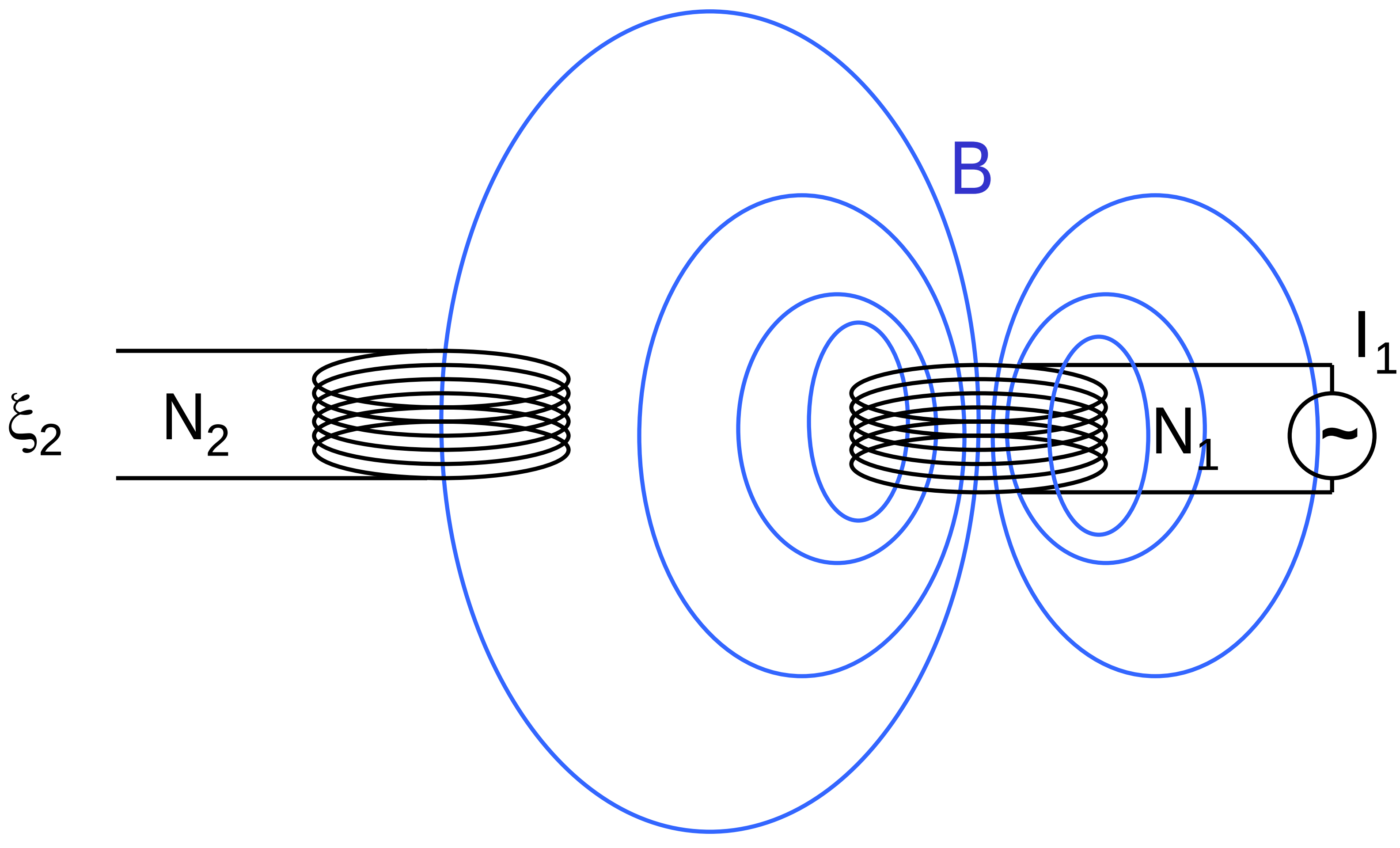
Electricity and Magnetism

- Reminder
  - Mutual- and Self-Inductance
  - Inductance and AC circuits
    - Phase shift between I(t) and V(t)
- Today
  - RL circuits
  - Energy in B-Field

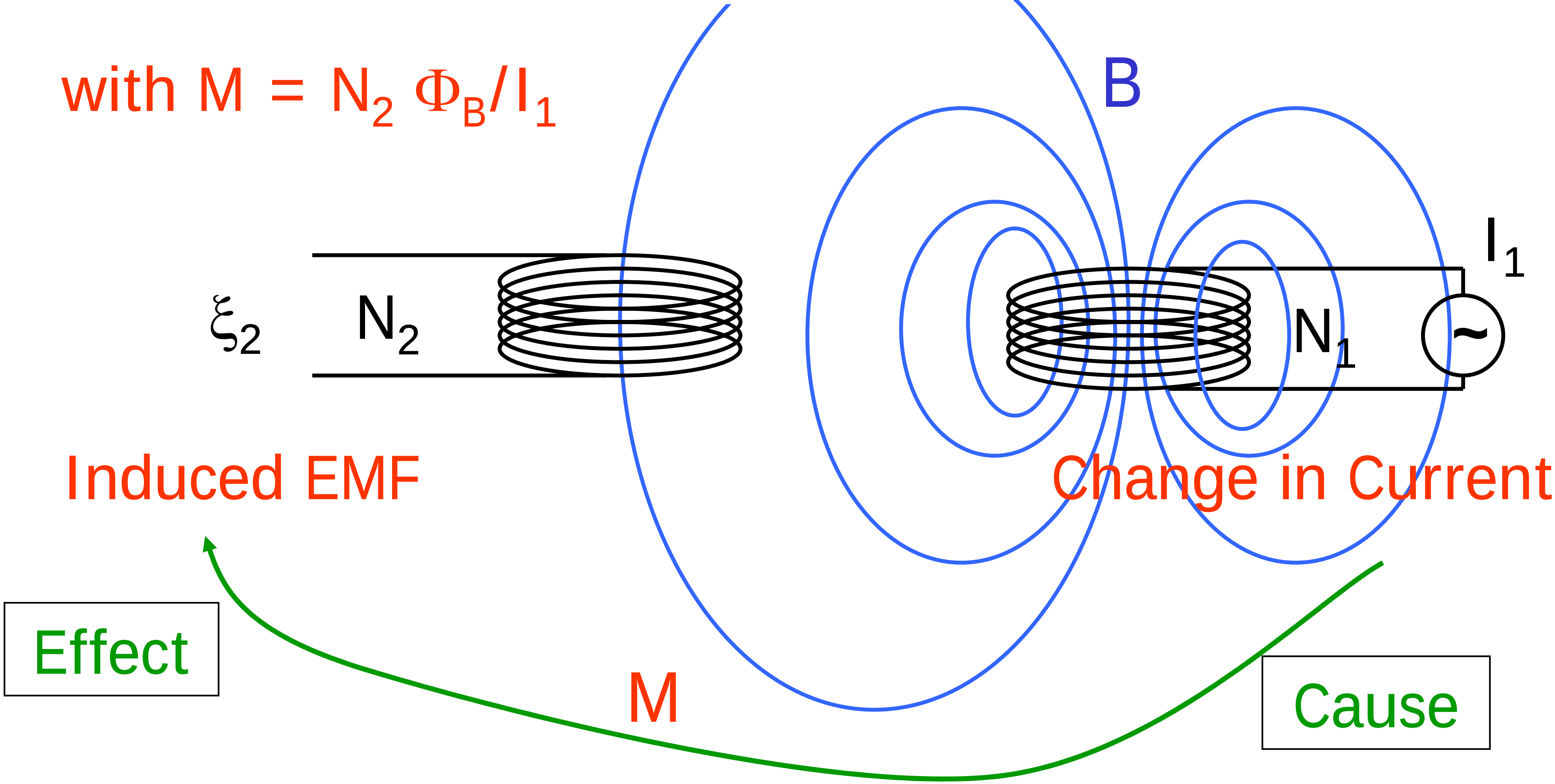
Inductance



Inductance

$\xi_2 = - N_2 \, d\Phi_B/dt = - M \, dI_1/dt$

with $M = N_2 \, \Phi_B / I_1$



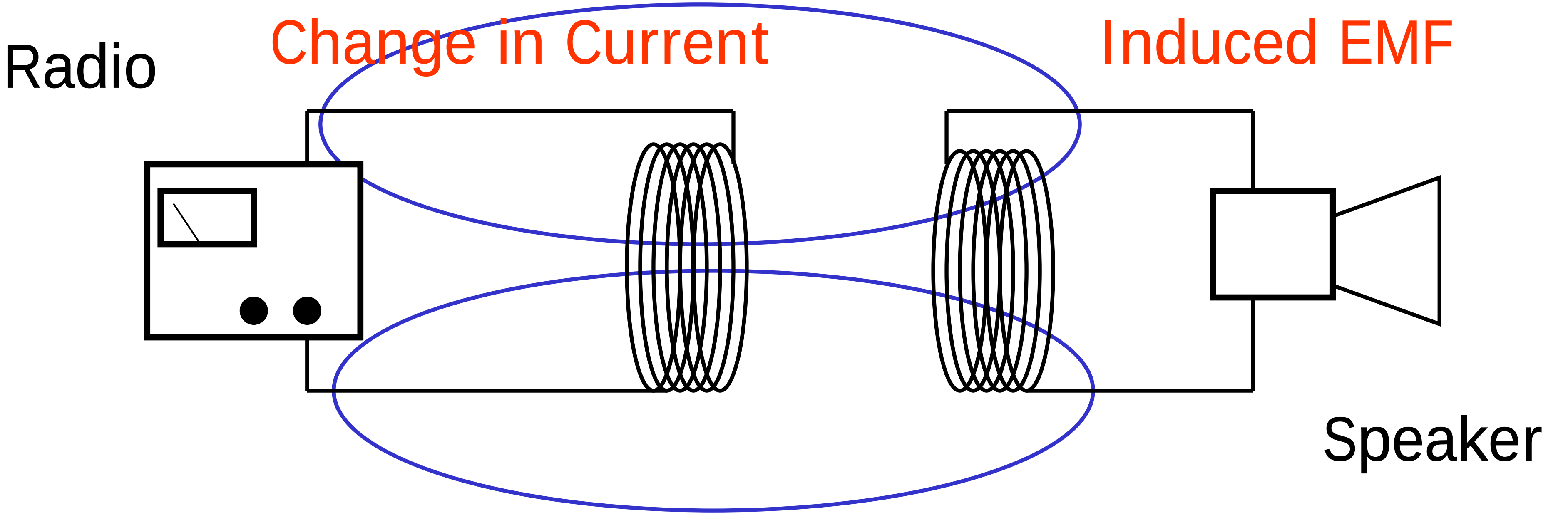
Mutal Inductance

- Coupling is symmetric: $M_{12} = M_{21} = M$
- M depends only on Geometry and Material
- Mutual inductance gives strength of coupling between two coils (conductors):

$$\xi_2 = - N_2 \, d\Phi_B/dt = - M \, dI_1/dt$$

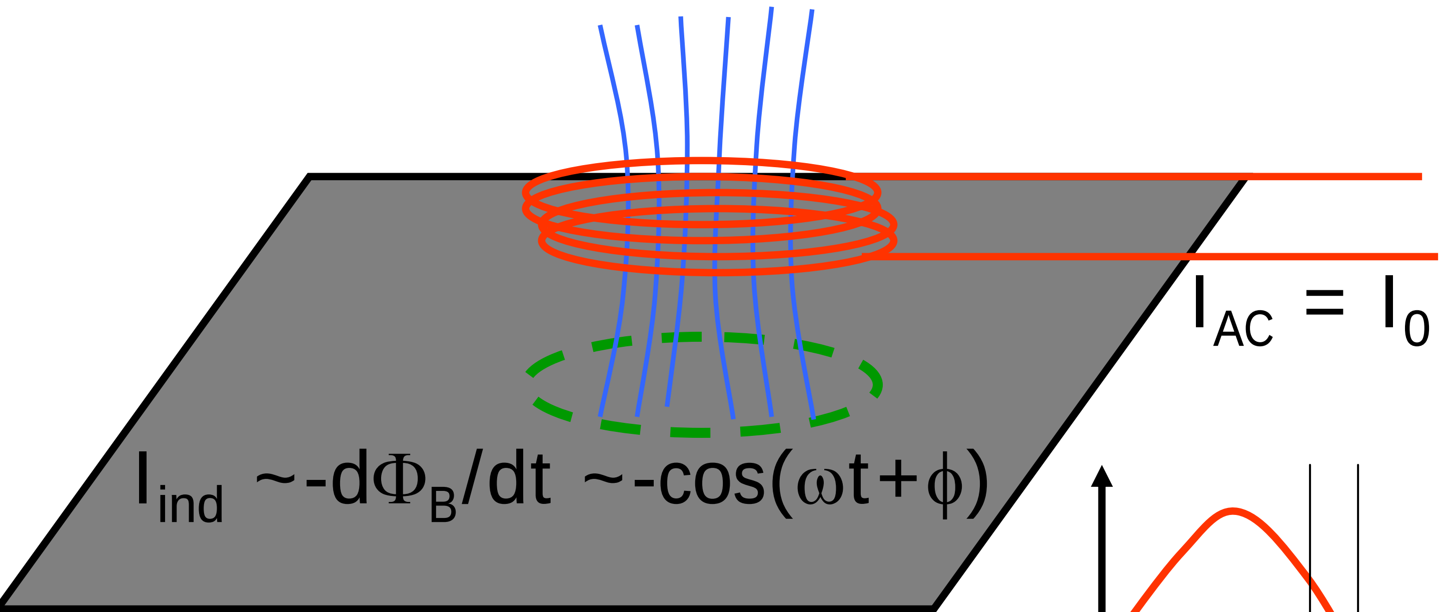
- M relates ξ_2 and I_1 (or ξ_1 and I_2)
- Units: [M] = V/(A/s) = V s /A = H ('Henry')

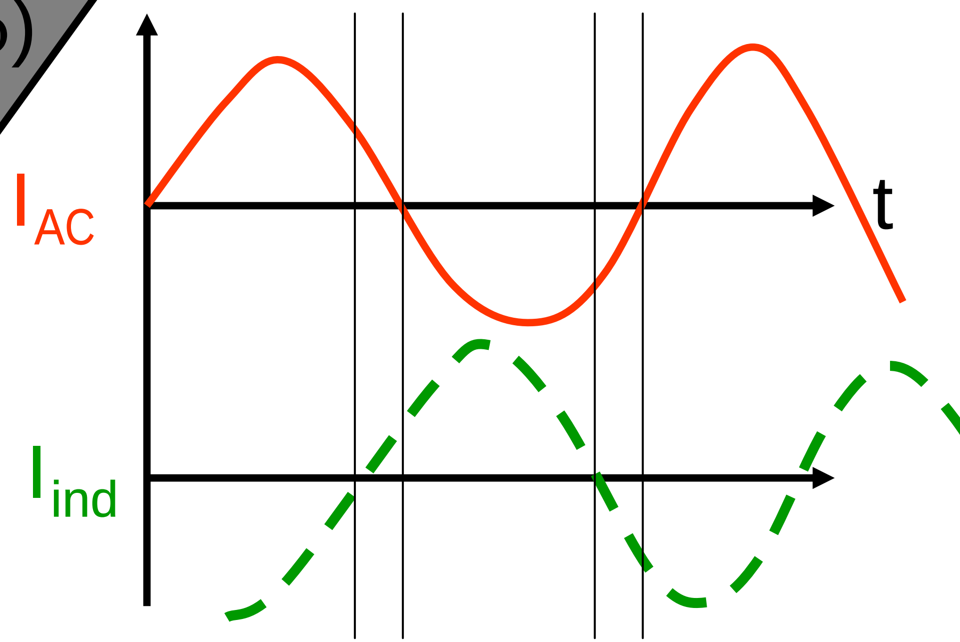
In-Class Demo: Two Coils



- Signal transmitted by varying Field
- Coupling depends on Geometry (angle, distance)

In-Class Demo: Levitating Coil



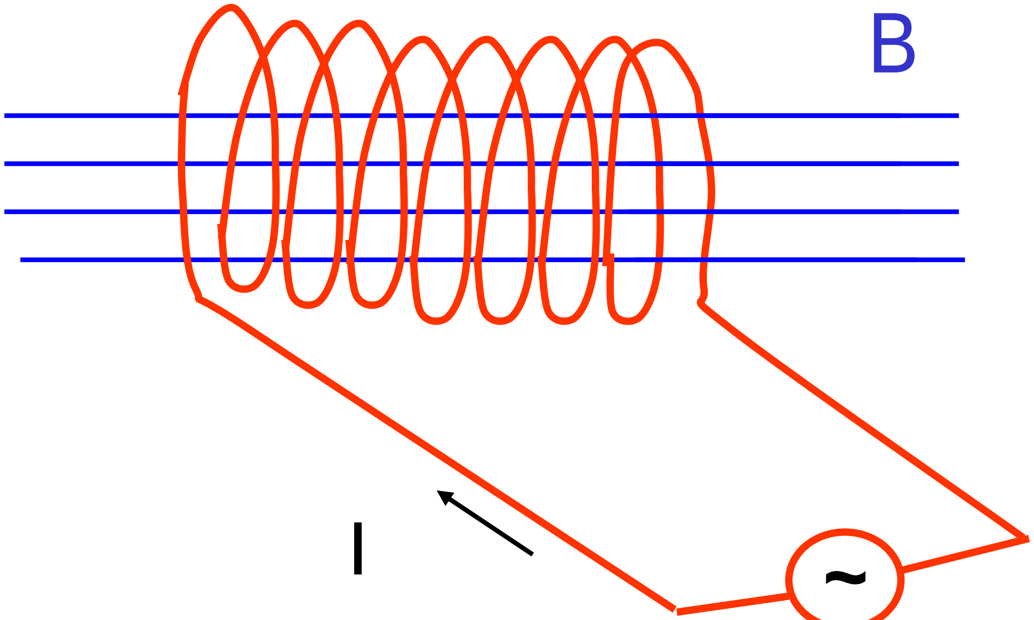


With delay ($\phi > 0$):

Net repulsion (currents are opposite most of the time)

Self Inductance

Circuit sees flux generated by it self



Def.: $L = N \Phi_B / I$

Self-Inductance

Example: Solenoid

Q: How big is L ?

A: $L = \mu_0 N^2 A/L$

Self Inductance

- L is also measured in [H]
- L connects induced EMF and variation in current:

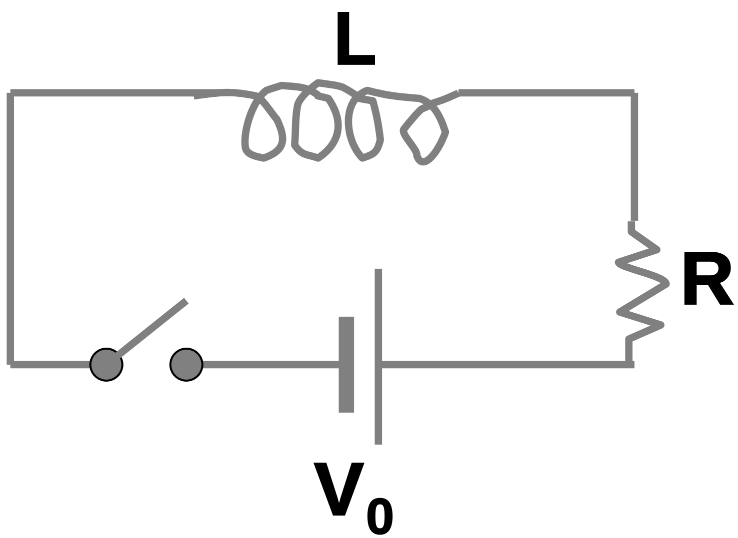
$$\xi = - L \, dI/dt$$

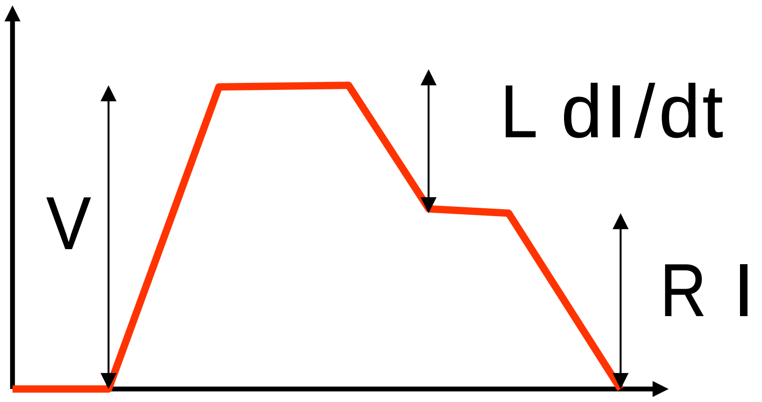
- Remember Lenz' Rule:

  Induced EMF will 'act against' change in current -> effective 'inertia'

- Delay between current and voltage

RL Circuits



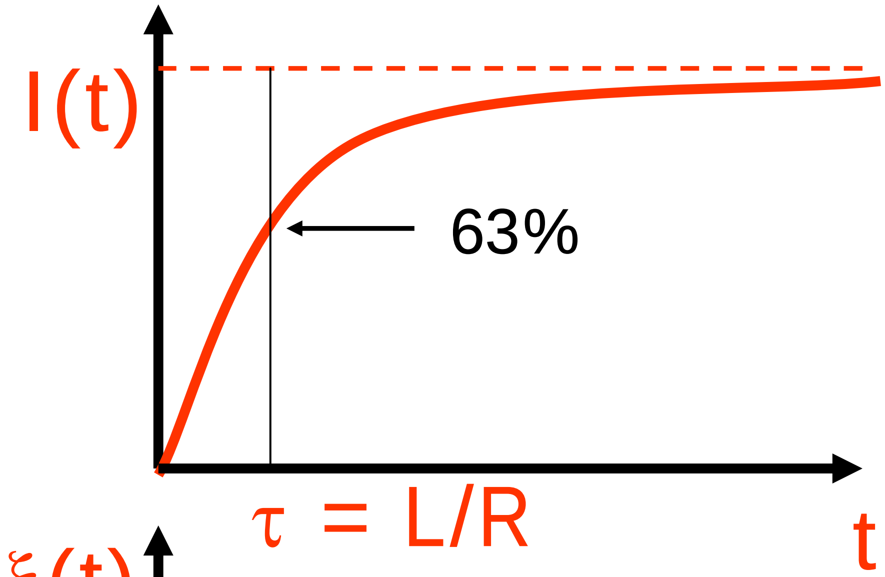


Kirchoffs Rule: $V_0 + \xi_{ind} = R\ I$ -> $V_0 = L\ dI/dt + R\ I$

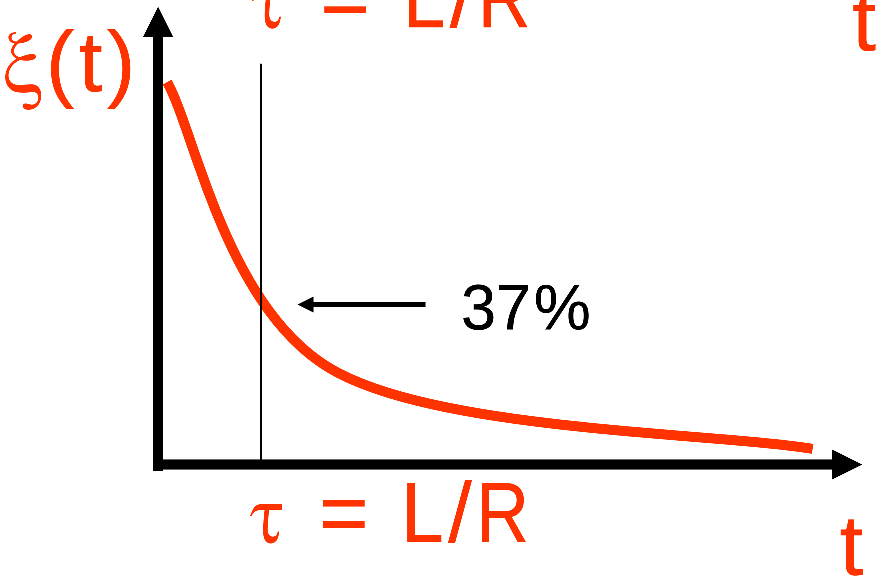
Q: What is $I(t)$? Note: Not an AC circuit

A: $I(t) = V_0/R\ [1 - \exp(-t/\tau)]$ with $\tau = L/R$

RL Circuits



$$I(t) = V_0/R\ [1 - \exp(-t/\tau)]$$

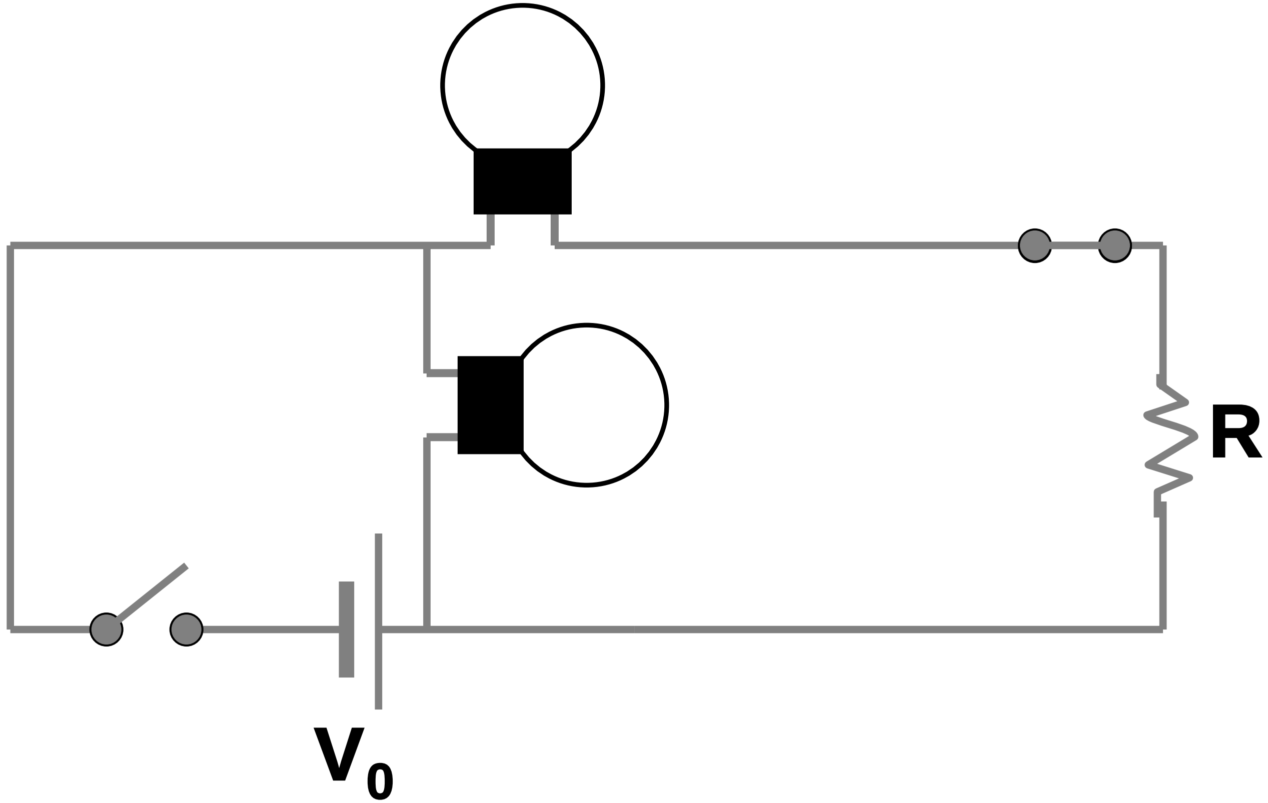


$$\xi(t) = V_0\ \exp(-t/\tau)$$

RL Circuits

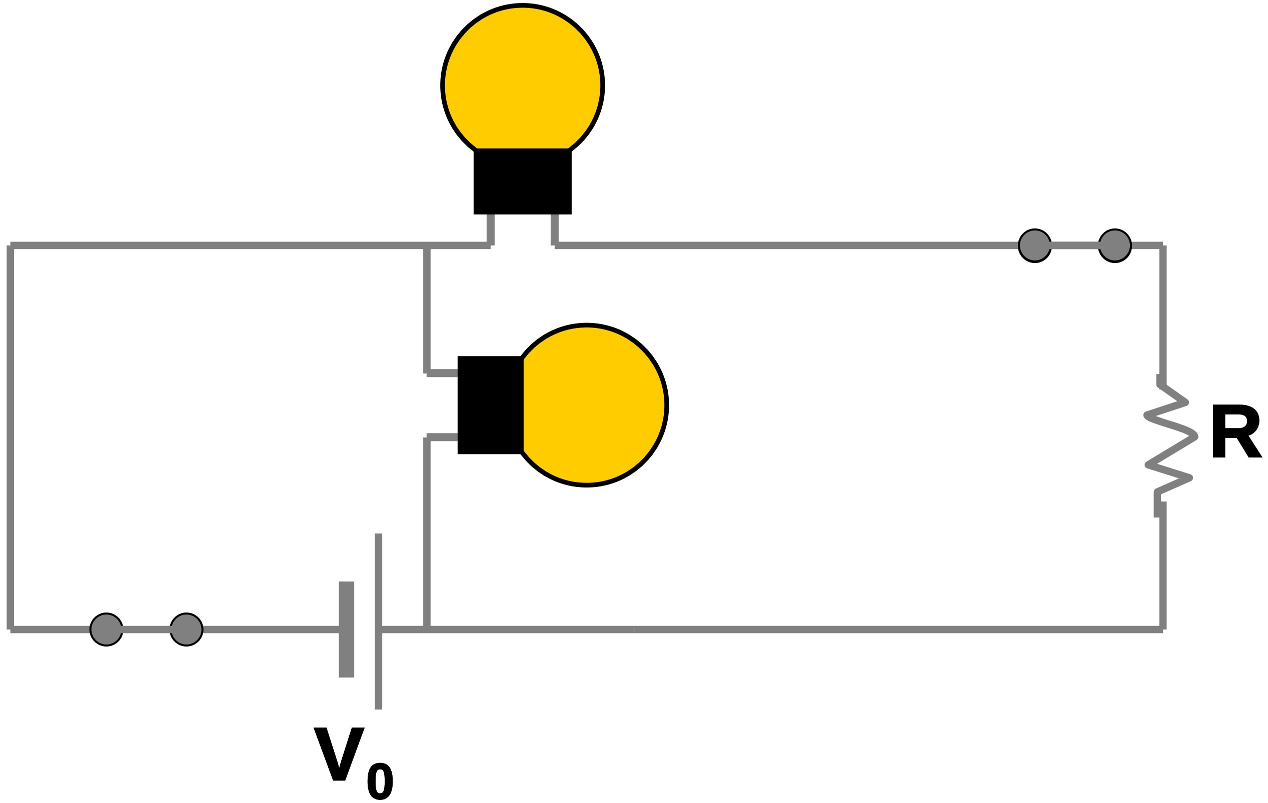
- Inductance leads to ‘delay’ in reaction of current to change of voltage V_0
- All practical circuits have some L and R
  - change in I never instantaneous

In-Class Demo: Large L



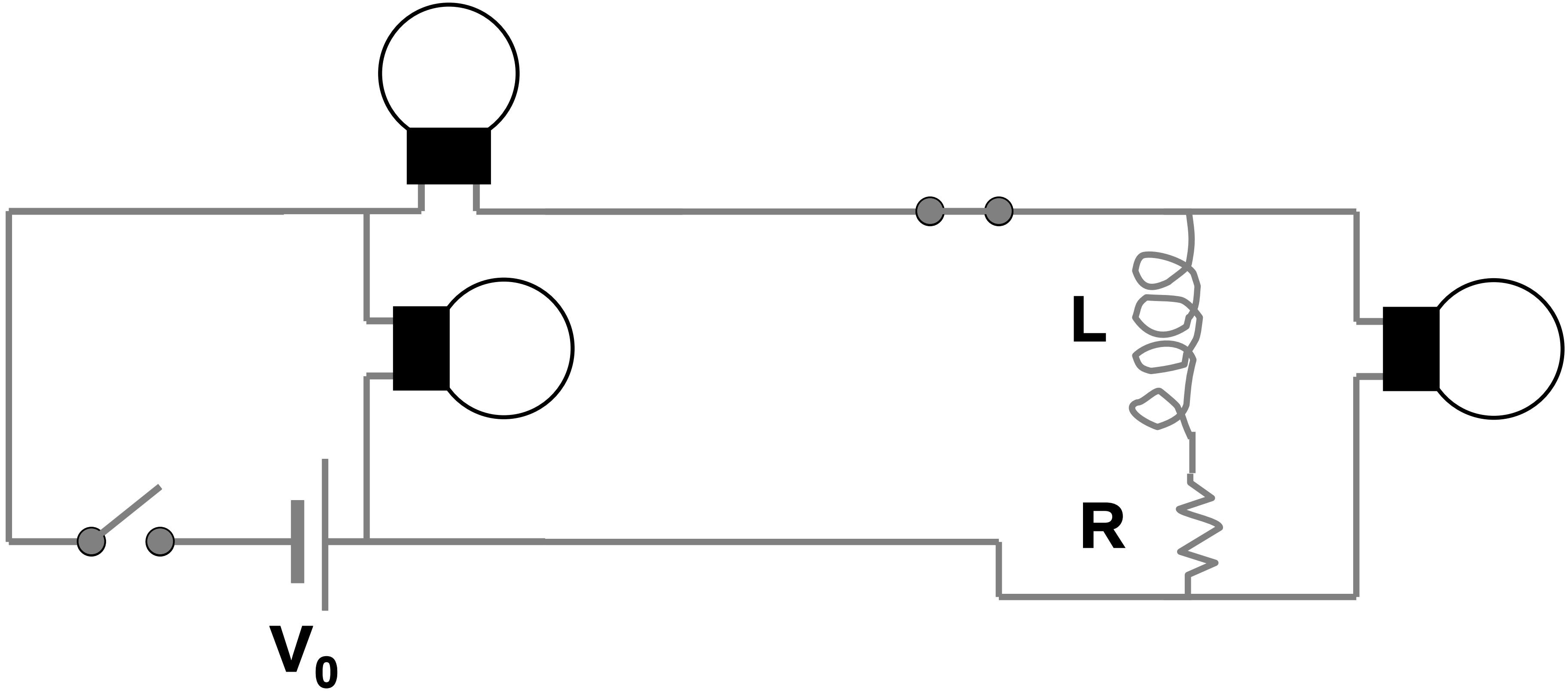
First: No L

In-Class Demo: Large L



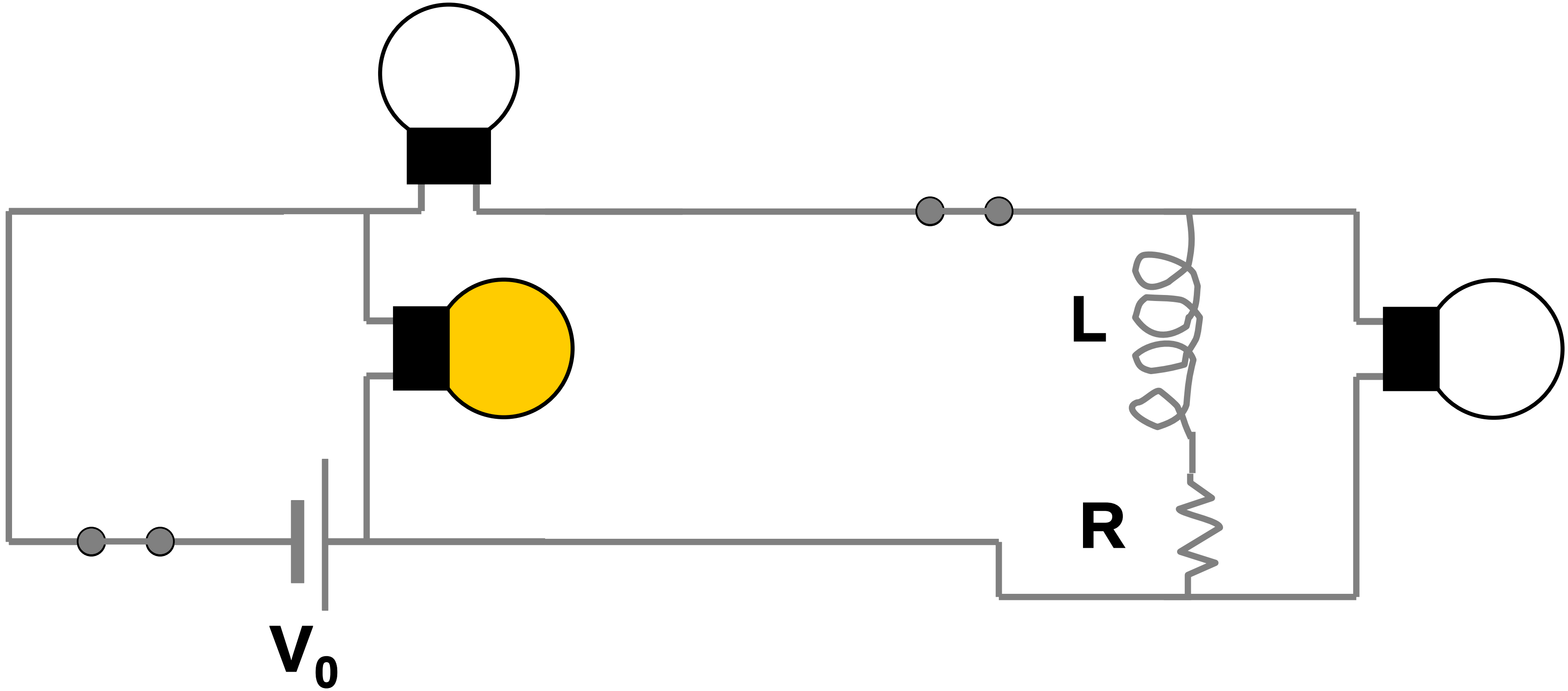
First: No L - no delay between I and V

In-Class Demo: Large L



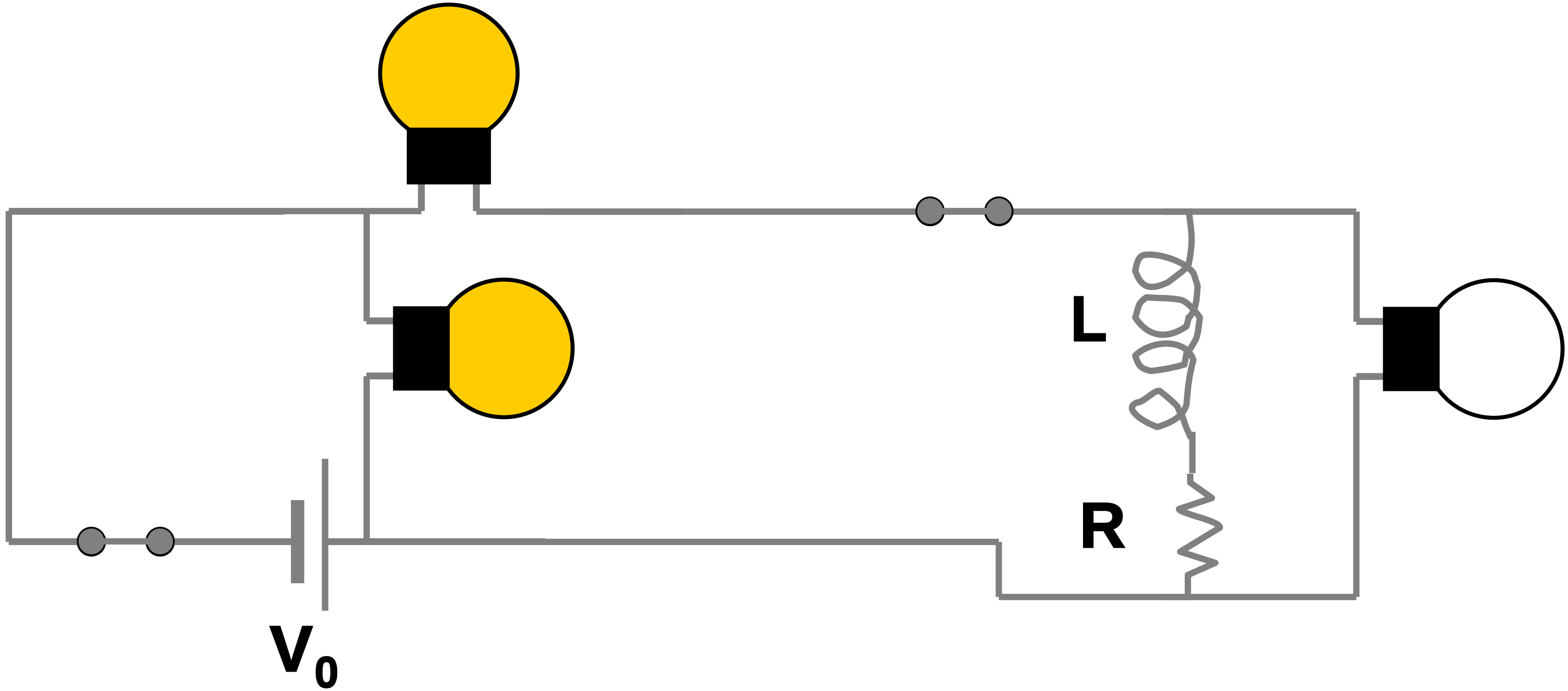
Then: Large L (30H)

In-Class Demo: Large L



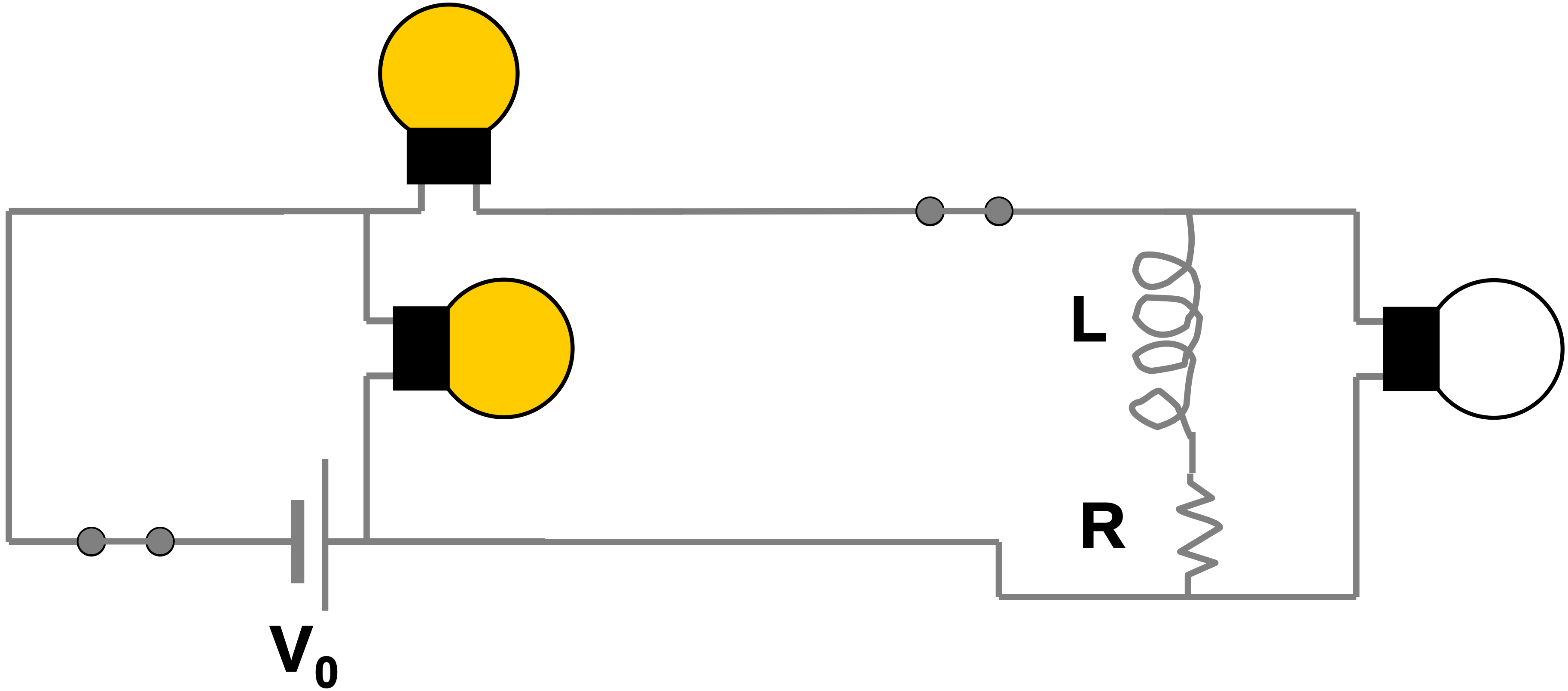
Then: Large L (30H)

In-Class Demo: Large L



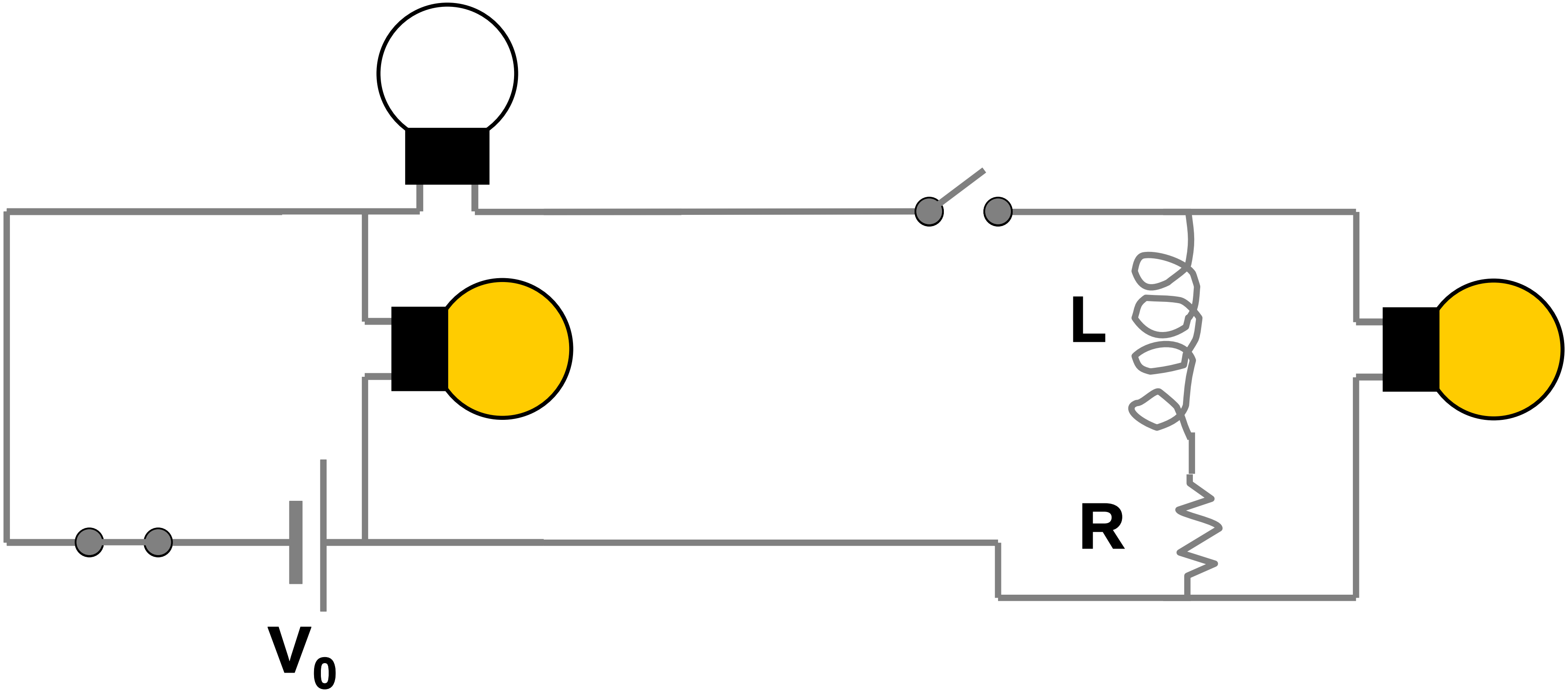
Then: Large L (30H)

In-Class Demo: Large L



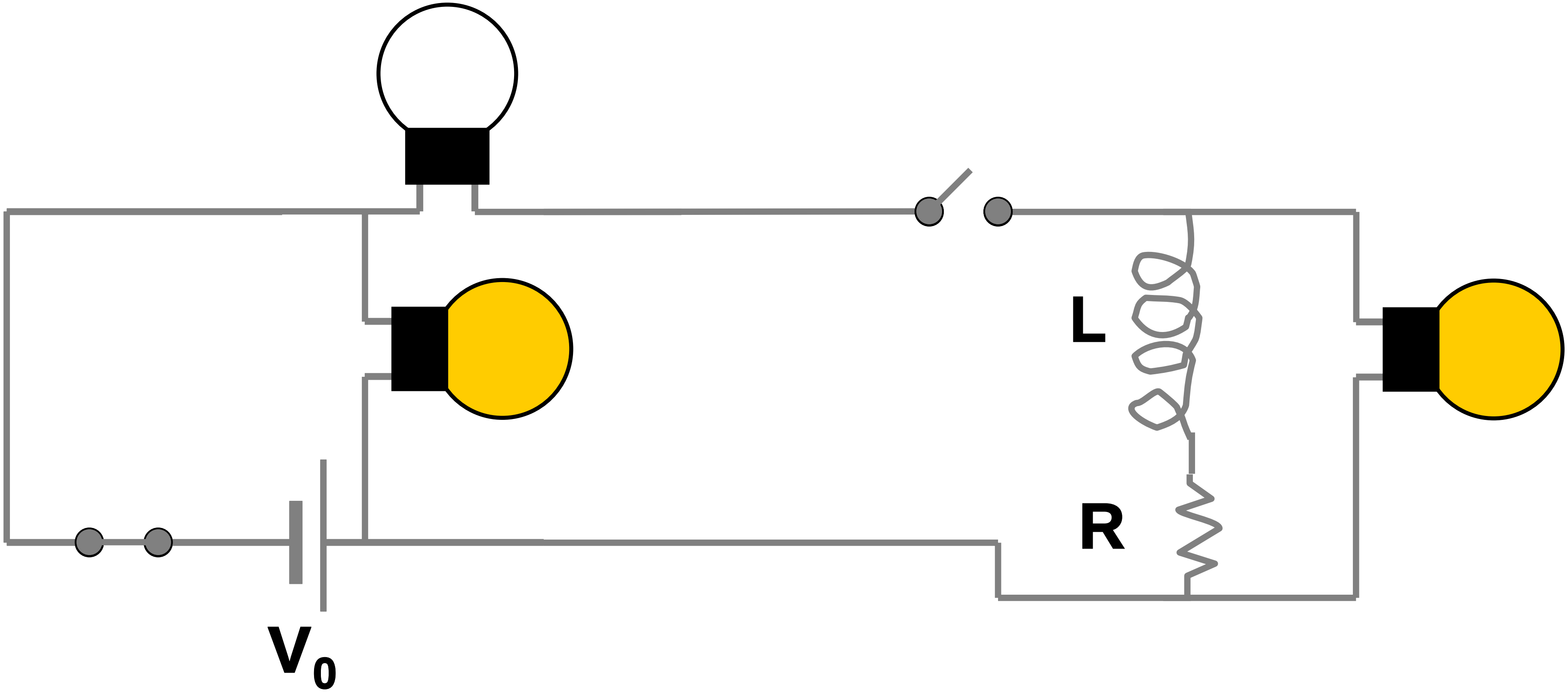
Then: Large L (30H) - Delay in I

In-Class Demo: Large L



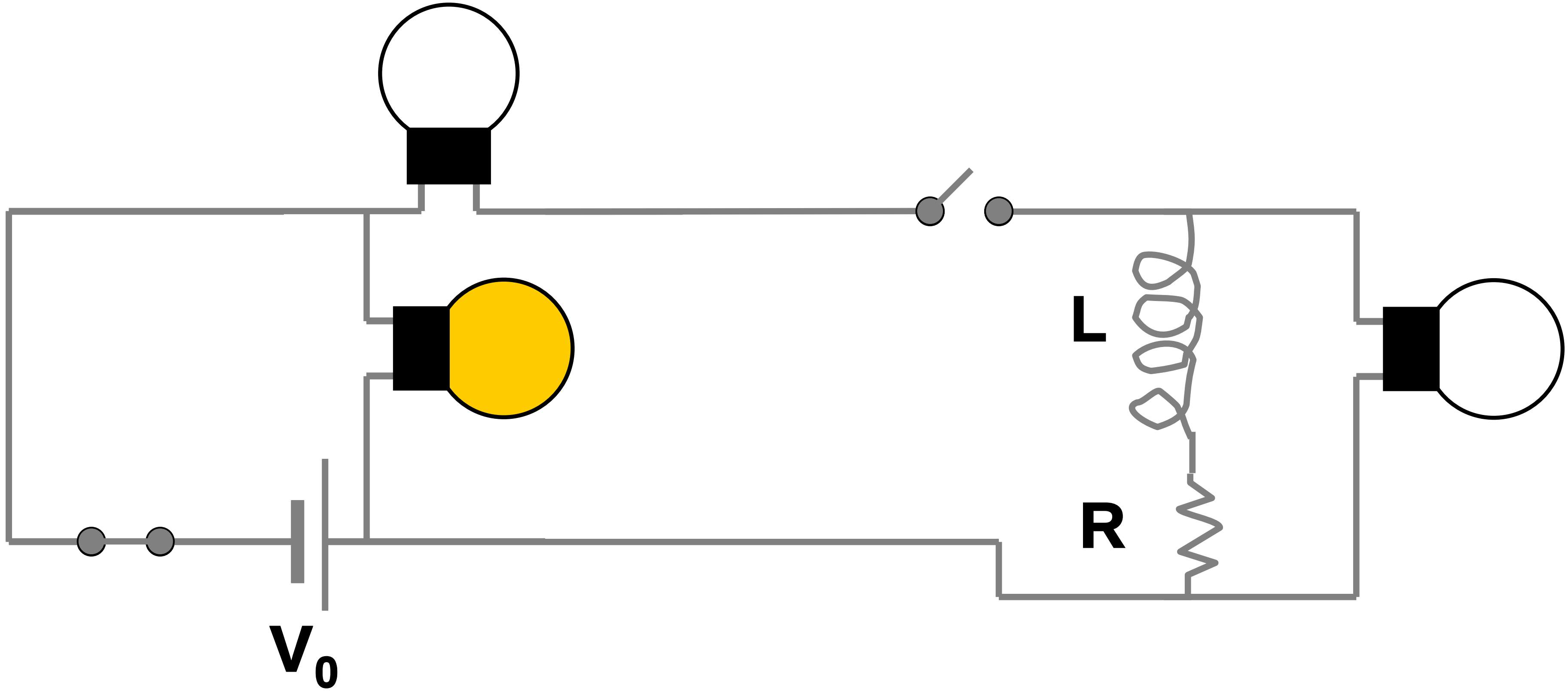
Then: Large L (30H)

In-Class Demo: Large L



Then: Large L (30H)

In-Class Demo: Large L

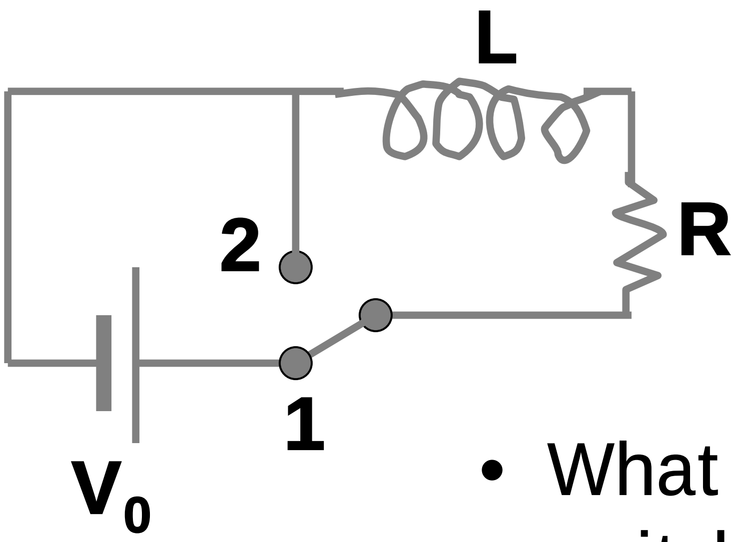


Then: Large L (30H)

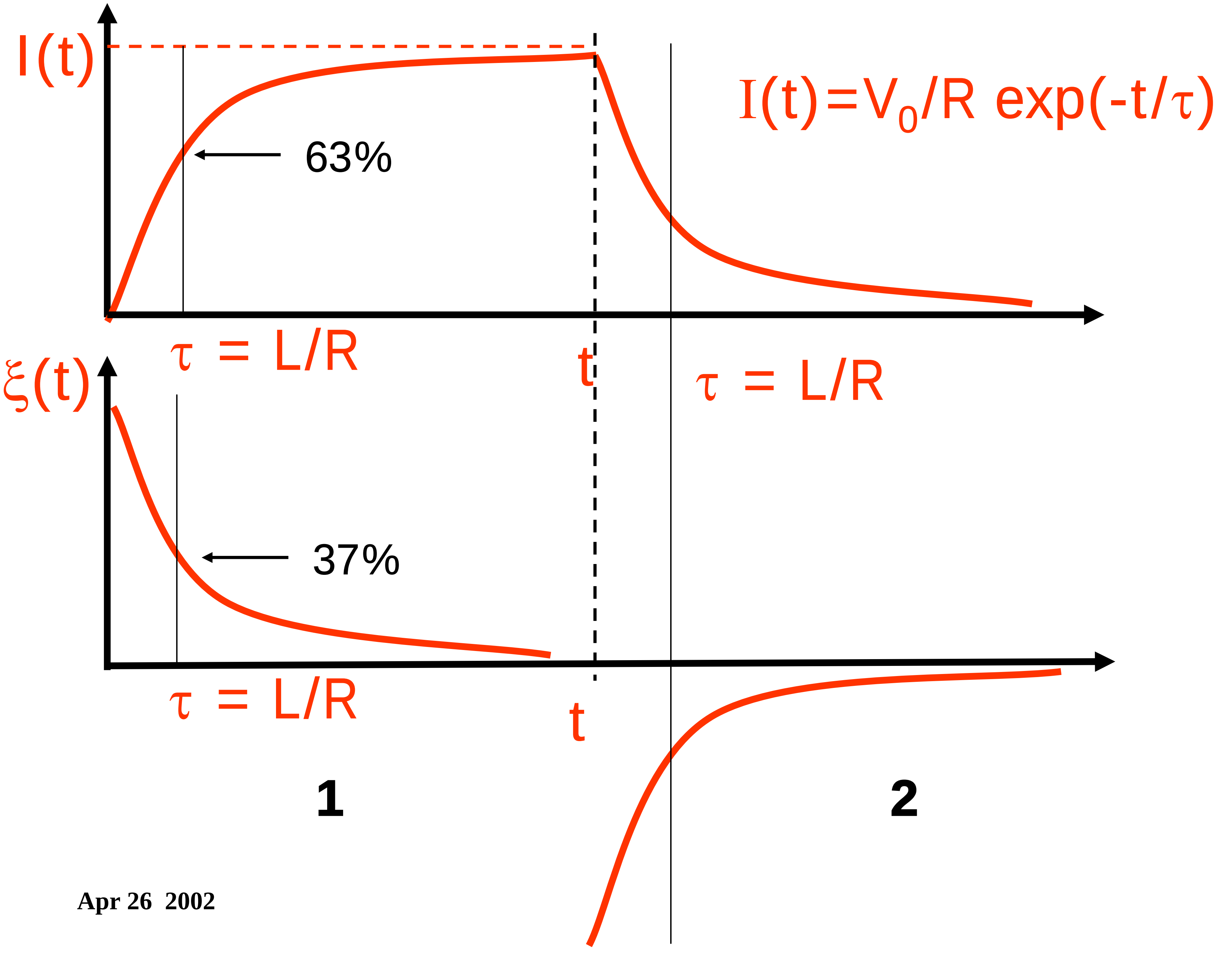
In-Class Demo: Large L

- L counteracts change in current both ways
  - Resists increase in I when closing switch
  - Resists decrease in I when opening switch
  - 'Back EMF'
- That's what causes spark when switching off e.g. appliance, light

‘Back EMF’



- What happens if we move switch to position 2?



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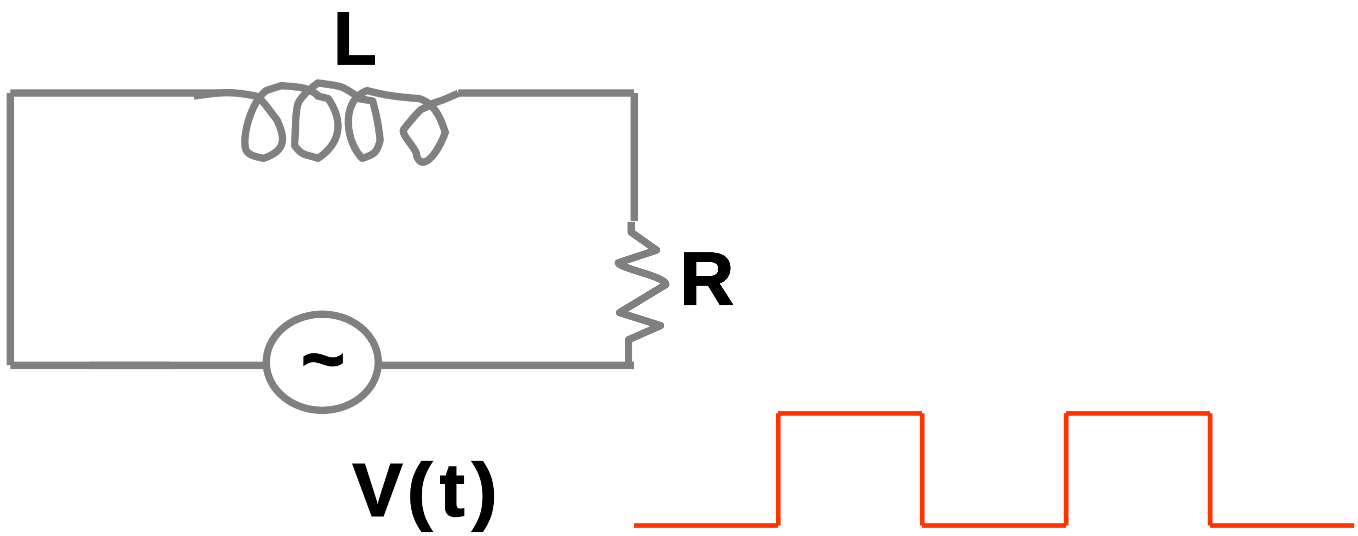
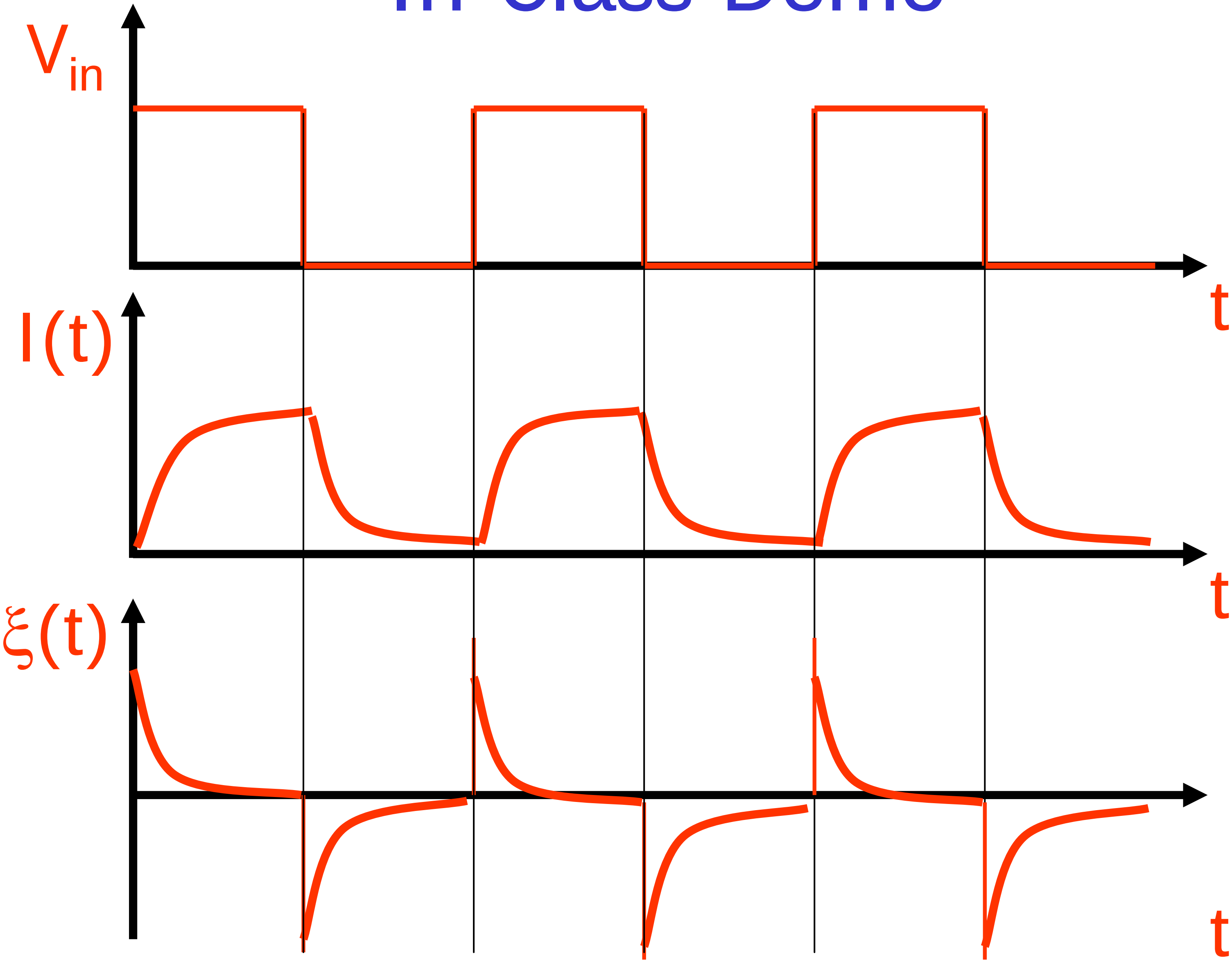
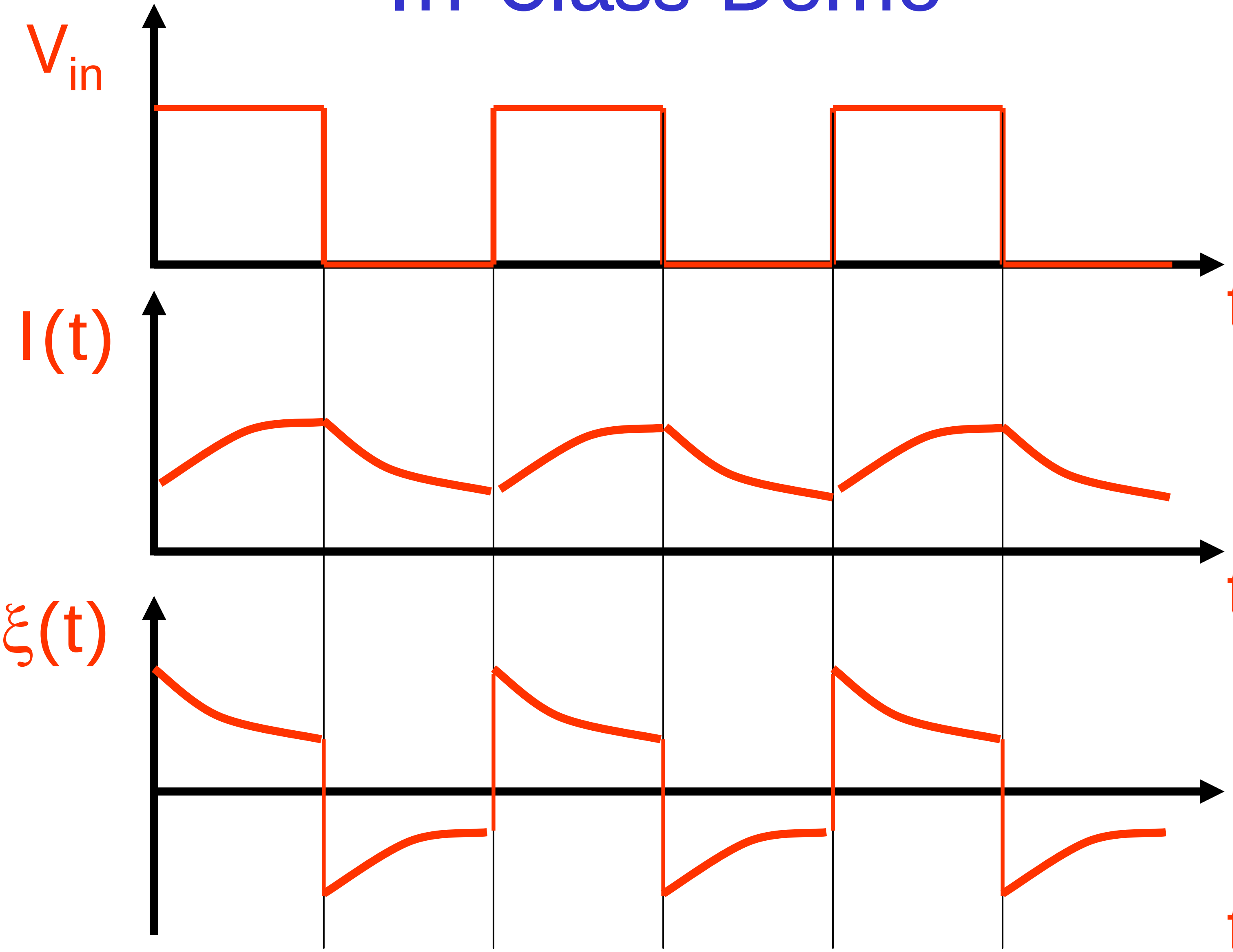
In-Class Demo: Square Wave V_0



In-Class Demo



In-Class Demo



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RL as low-pass filter

- Again, like RC circuits, RL circuits act as low-pass filters
- Sharp edges/high frequencies are removed
  - > In-Class Demo…
- RC circuit: Energy gets stored in C when Voltage switched on, released when Voltage switched off
- Energy storage in RL circuits?

Energy Storage in Inductor

- Power $P = |\xi\ I| = L\ dI/dt\ I = dU/dt$

  -> $dU = L\ dI\ I$

  -> $U = \frac{1}{2} L I^2$

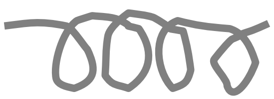
- Where is the Energy stored?

  $U/\text{Volume} = \frac{1}{2} B^2/\mu_0$

Summary of Circuit Components

V $V(t)$

R $V_R = IR$



L $V_L = L\, dI/dt$

C $V_C = 1/C \int I\, dt$