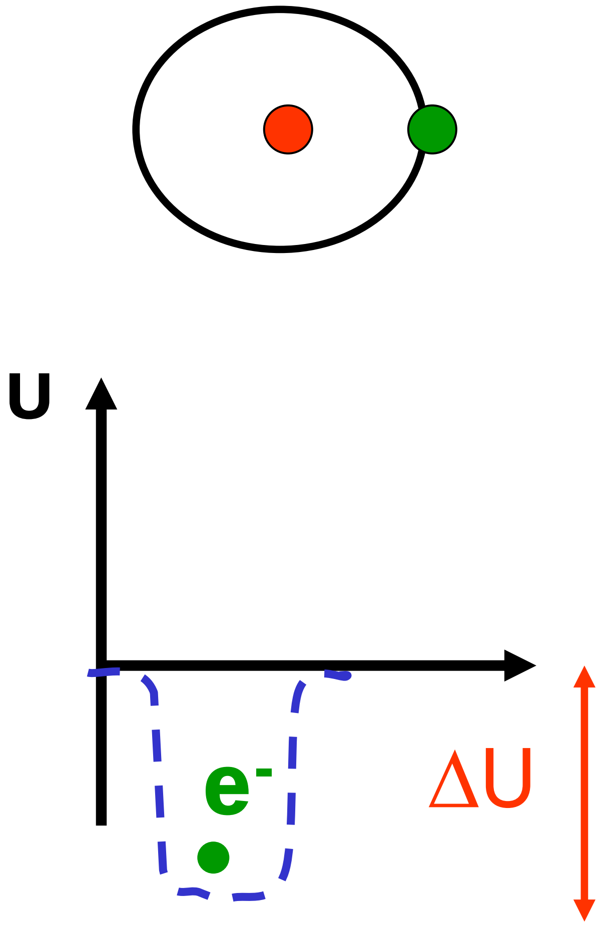
Electricity and Magnetism

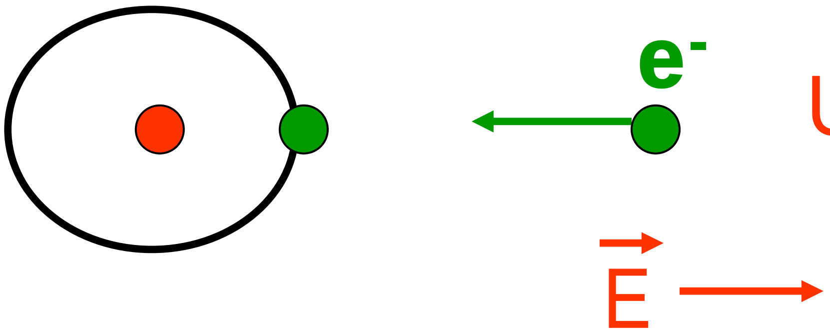
- Review for Quiz #3
  - Exp EB
  - Force in B-Field
    - on free charge
    - on current
  - Sources of B-Field
    - Biot-Savart
    - Ampere's Law
  - Exp MF
  - Magnetic Induction
    - Faradays Law
    - Lenz' Rule

Electrical Breakdown

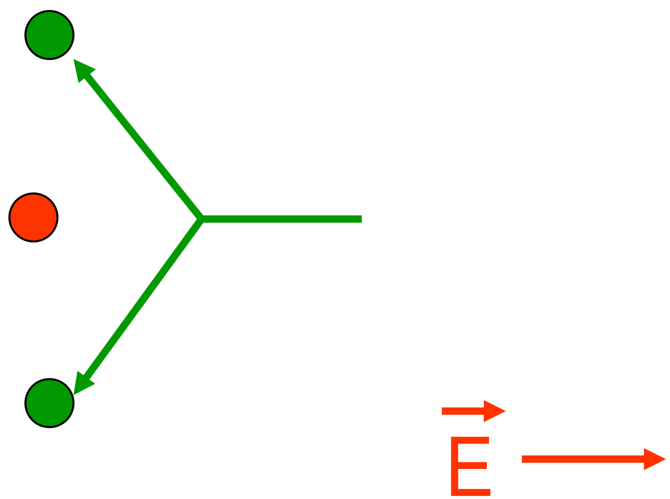


- Need lot's of free charges
- But electrons stuck in potential well of nucleus
- Need energy ΔU to jump out of well
- How to provide this energy?

Impact Ionization

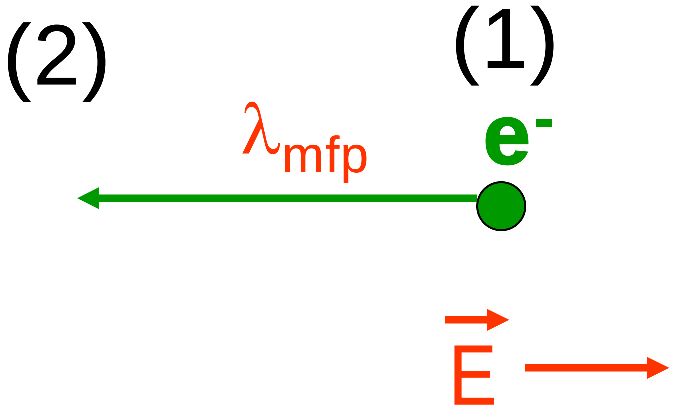


$U_{kin} > \Delta U$



- Define $V_{ion} = \Delta U/q$
  Ionization potential
- One e⁻ in, two e⁻ out
- Avalanche?

Impact Ionization



λ_{mfp} : Mean Free Path

- To get avalanche we need:

  ΔU_{kin} between collisions (1) and (2) $> V_{ion} * e$

- Acceleration in uniform Field

  $\Delta U_{kin} = V_2 - V_1 = e\,E\,d_{12}$

- Avalanche condition then

  $E > V_{ion} / \lambda_{mfp}$

Impact Ionization

How big is Mean Free Path?

(i) If Density n is big -> λ_{mfp} small

(ii) If size σ of molecules is big -> λ_{mfp} small

$$\longrightarrow \quad \boxed{\lambda_{mfp} = 1/(n\,\sigma)}$$

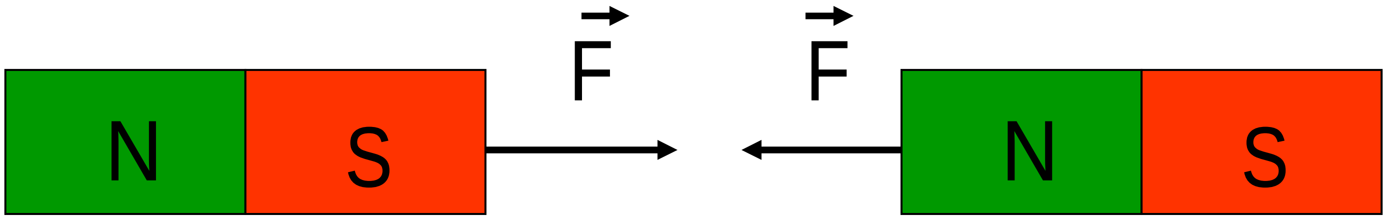
Avalanche if $E > V_{ion}/\lambda_{mfp} = V_{ion}\, n\, \sigma$

Magnetism

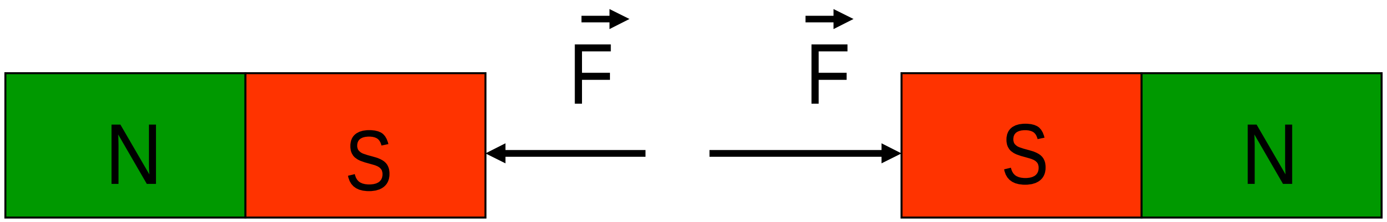
- Observed New Force between
  - two Magnets
  - Magnet and Iron
  - Magnet and wire carrying current
  - Wire carrying current and Magnet
  - Two wires carrying currents
- Currents (moving charges) can be subject to and source of Magnetic Force

Magnetic Force

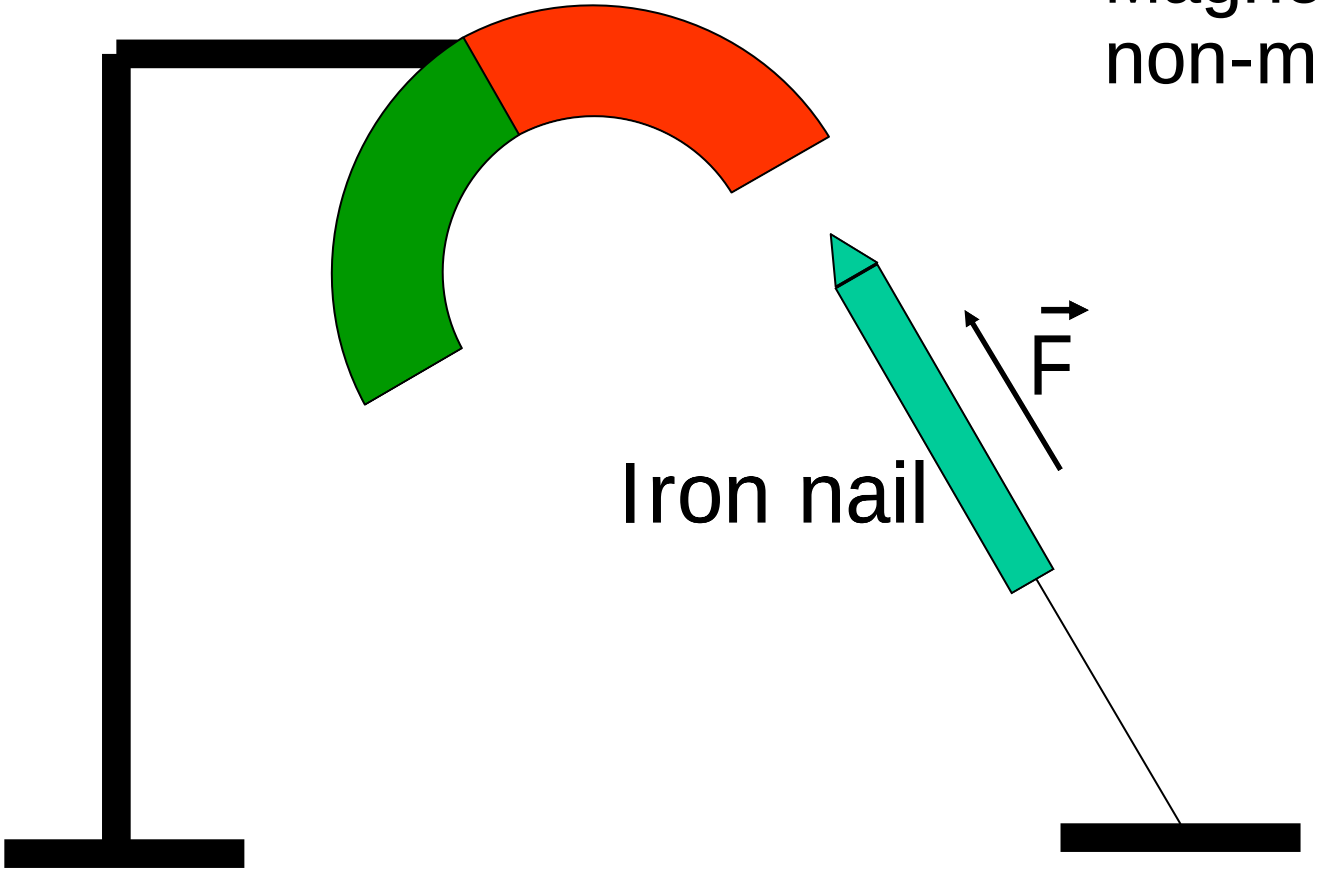
- Force between Magnets
- Unlike Poles attract



- Like Poles repel

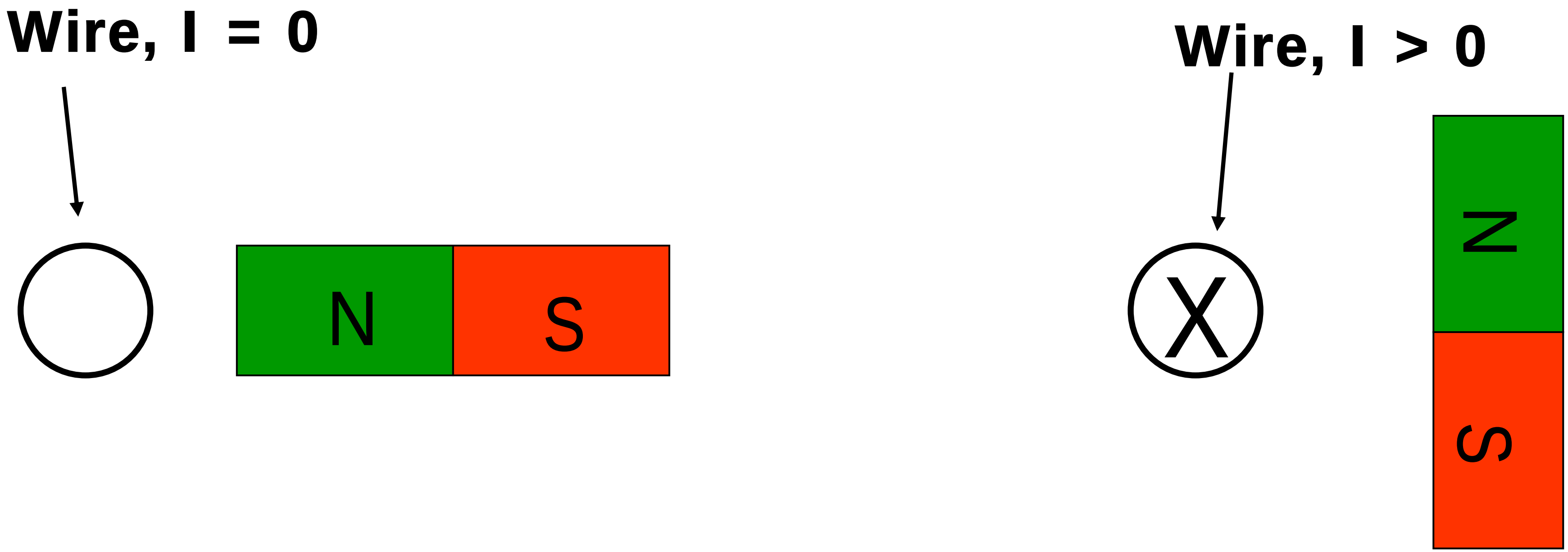


Magnetic Force

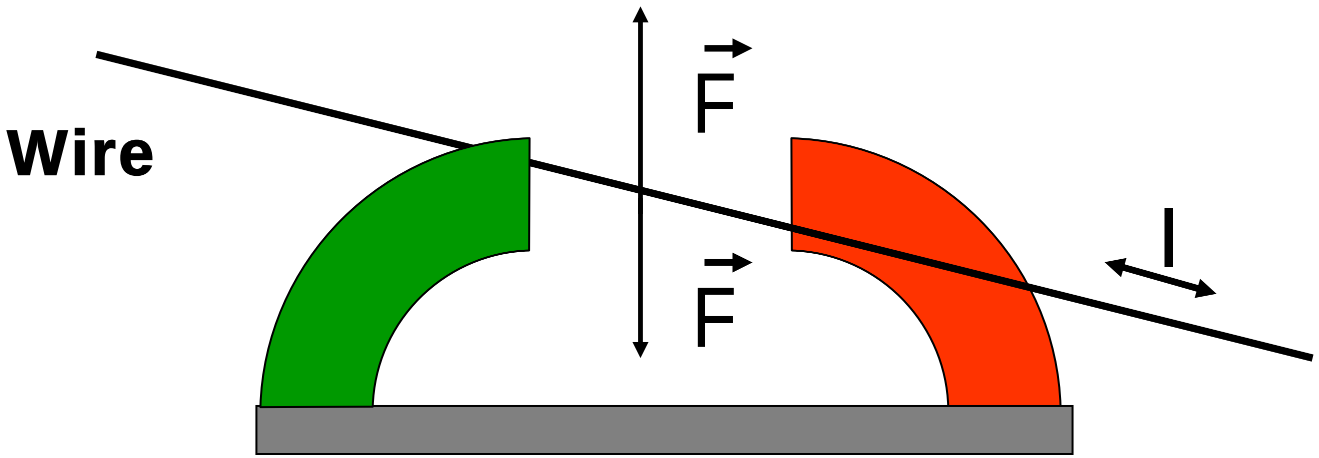


- Magnets also attract non-magnets!

Current and Magnet

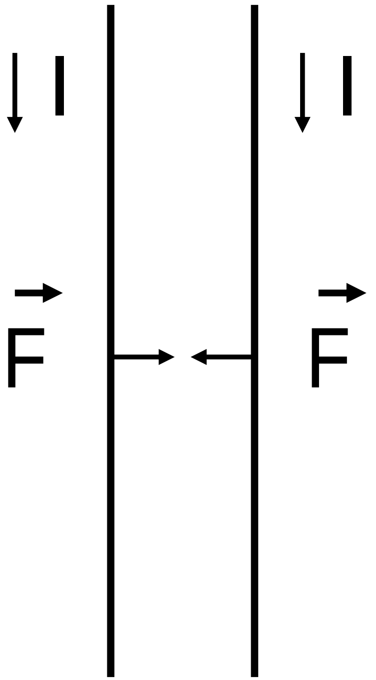


Magnet and Current

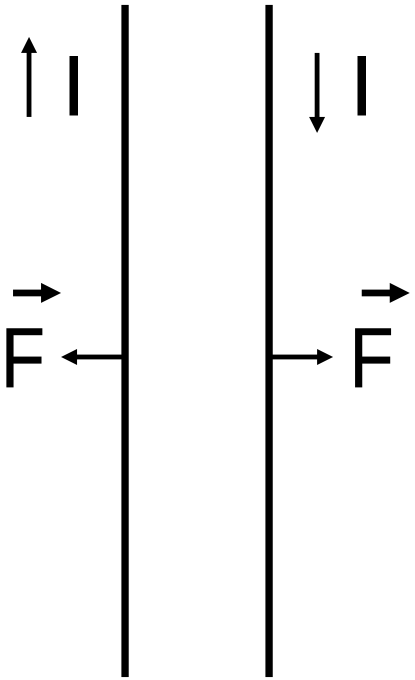


- Force on wire if I != 0
- Direction of Force depends on Sign of I
- Force perpendicular to I

Current and Current



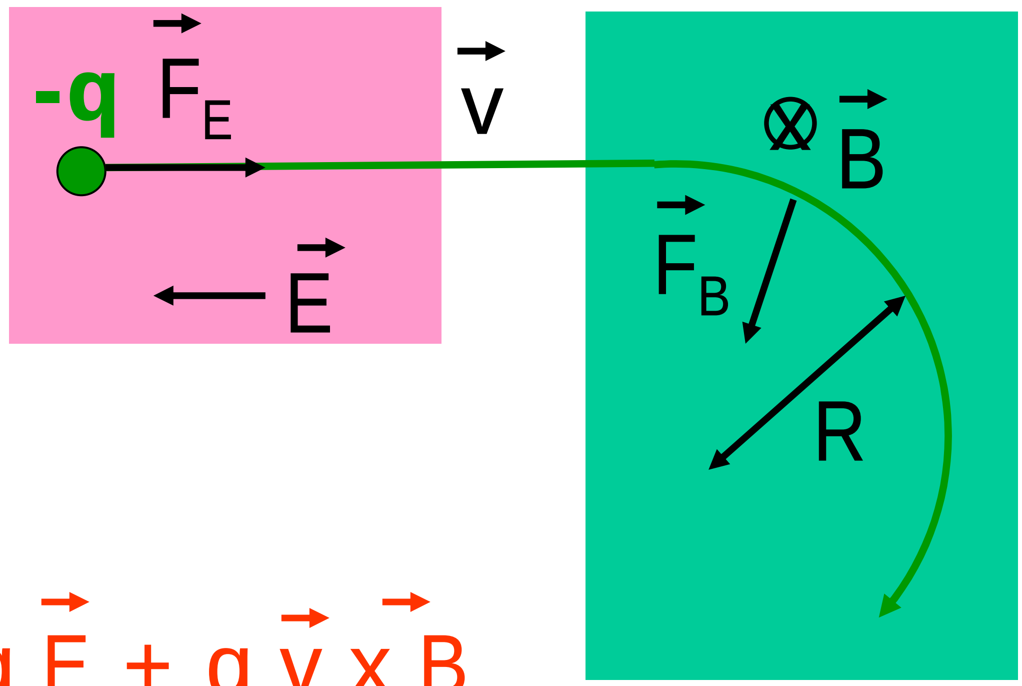
Attraction



Repulsion

Experiment MF

Force on moving charge



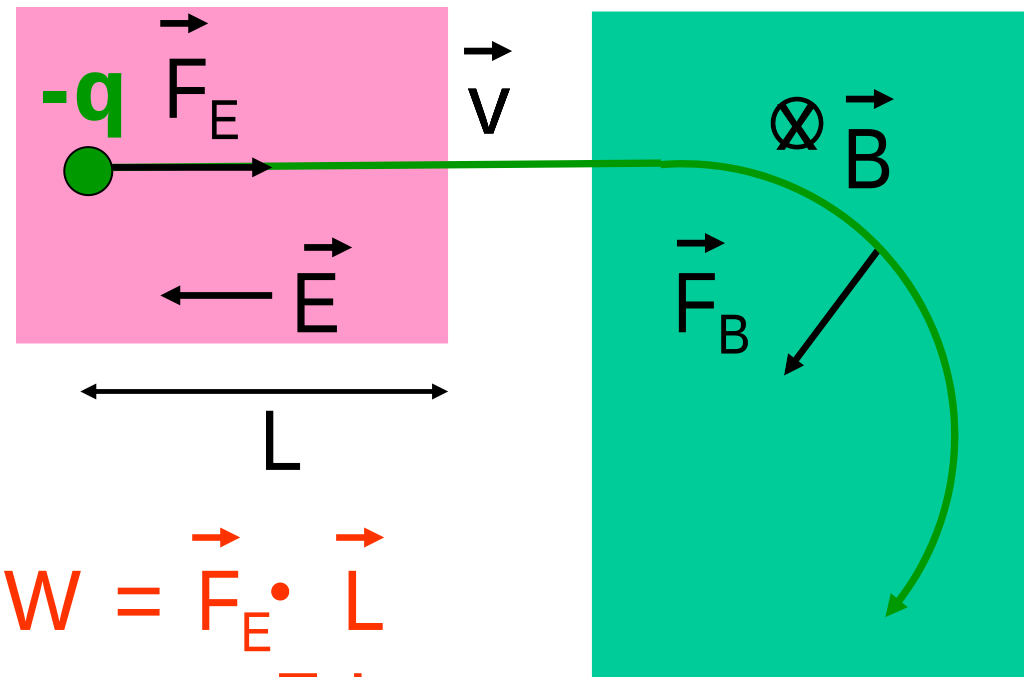
$$\vec{F}_L = q\,\vec{E} + q\,\vec{v} \times \vec{B}$$

Lorentz-Force

$$R = m\,v/(q\,B)$$

Cyclotron Radius

Work done on moving charge



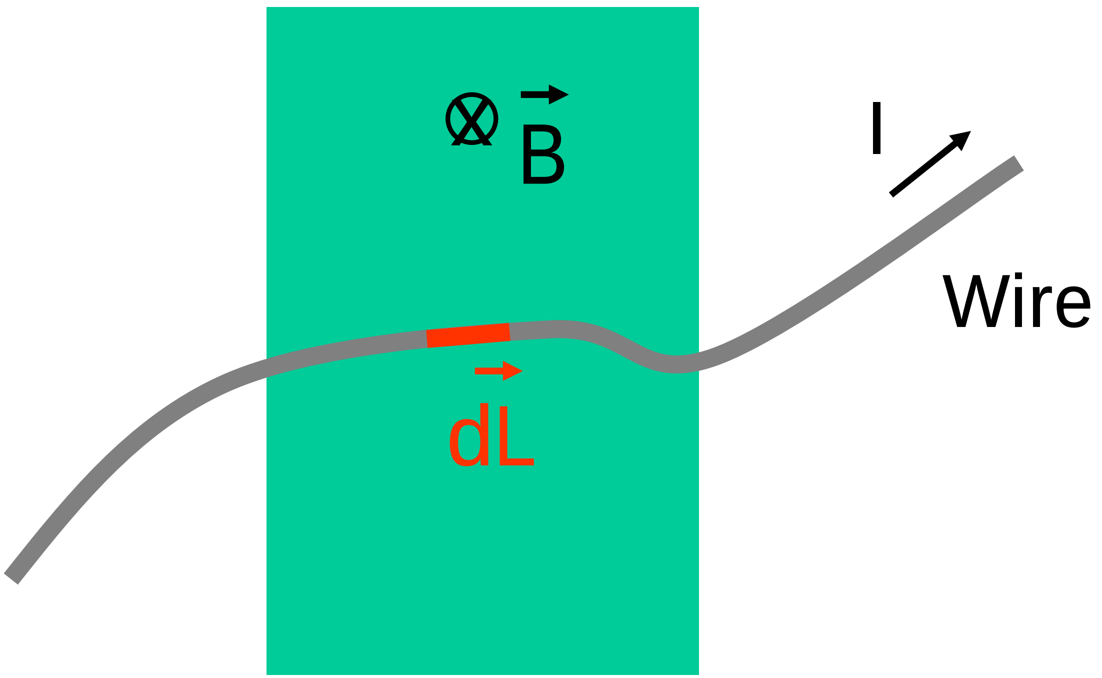
$$W = \vec{F}_E \cdot \vec{L}$$
$$= q\,E\,L$$

$$dW = \vec{F}_B \cdot d\vec{L}$$
$$= (q\,\vec{v} \times \vec{B}) \cdot d\vec{L}$$

$$= (q\,d\vec{L}/dt \times \vec{B}) \cdot d\vec{L}$$
$$= 0$$

Magnetic Field does no Work!

Force on Wire carrying current I

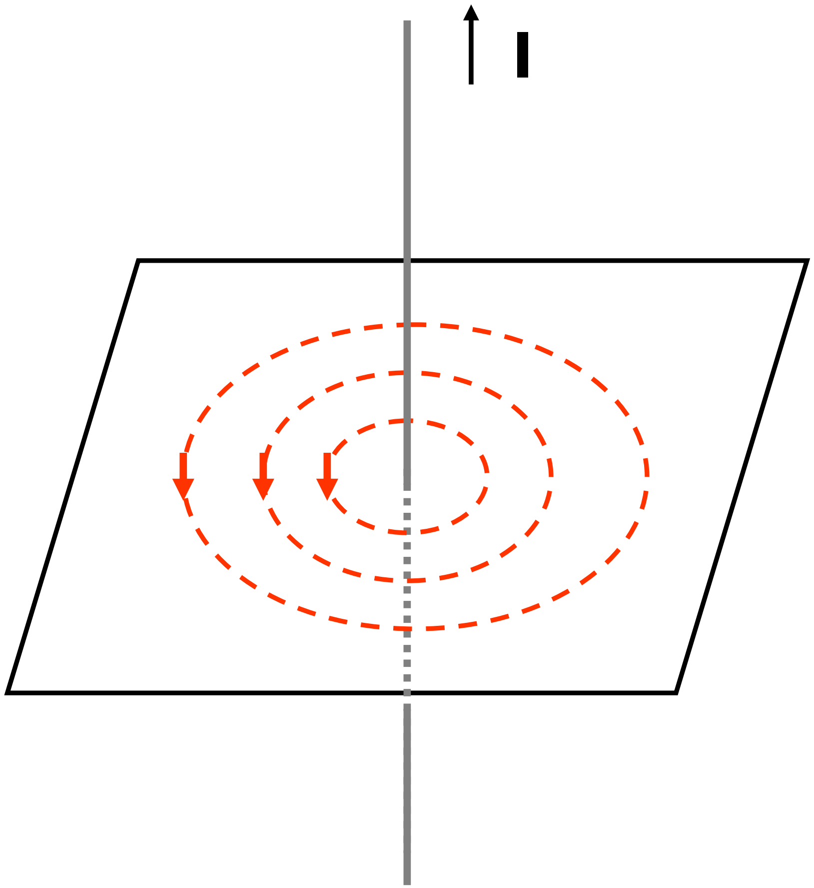


$$d\vec{F}_B = dq\ \vec{v} \times \vec{B}$$

$$= dq\ d\vec{L}/dt \times \vec{B}$$

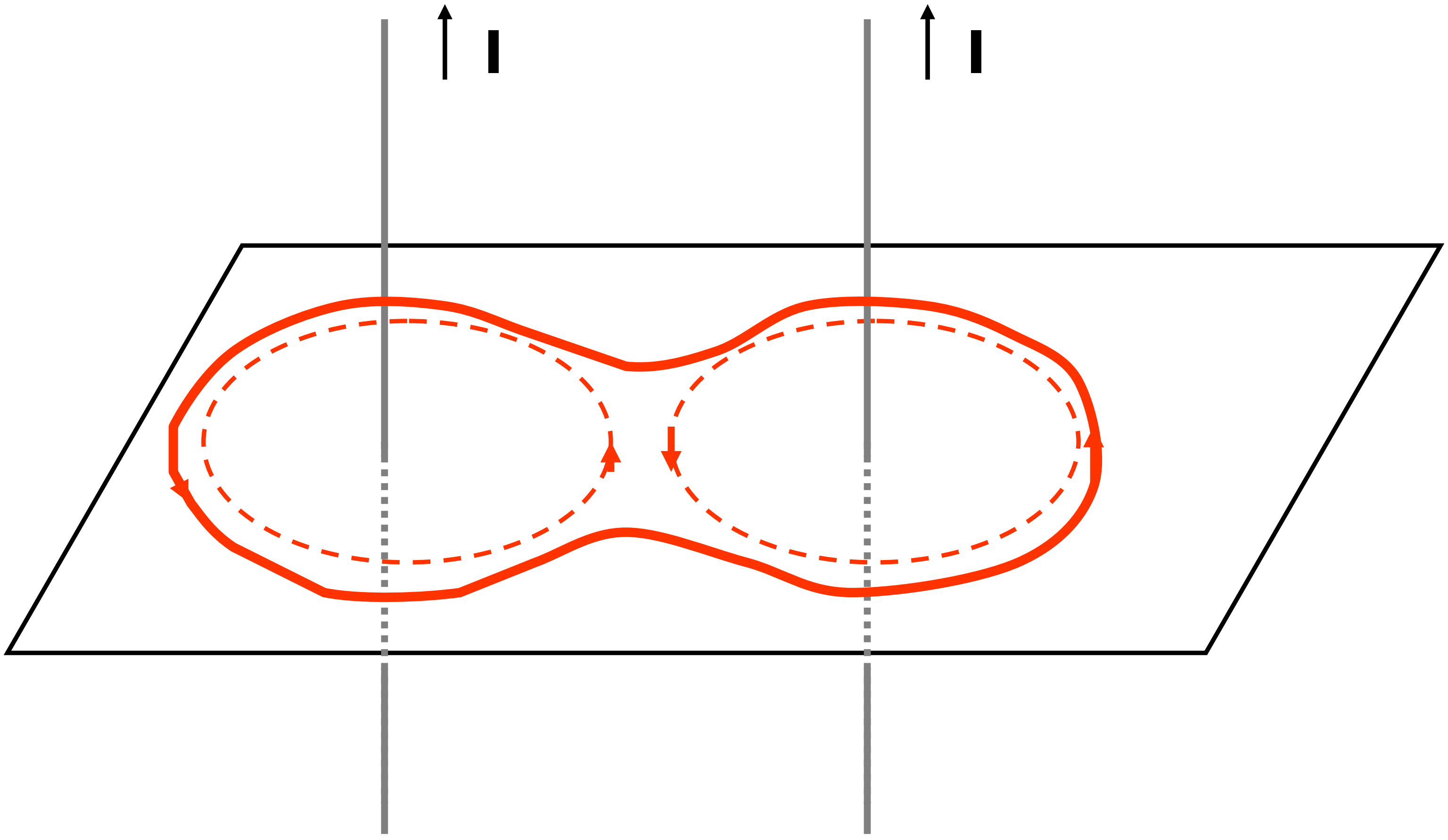
$$= \underline{I\ d\vec{L} \times \vec{B}}$$

Currents and B-Field



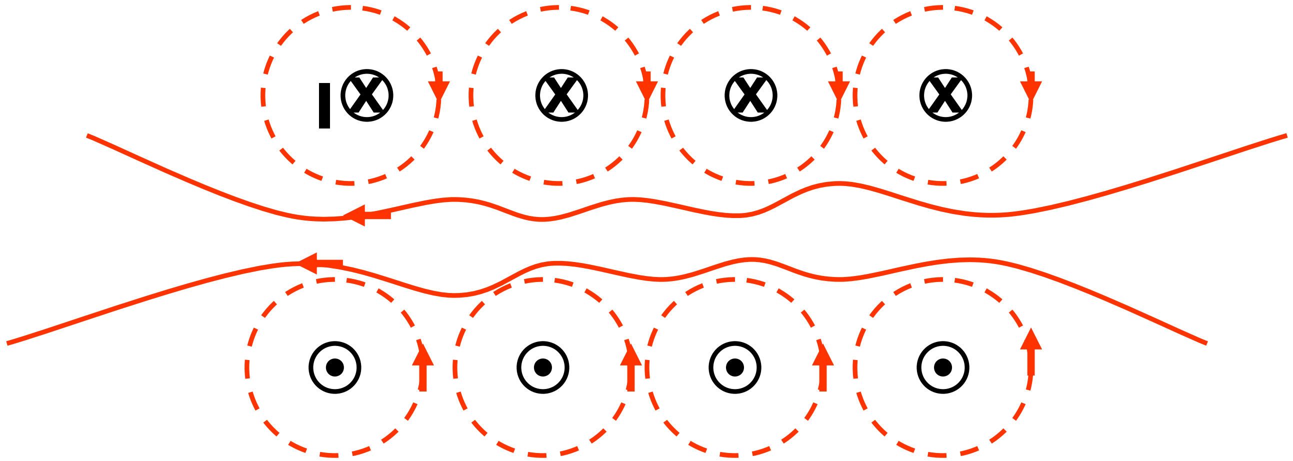
- Current as Source of B
- Magnetic Field lines are always closed
  - no Magnetic Charge (Monopole)
- Right Hand Rule

Currents and B-Field



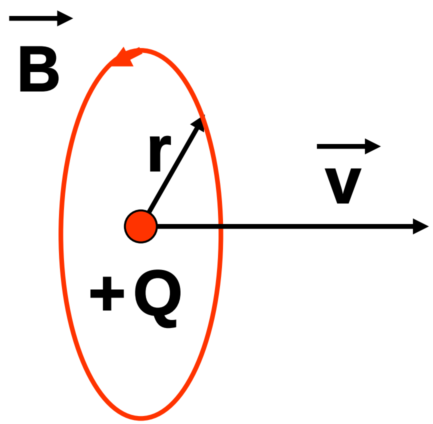
- Superposition Principle!

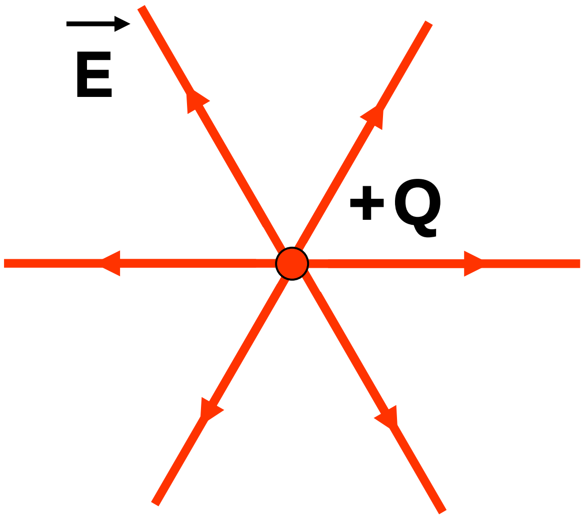
Currents and B-Field



- Solenoid: Large, uniform B inside
- Superposition Principle!

Magnetic Field vs Electric Field





$\vec{B} = \mu_0/(4\,\pi)\; Q/r^2\; \vec{v} \times \hat{r}$

$\vec{E} = 1/(4\,\pi\,\varepsilon_0)\; Q/r^2\; \hat{r}$

$\mu_0 = 4\,\pi \bullet 10^{-7}$ T m /A

$\varepsilon_0 = 8.85 \bullet 10^{-12}\; C^2/(Nm^2)$

$1/(\mu_0\,\varepsilon_0) = (3 \bullet 10^8 \text{ m/s})^2 = c^2$ Speed of Light

Deep connection between B and E Field

Magnetic Field for Current I

$d\vec{B} = \mu_0/(4\,\pi)\; dQ/r^2\; \vec{v} \times \hat{r}$ for charge dQ

$I = dQ/dt \quad -> \quad dQ\,\vec{v} = dQ\,d\vec{l}/dt = I\,d\vec{l}$

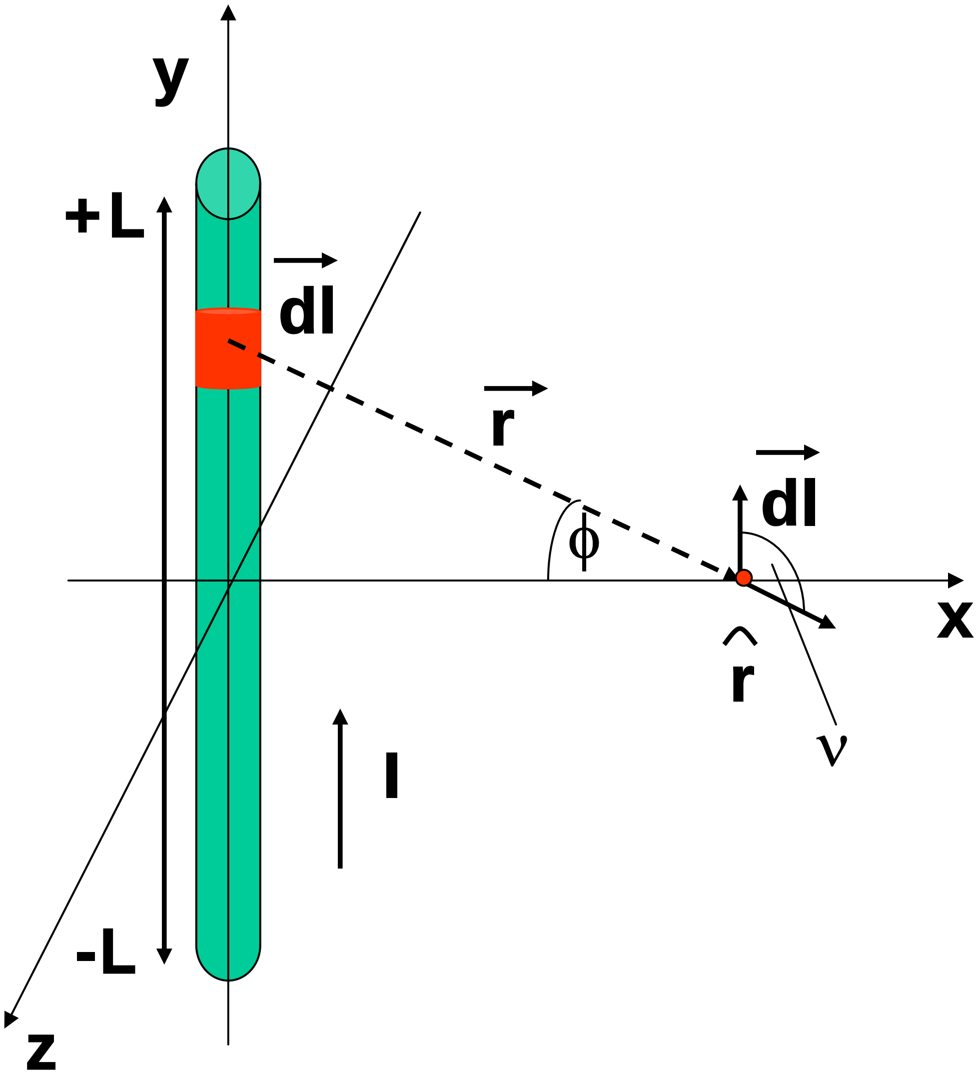
$dB = \mu_0/(4\,\pi)\;\; I\;\; d\vec{l} \times \hat{r}/r^2$

Law of Biot-Savart

Magnetic Field dB for current through segment dl

For total B-Field: Integrate over all segments dl

Magnetic Field for Current I



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$\vec{B}_{tot} = \int_{wire} \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0}{2\pi} I \frac{L}{x(x^2 + L^2)^{\frac{1}{2}}}$$

$$= \frac{\mu_0}{2\pi} \frac{I}{x} \quad \text{for} \quad L >> x$$

Magnetic Field for Current I

- For quiz:
  - No long calculations
  - But need to understand how to use Biot-Savart to find direction of $\vec{B}$

Remember: Gauss' Law

Electric Flux ϕ_E →

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Integral over any closed surface

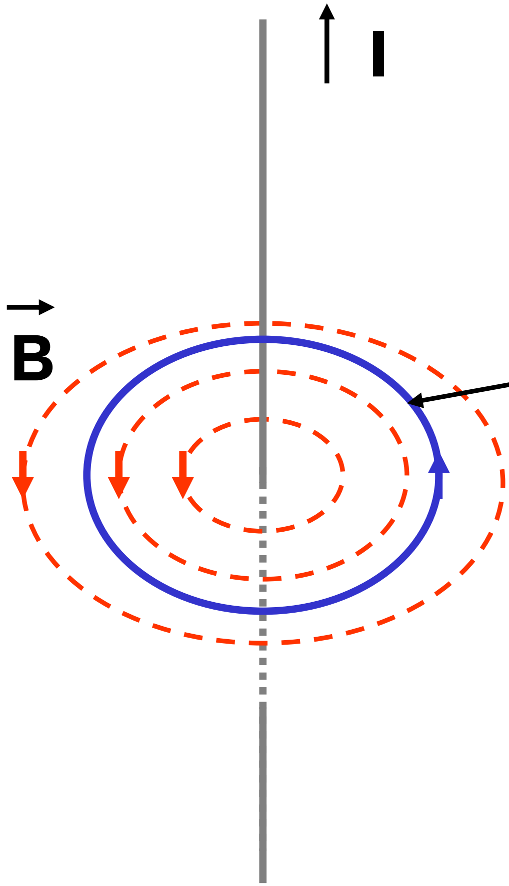
Electric Charge is the Source of Electric Field

Gauss' Law for Magnetic Fields

$$\Phi_B = \oint_A \vec{B} \cdot d\vec{A} = 0$$

- Magnetic Flux through closed surface is 0
- This says: There are no magnetic monopoles
- Important Law – one of Maxwell's equations
- Unfortunately of limited practical use

Ampere's Law



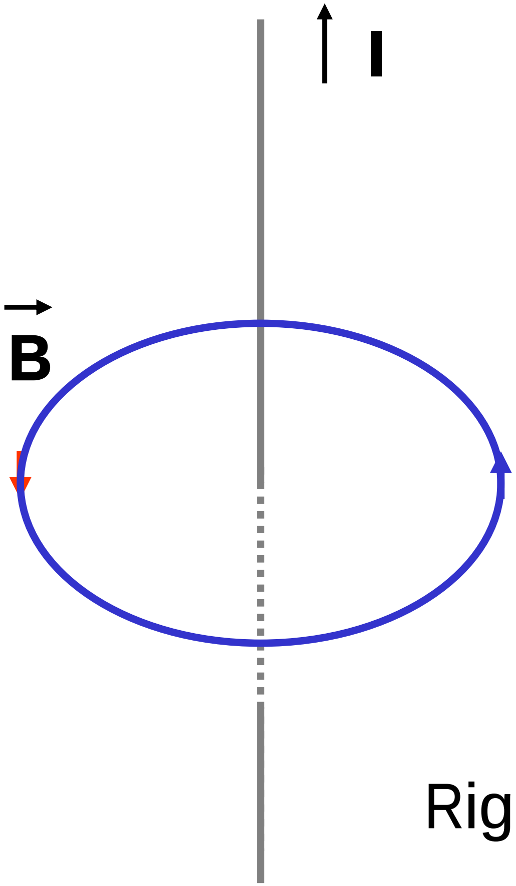
- Ampere's idea:

Relate Field $\vec{B}$ to its Source: I

Closed Line instead of closed surface!

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \, I_{encl}$$

Ampere's Law



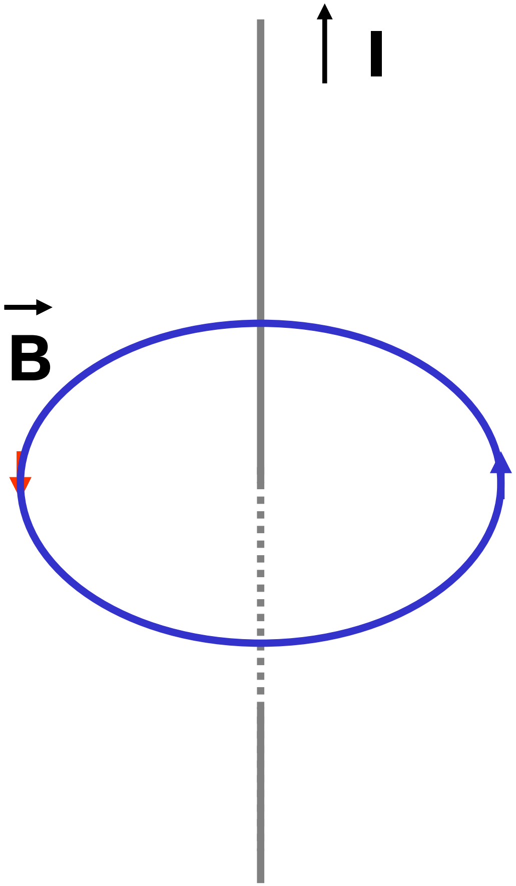
Ampere's Law helps because we can choose integration path!

$$\vec{B} \perp d\vec{l} \Rightarrow \vec{B} \cdot d\vec{l} = 0$$

$$\vec{B} || d\vec{l} \Rightarrow \vec{B} \cdot d\vec{l} = B\ dl$$

Right-Hand rule for relating sign of dl and I

Ampere's Law



Ampere's Law helps because we can choose integration path!

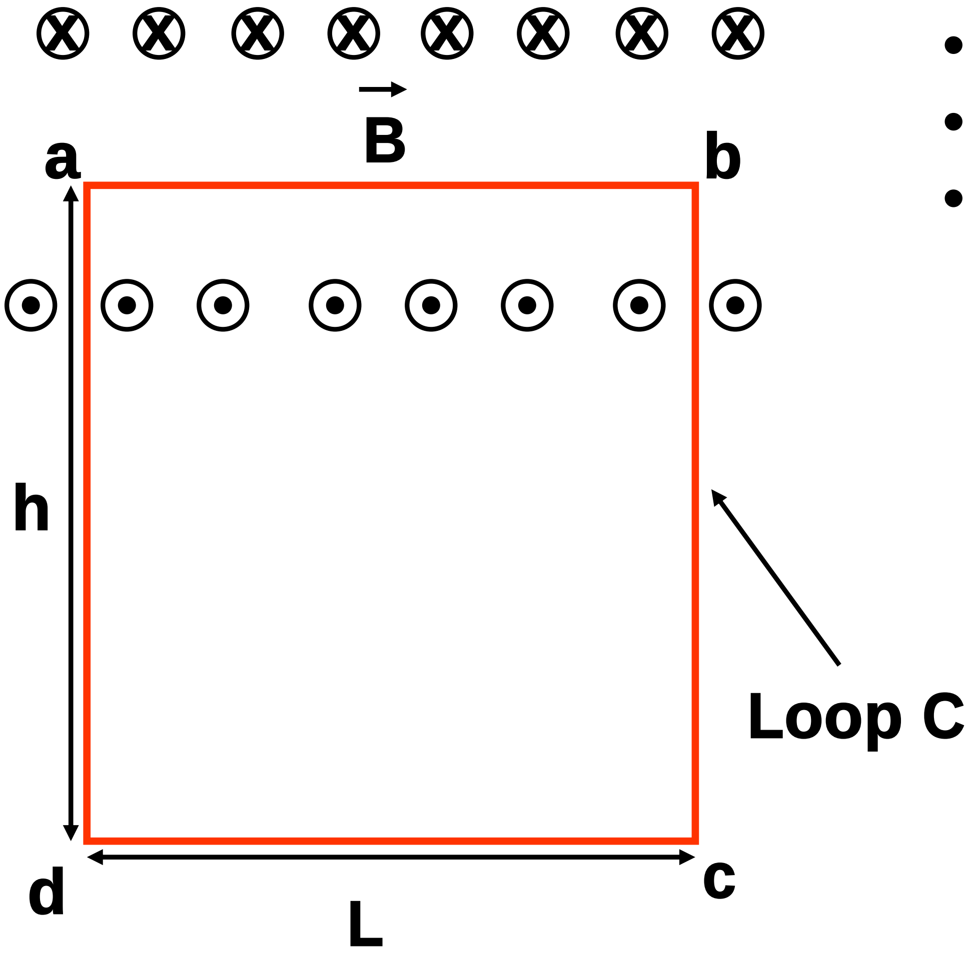
$$\oint_L \vec{B} \cdot d\vec{l} =$$

$$B \oint_L d\vec{l} =$$

$$B \; 2\pi \; r = \mu_0 I_{encl}$$

$$\Rightarrow \quad B = \mu_0 \frac{I}{2\pi r}$$

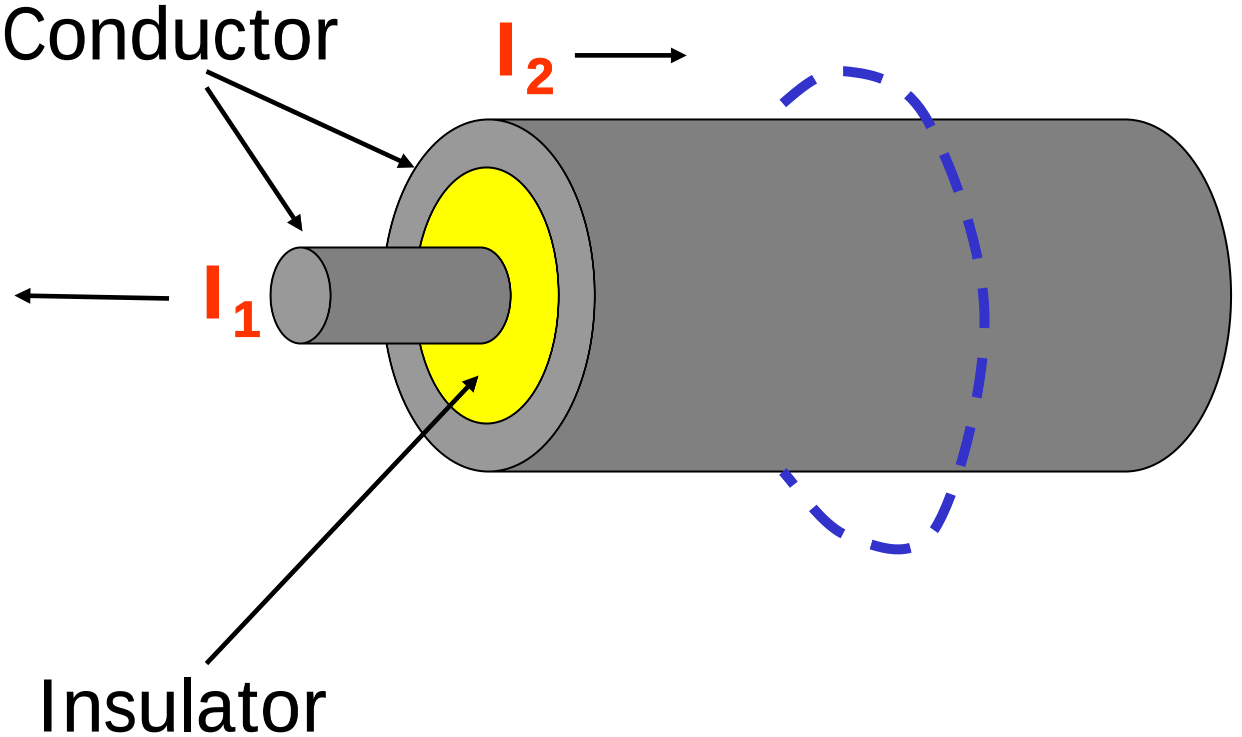
Field of a Solenoid



- Current I
- n turns per unit length
- (infinite length)

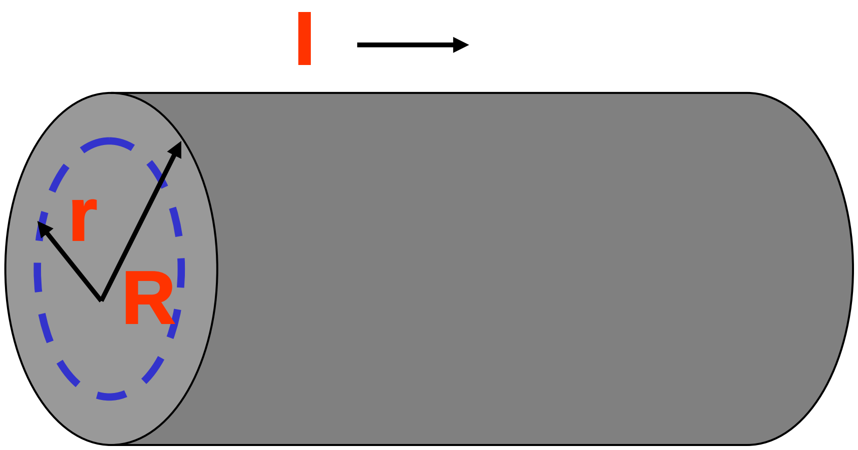
$$B = \mu_0 \, I \, n$$

Coaxial Cable



Outside field vanishes for $I_2 = I_1$

Cylindrical Conductor



- Uniform Current-Density J
- Radius R
- J = I/(π R^2)

$$r < R \qquad \oint_L \vec{B} \cdot d\vec{l} = \mu_0 \, I_{encl}$$

$$B(r) \oint_L d\vec{l} = \mu_0 J \, \pi r^2$$

$$B(r) \, 2\pi r = \mu_0 \frac{I}{\pi R^2} \pi r^2$$

$$\Rightarrow B(r) = \mu_0 \frac{I}{2\pi R^2} \, r$$

$$r > R \qquad \oint_L \vec{B} \cdot d\vec{l} = \mu_0 \, I_{encl}$$

$$\Rightarrow B(r) 2\pi r = \mu_0 \, I$$

$$\Rightarrow B(r) = \mu_0 \frac{I}{2\pi r}$$

Magnetic Induction

- Currents give rise to B-Field
- Q: Can B-Field give rise to current?
- A: Only if Magnetic Flux changes with time!
- Took a very long time to realize…

Faradays Law

$$\Phi_B = \int_A \vec{B} \cdot d\vec{A}$$

Magnetic Flux
(usually, A not closed surface)

$$\xi_{ind} = -\frac{d\Phi_B}{dt}$$

Faradays Law

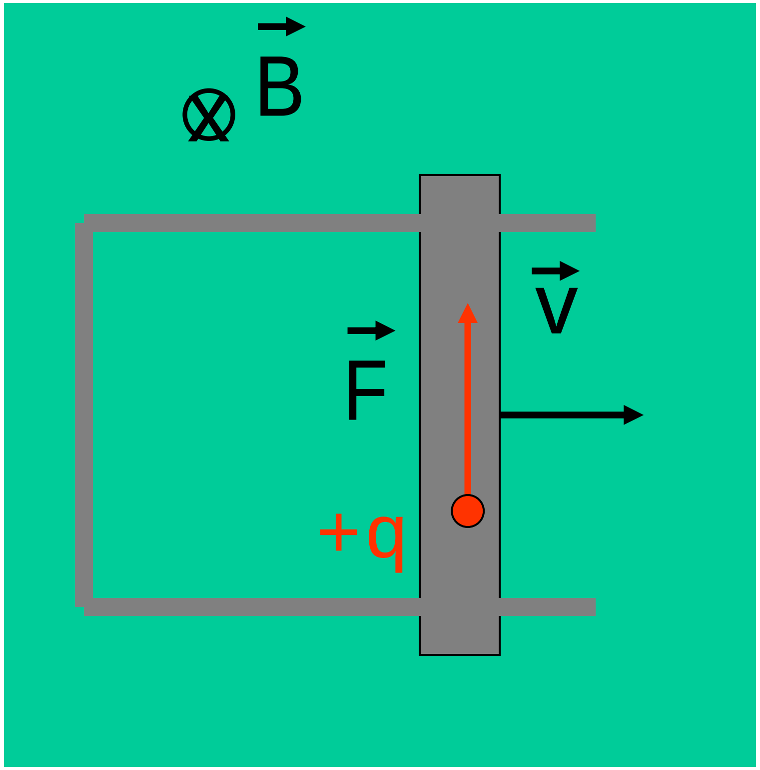
$$\Rightarrow I_{ind} = \frac{\xi_{ind}}{R}$$

Faradays Law

- Φ_B can change because
  - $|B|$ changes
  - Angle between $\vec{B}$ and $\vec{A}$ changes
  - $|A|$ (size of circuit in B) changes

Faradays Law

Moving circuit:
Induced EMF is consequence of force on moving charges



Lenz' Rule

$$\xi_{ind} = -\frac{d\Phi_B}{dt}$$

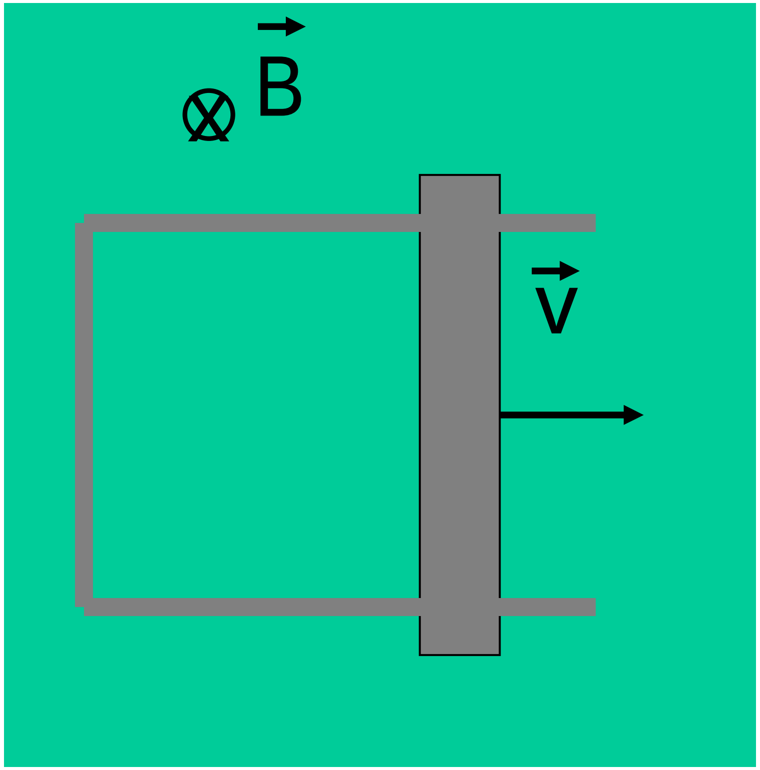
$$\Rightarrow I_{ind} = \frac{\xi_{ind}}{R}$$

Lenz' Rule:

Sign of I_{ind} such that it opposes the flux change that generated it

Use of Faradays Law

- To find I_{ind}:
  - Calculate Φ_B
  - Find, what makes Φ_B change
  - Find sign of I_{ind} using Lenz' rule

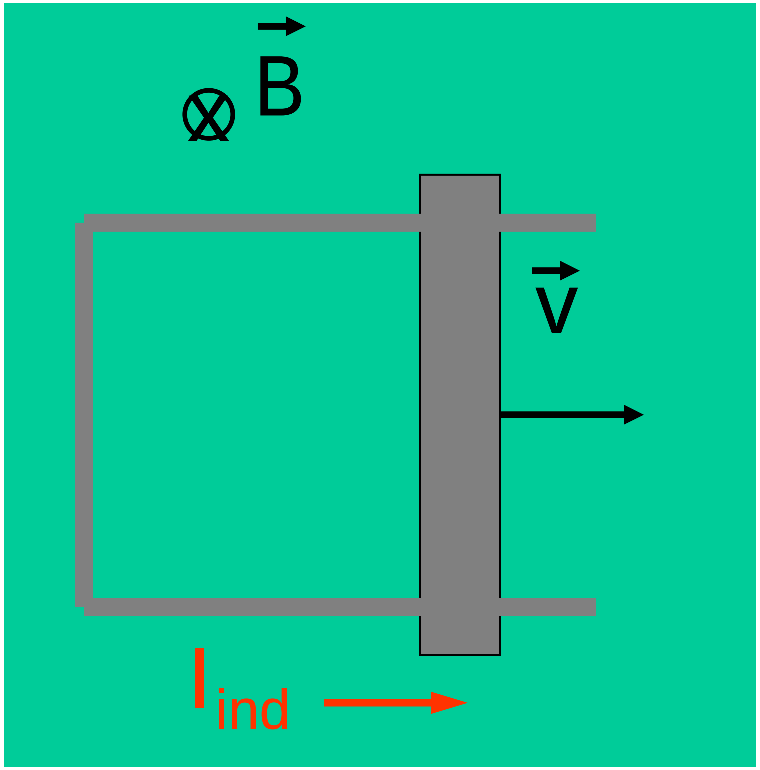


$$\Phi_B = B\ h\ l(t)$$

$$|\xi_{ind}| = -|\frac{d\Phi_B}{dt}| = Bh|\frac{dl}{dt}| = B\ h\ v$$

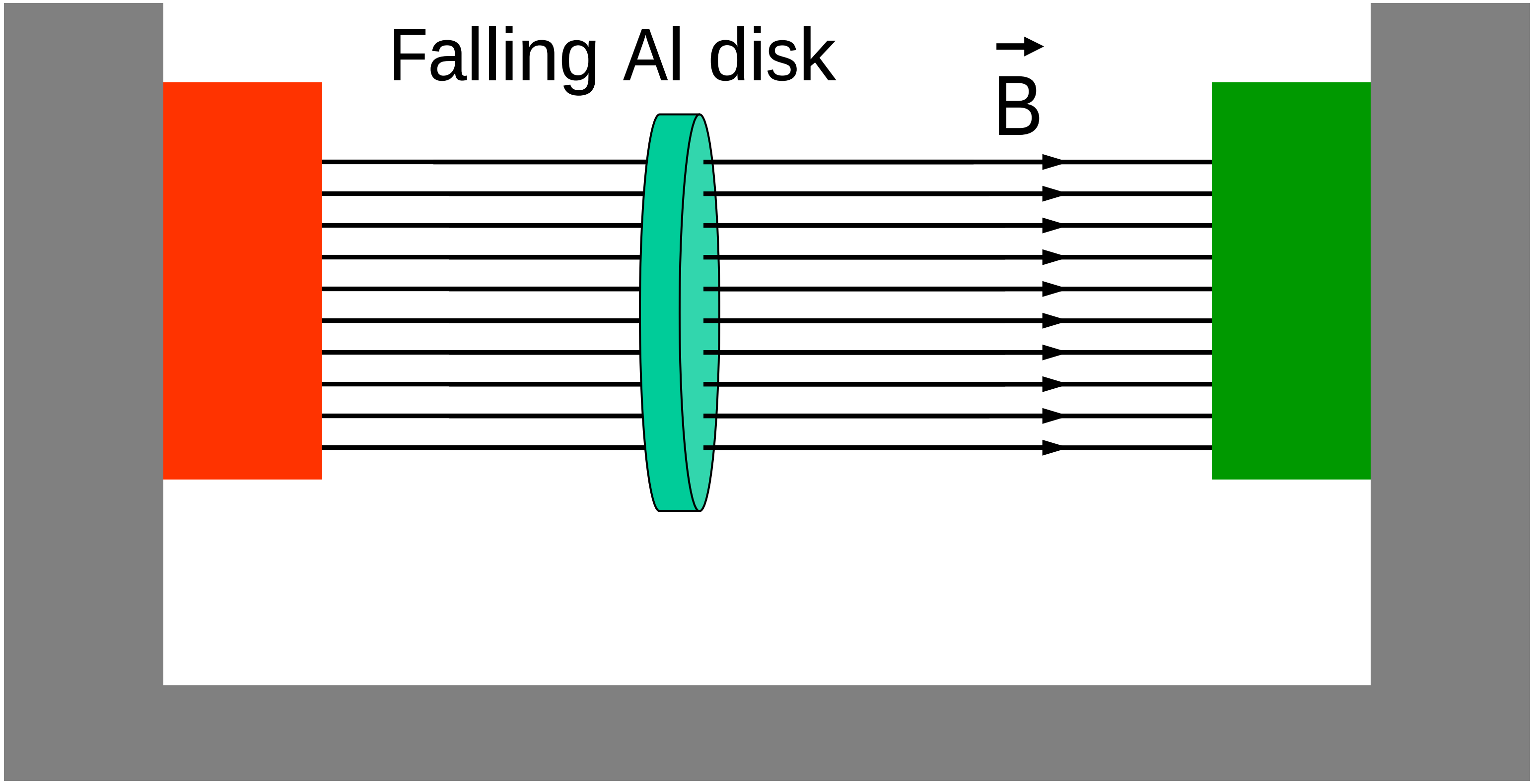
Use of Faradays Law

- Sign of current:
  Opposing change of Φ_B
  -> Reducing B



Lenz' Rule: Effect of I_{ind} current opposing $d\Phi_B/dt$ is like 'drag' or 'inertia'

My favorite Demo



- Falling Al ring is slowed down in B-Field
- Induced Eddy-currents
- Energy converted to heat

Faradays Law

Moving circuit:
Induced EMF is consequence of force on moving charges

What about changing B?