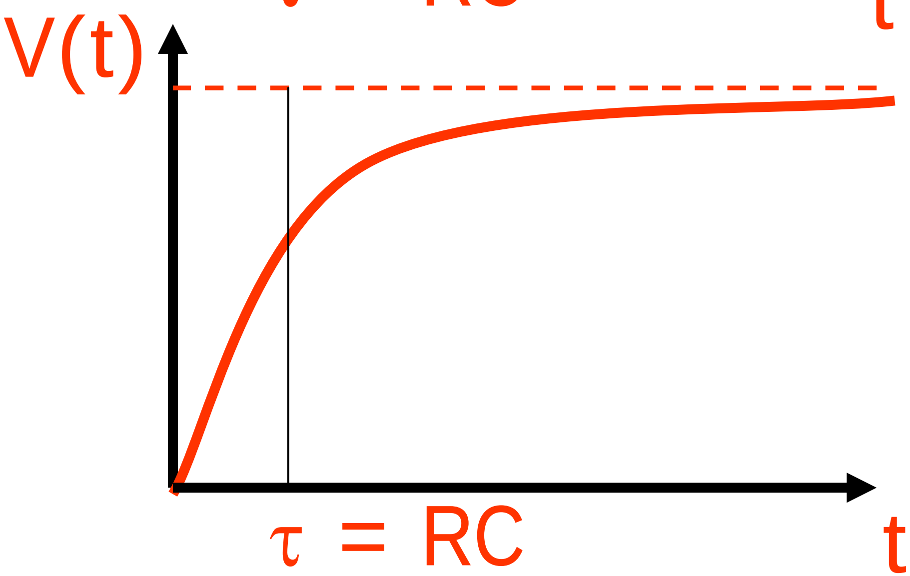
$$I(t) = dq/dt = \xi/R \exp(-t/\tau)$$

$$V(t) = \xi[1-\exp(-t/\tau)]$$

Charging C







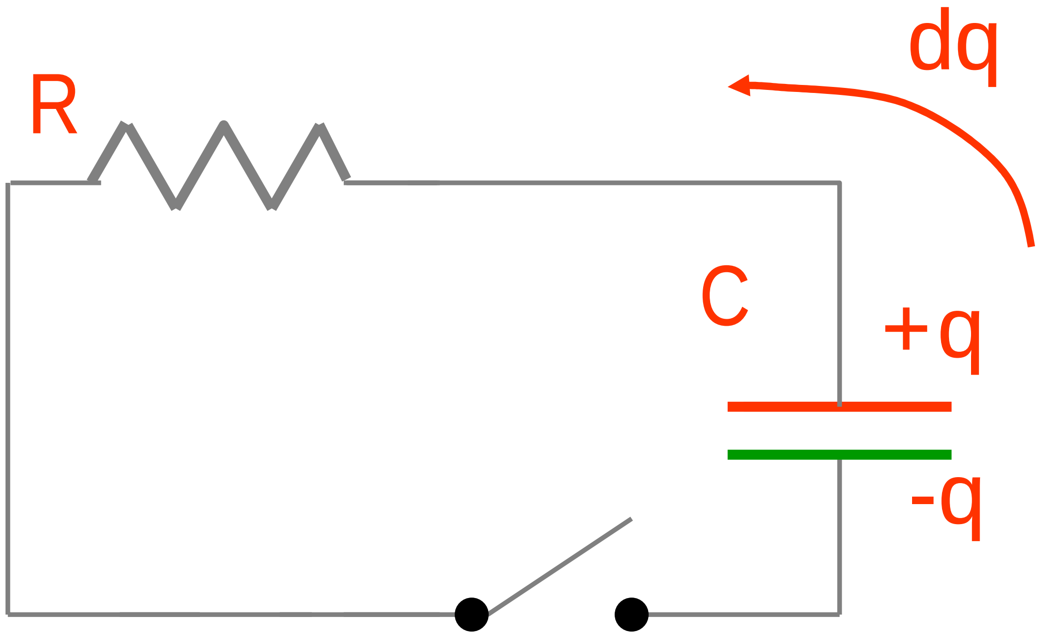
At $t = \tau$,

$q(t) = (1-1/e)\, q_{max}$

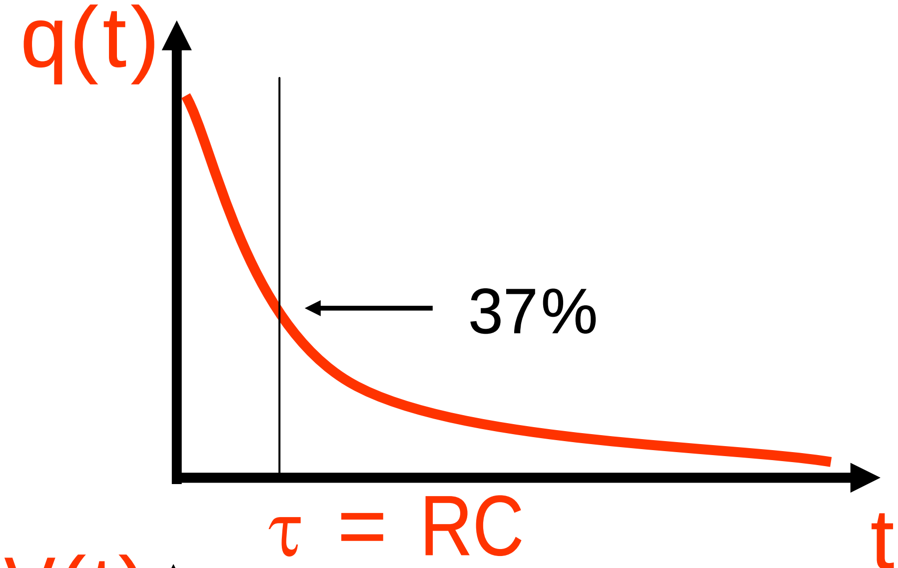
$= \underline{63\ \%\ q_{max}}$

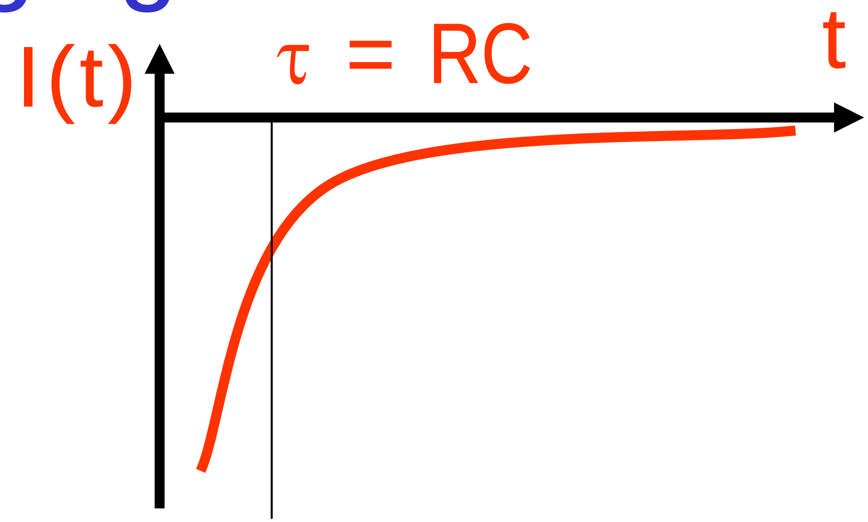
In-Class Demo

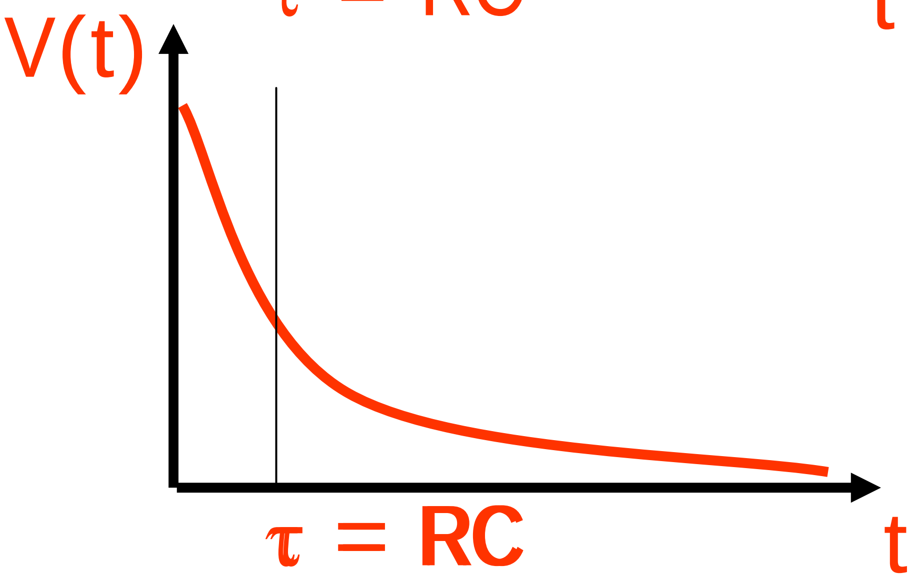
Discharging a capacitor



Discharging C





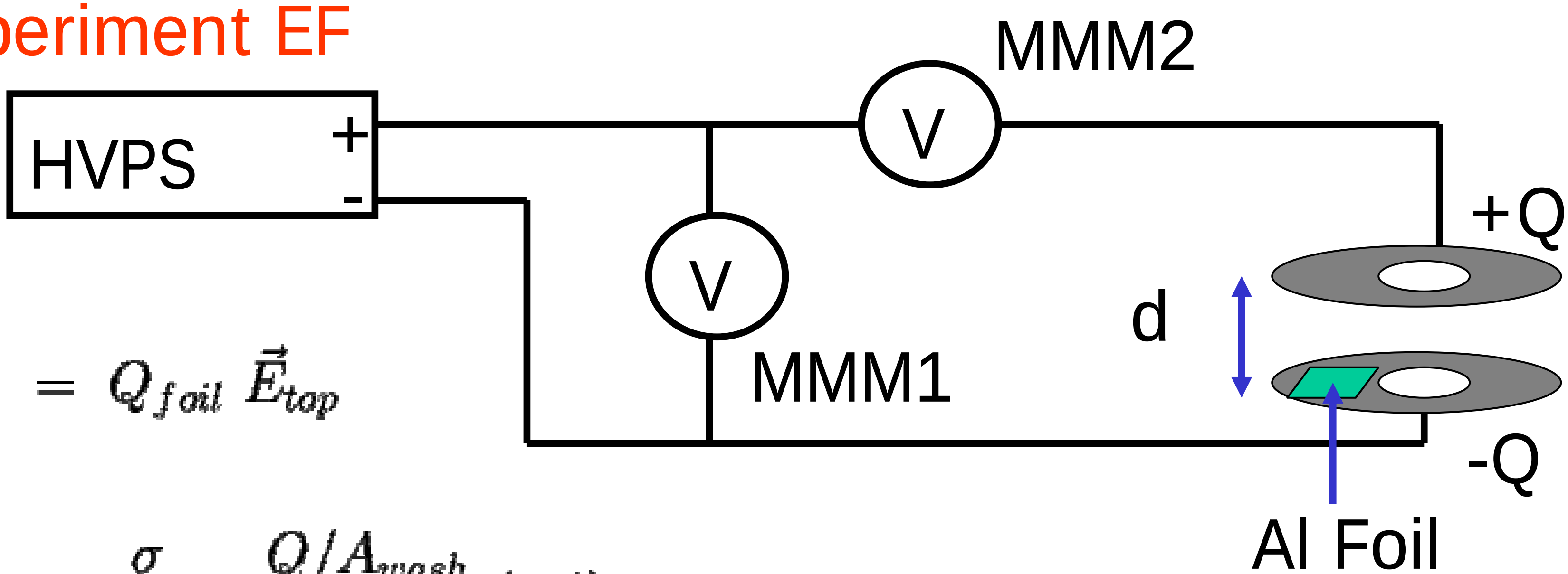


$$q(t) = \xi C \exp(-t/\tau)$$

$$I(t) = dq/dt = -\xi/R \exp(-t/\tau)$$

$$V(t) = \xi \exp(-t/\tau)$$

Experiment EF



$$\vec{F}_{on\,foil} = Q_{foil}\,\vec{E}_{top}$$

$$\vec{E}_{top} = \frac{\sigma}{2\epsilon_0} = \frac{Q/A_{wash}}{2\epsilon_0}\,(-\hat{y})$$

$$Q_{foil} = -Q\frac{A_{foil}}{A_{wash}}$$

Q = CV = ε_0A/d V

$$\Rightarrow \vec{F}_{on\,foil} = -Q\frac{A_{foil}}{A_{wash}}\,\frac{Q}{A_{wash}2\epsilon_0}(-\hat{y})$$

$$\Rightarrow \vec{F}_{on\,foil} = \frac{(\epsilon_0 V A_{wash}/d)^2}{A_{wash}^2}\,\frac{A_{foil}}{2\epsilon_0}\,\hat{y}$$

$$= \frac{Q^2}{A_{wash}^2}\,\frac{A_{foil}}{2\epsilon_0}\hat{y}$$

$$= \frac{\epsilon_0 V^2}{2d^2}A_{foil}\,\hat{y}$$