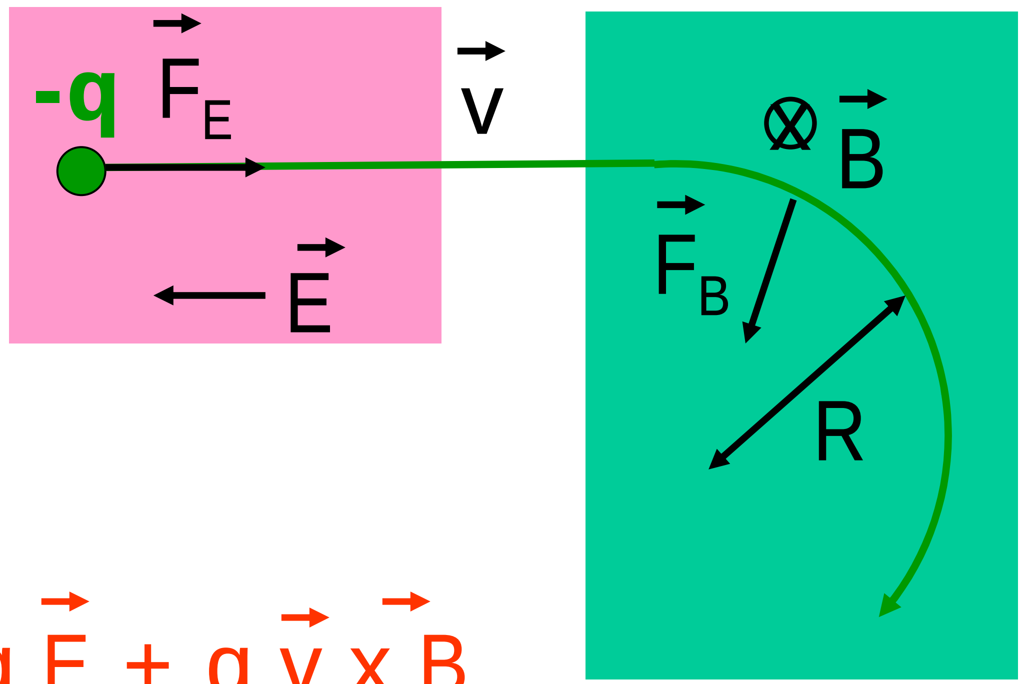
Electricity and Magnetism

- Recap
  - Forces in B-Field
- Today
  - Sources of B-Field
    - Law of Biot-Savart
    - Ampere's Law

Force on moving charge



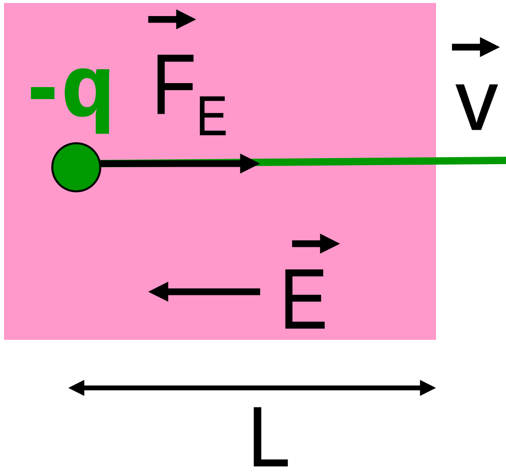
$$\vec{F}_L = q\,\vec{E} + q\,\vec{v} \times \vec{B}$$

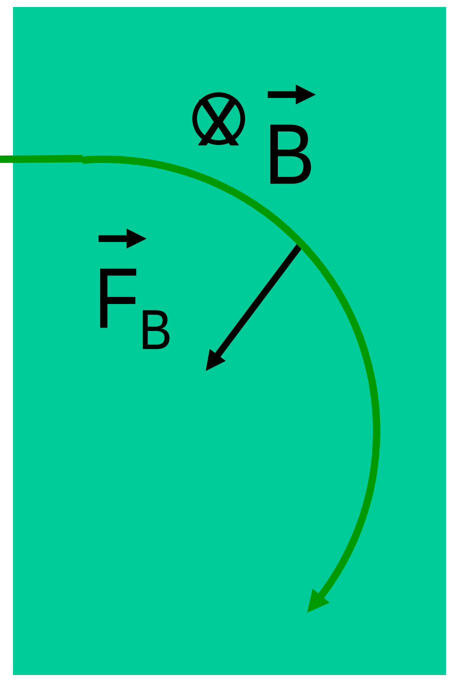
Lorentz-Force

$$R = m\,v/(q\,B)$$

Cyclotron Radius

Work done on moving charge





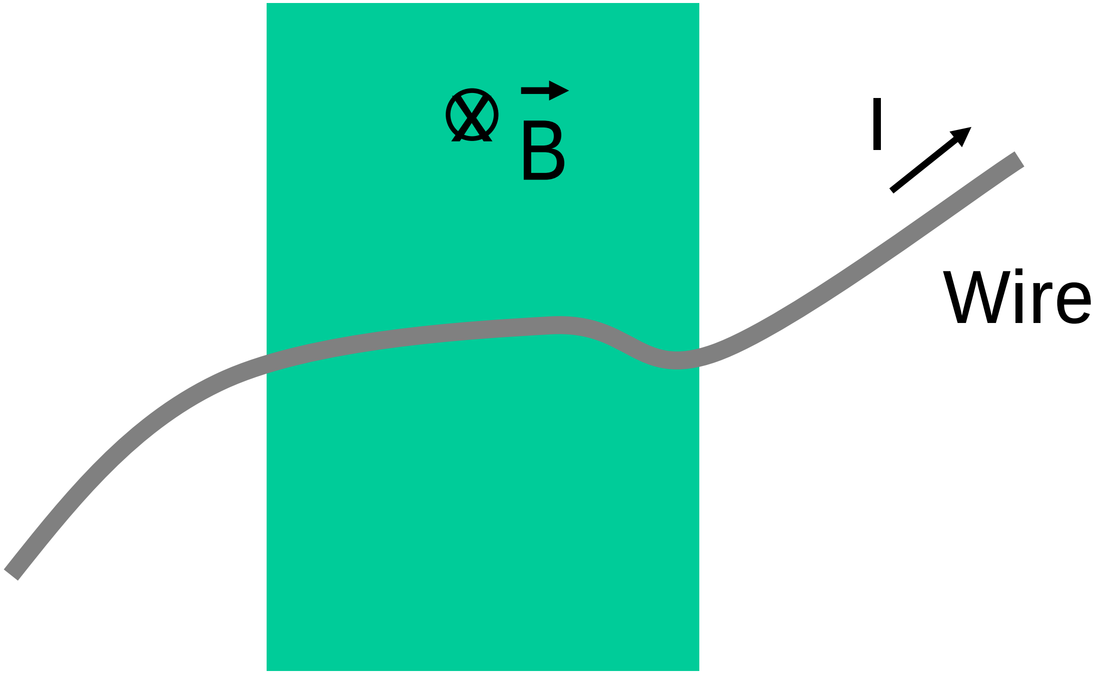
$$W = \vec{F}_E \cdot \vec{L}$$
$$= q\,E\,L$$

$$dW = \vec{F}_B \cdot d\vec{L}$$
$$= (q\,\vec{v} \times \vec{B}) \cdot d\vec{L}$$

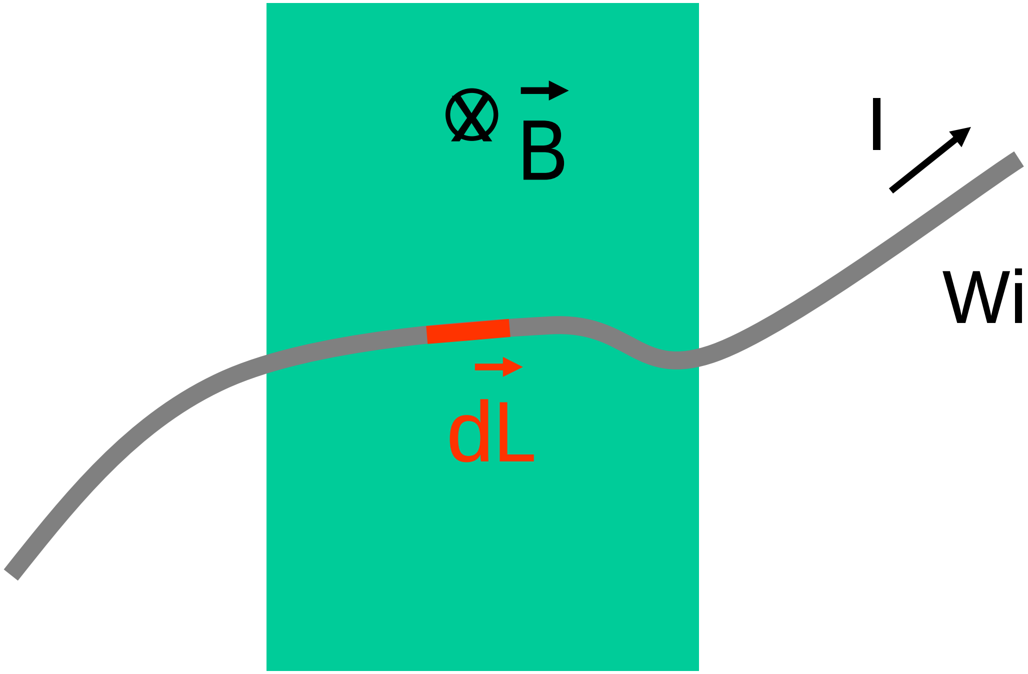
$$= (q\,d\vec{L}/dt \times \vec{B}) \cdot d\vec{L}$$
$$= 0$$

Magnetic Field does no Work!

Force on Wire carrying current I



Force on Wire carrying current I

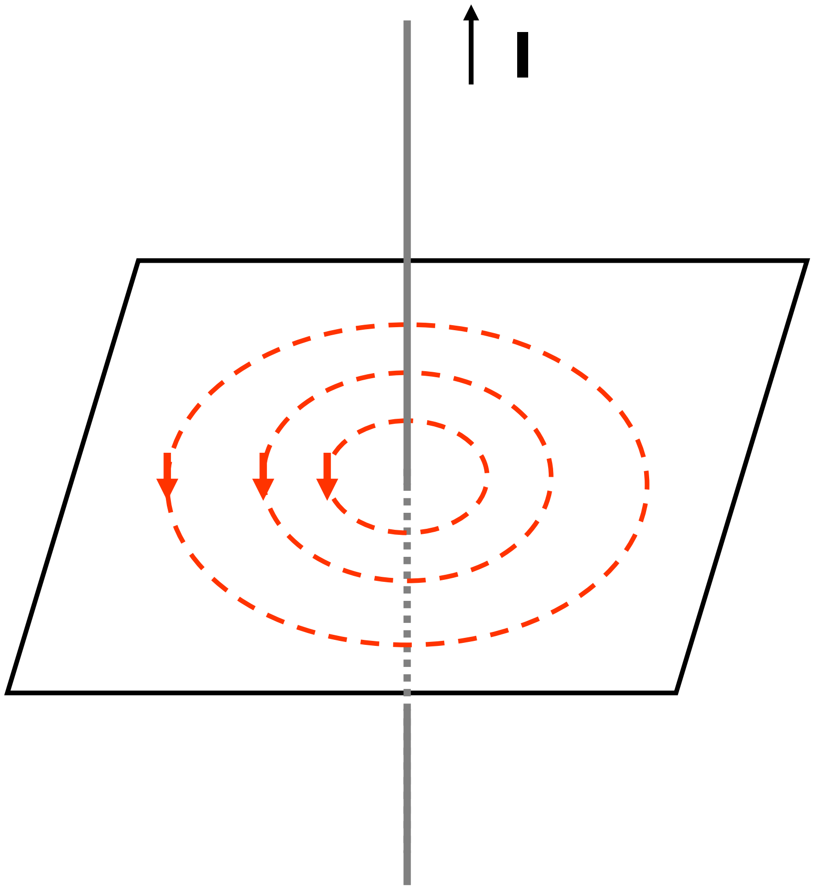


$$d\vec{F}_B = dq\, \vec{v} \times \vec{B}$$

$$= dq\, d\vec{L}/dt \times \vec{B}$$

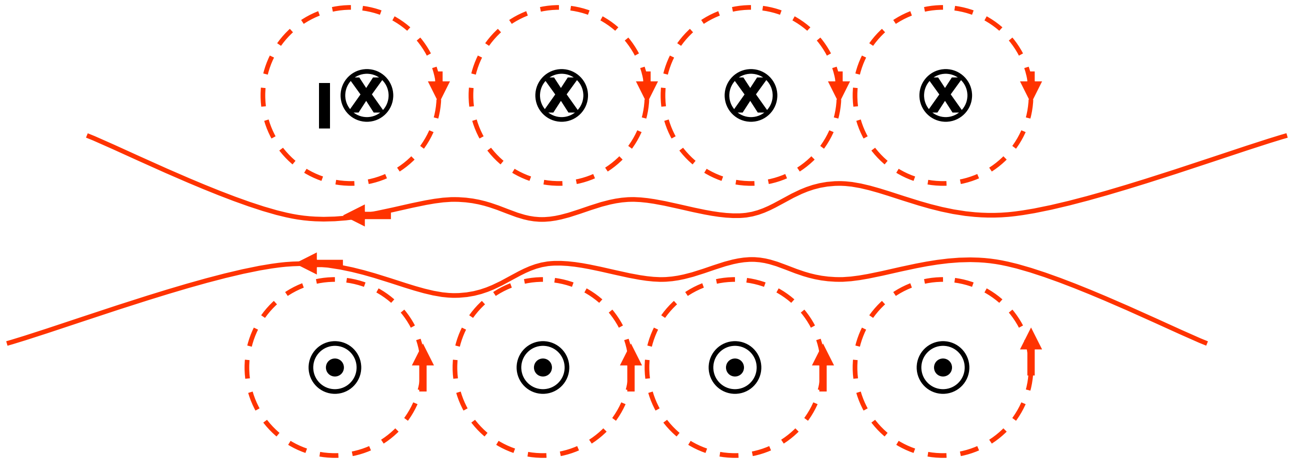
$$= \underline{I\, d\vec{L} \times \vec{B}}$$

Currents and B-Field



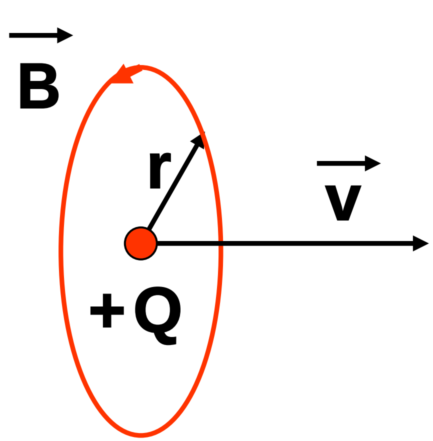
- Current as Source of B
- Magnetic Field lines are always closed
  - no Magnetic Charge (Monopole)
- Right Hand Rule

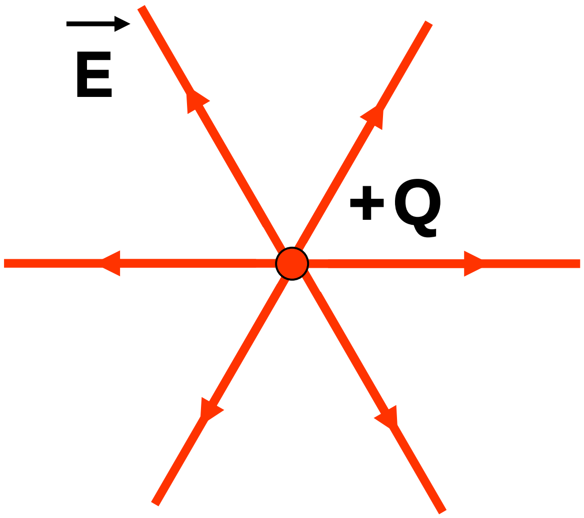
Currents and B-Field



- Solenoid: Large, uniform B inside
- Superposition Principle!

Magnetic Field vs Electric Field





$$\vec{B} = \mu_0/(4\,\pi)\; Q/r^2\; \vec{v} \times \hat{r}$$

$$\vec{E} = 1/(4\,\pi\,\varepsilon_0)\; Q/r^2\; \hat{r}$$

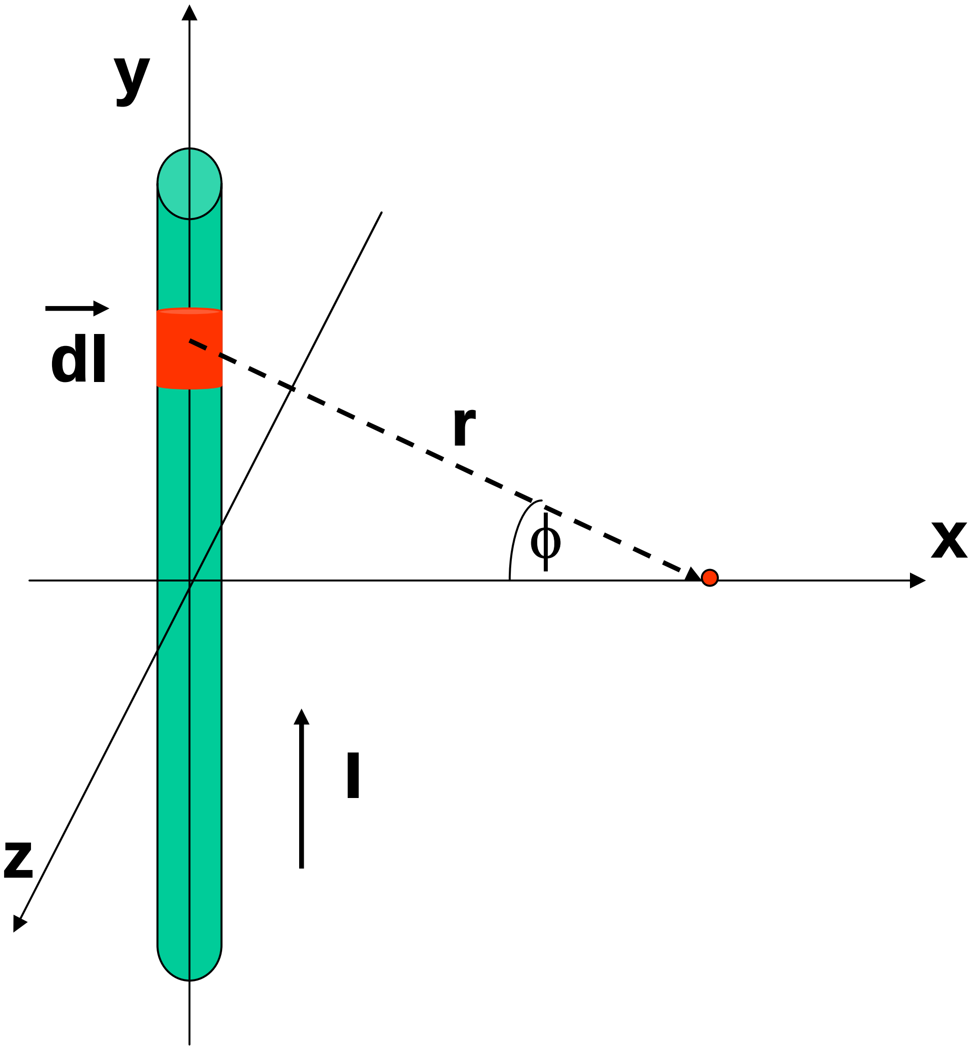
$\mu_0 = 4\,\pi \bullet 10^{-7}$ T m /A

$\varepsilon_0 = 8.85 \bullet 10^{-12}\ C^2/(Nm^2)$

$1/(\mu_0\,\varepsilon_0) = (3 \bullet 10^8\ m/s)^2 = c^2$ Speed of Light

Deep connection between B and E Field

Magnetic Field for Current I



Magnetic Field for Current I

$d\vec{B} = \mu_0/(4\pi)\, dQ/r^2\, \vec{v} \times \hat{r}$ for charge dQ

$I = dQ/dt \quad \rightarrow \quad dQ\,\vec{v} = dQ\, d\vec{l}/dt = I\, d\vec{l}$

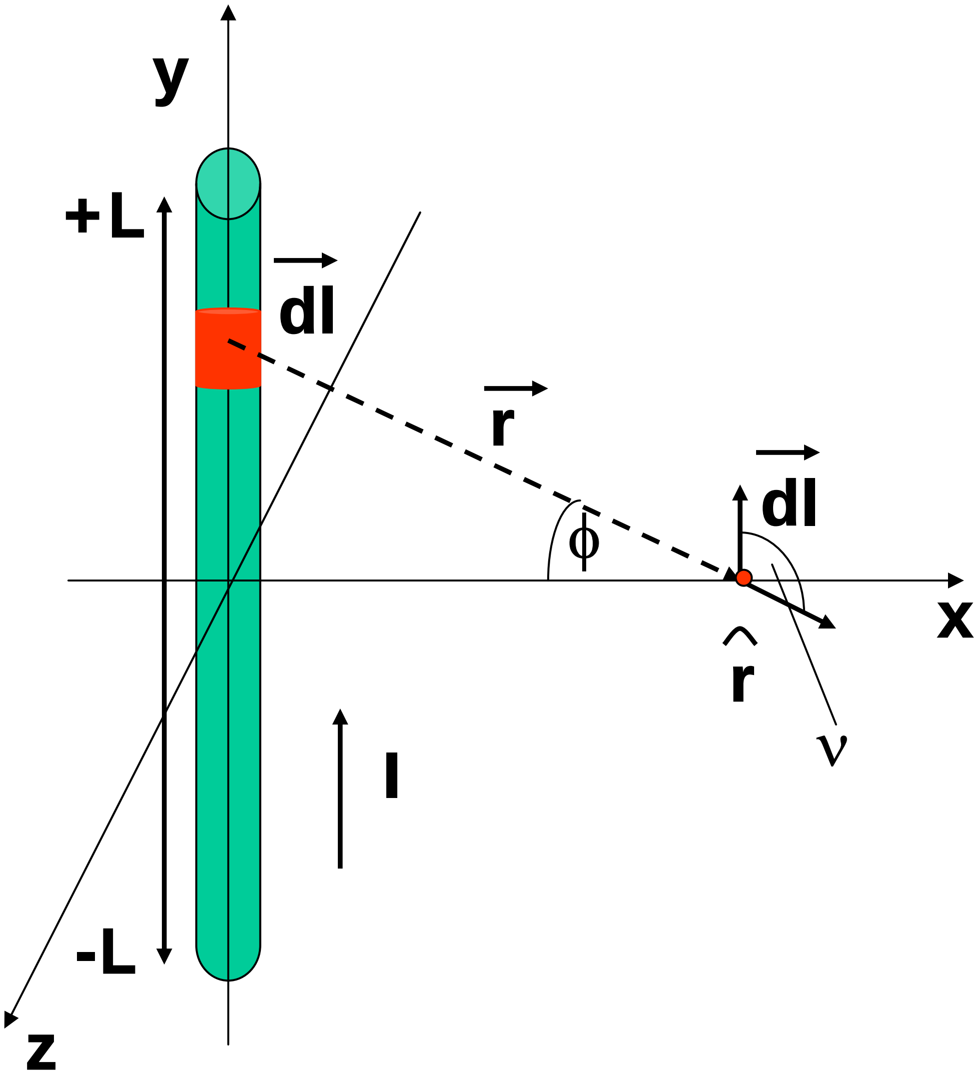
$$dB = \mu_0/(4\pi)\; I\; d\vec{l} \times \hat{r}/r^2$$

Law of Biot-Savart

Magnetic Field dB for current through segment dl

For total B-Field: Integrate over all segments dl

Magnetic Field for Current I



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$\vec{B}_{tot} = \int_{wire} \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0}{2\pi} I \frac{L}{x(x^2 + L^2)^{\frac{1}{2}}}$$

$$= \frac{\mu_0}{2\pi} \frac{I}{x} \quad \text{for} \quad L >> x$$

Magnetic Field for Current I

- That was painful…
- Long calculation for such a simple case
- Is there a simpler way?
- Recall Electrostatics!

Remember: Gauss' Law

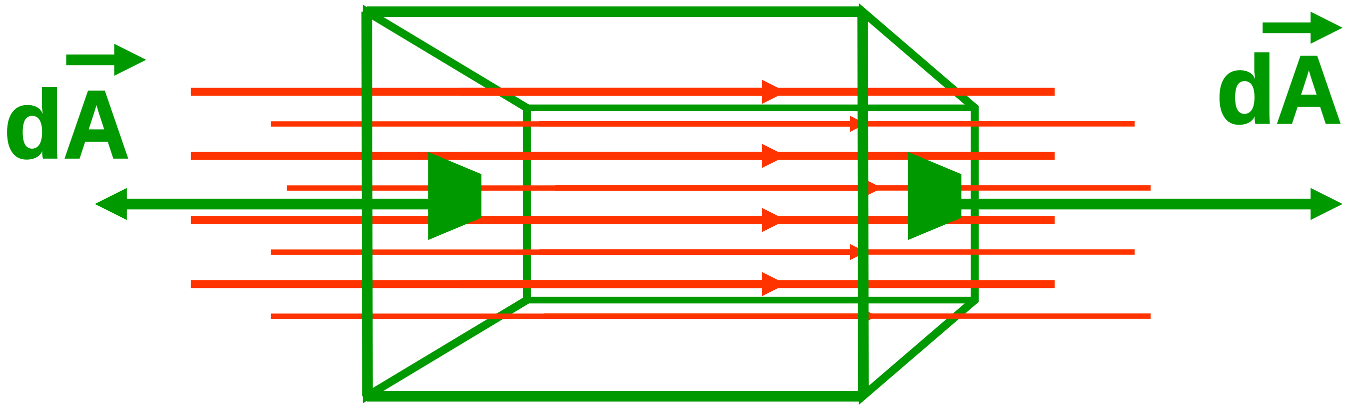
Electric Flux ϕ_E →

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Integral over any closed surface

Electric Charge is the Source of Electric Field

Gauss' Law



- No charge inside close surface:

  Flux in = -Flux out : $\Phi = 0$

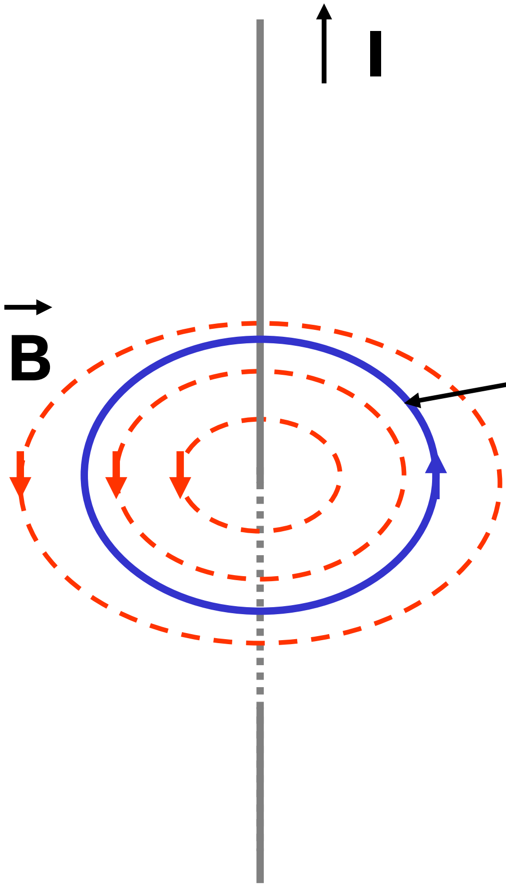
- There are no magnetic charges:
- Magnetic Flux $\Phi_B = 0$ for any close surface

Gauss' Law for Magnetic Fields

$$\Phi_B \;=\; \oint_A \vec{B} \cdot d\vec{A} = 0$$

- Magnetic Flux through closed surface is 0
- This says: There are no magnetic monopoles
- Important Law – one of Maxwell's equations
- Unfortunately of limited practical use

Ampere's Law



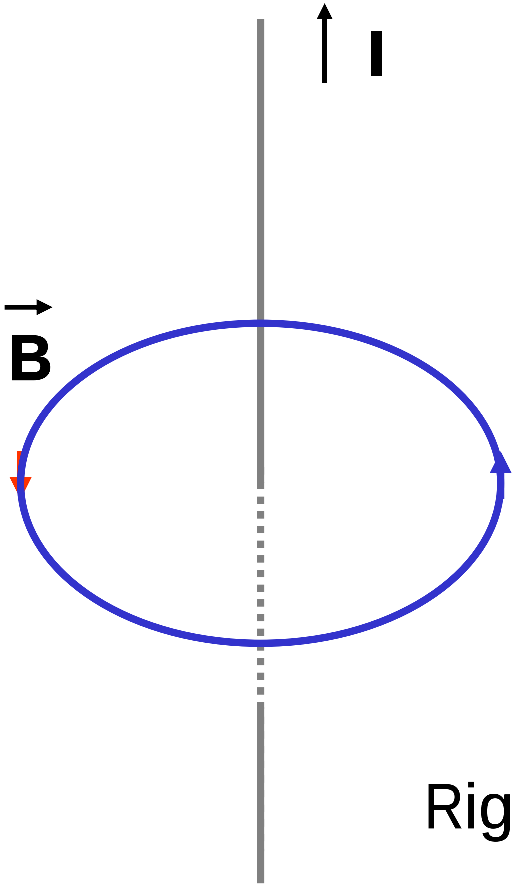
- Ampere's idea:

Relate Field $\vec{B}$ to its Source: I

Closed Line instead of closed surface!

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \, I_{encl}$$

Ampere's Law



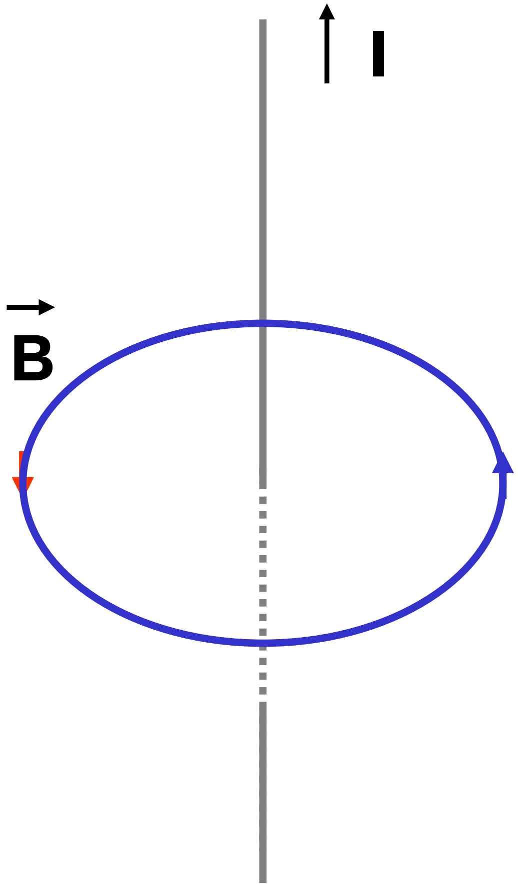
Ampere's Law helps because we can choose integration path!

$$\vec{B} \perp d\vec{l} \Rightarrow \vec{B} \cdot d\vec{l} = 0$$

$$\vec{B} || d\vec{l} \Rightarrow \vec{B} \cdot d\vec{l} = B\, dl$$

Right-Hand rule for relating sign of dl and I

Ampere's Law



Ampere's Law helps because we can choose integration path!

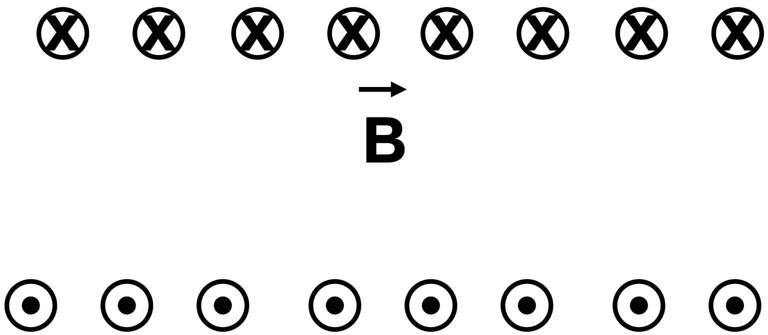
$$\oint_L \vec{B}\cdot d\vec{l} =$$

$$B \oint_L d\vec{l} =$$

$$B\ 2\pi\ r = \mu_0 I_{encl}$$

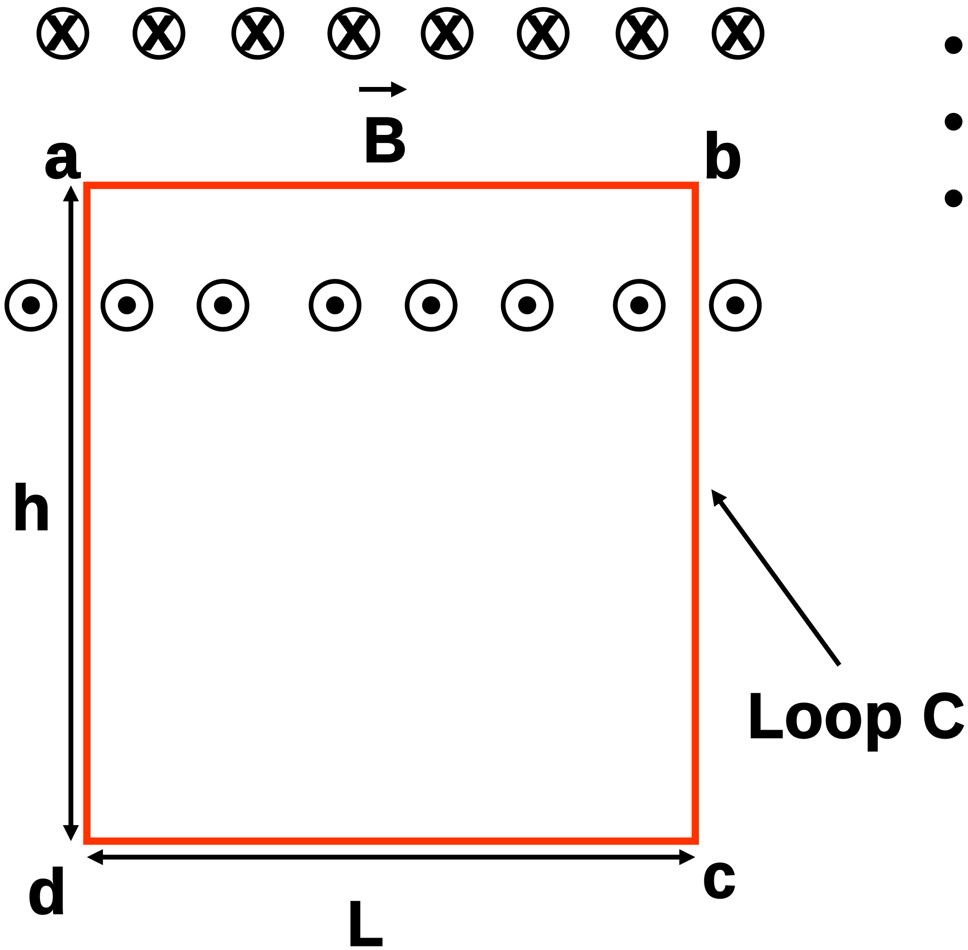
$$\Rightarrow\ B = \mu_0 \frac{I}{2\pi r}$$

Field of a Solenoid



- Current I
- n turns per unit length

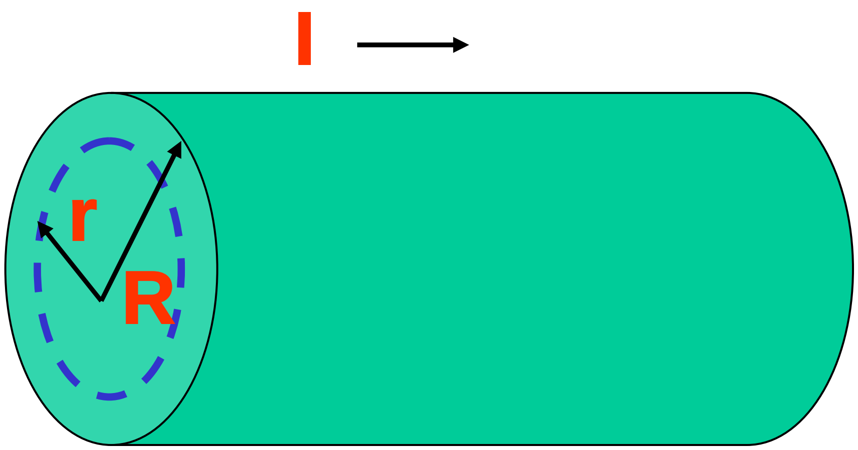
Field of a Solenoid



- Current I
- n turns per unit length
- (infinite length)

$$B = \mu_0 I n$$

Cylindrical Conductor



- Uniform Current-Density J
- Radius R
- $J = I/(\pi R^2)$

$r < R$

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \, I_{encl}$$

$$B(r) \oint_L d\vec{l} = \mu_0 J \, \pi r^2$$

$$B(r) \; 2\pi r = \mu_0 \frac{I}{\pi R^2} \pi r^2$$

$$\Rightarrow B(r) = \mu_0 \frac{I}{2\pi R^2} \; r$$

$r > R$

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \, I_{encl}$$

$$\Rightarrow B(r) 2\pi r = \mu_0 \, I$$

$$\Rightarrow B(r) = \mu_0 \frac{I}{2\pi r}$$