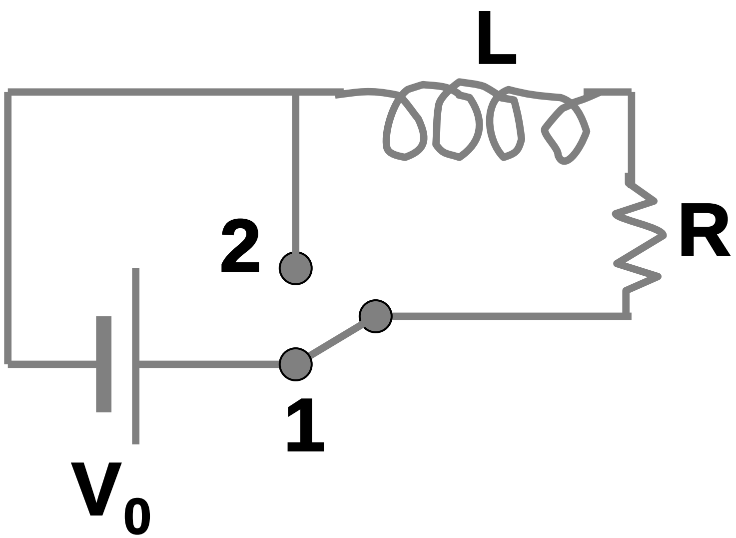
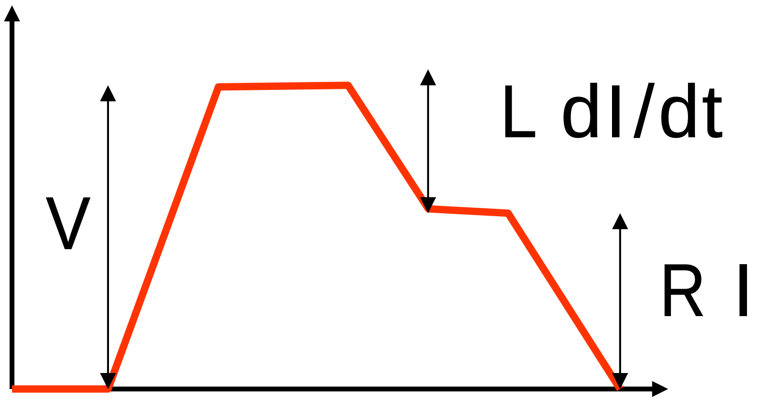
Electricity and Magnetism

- Reminder
  - RL Circuits
  - Energy storage in Inductor
- Today
  - RLC circuits
  - Resonance in RLC AC circuit

RL Circuits

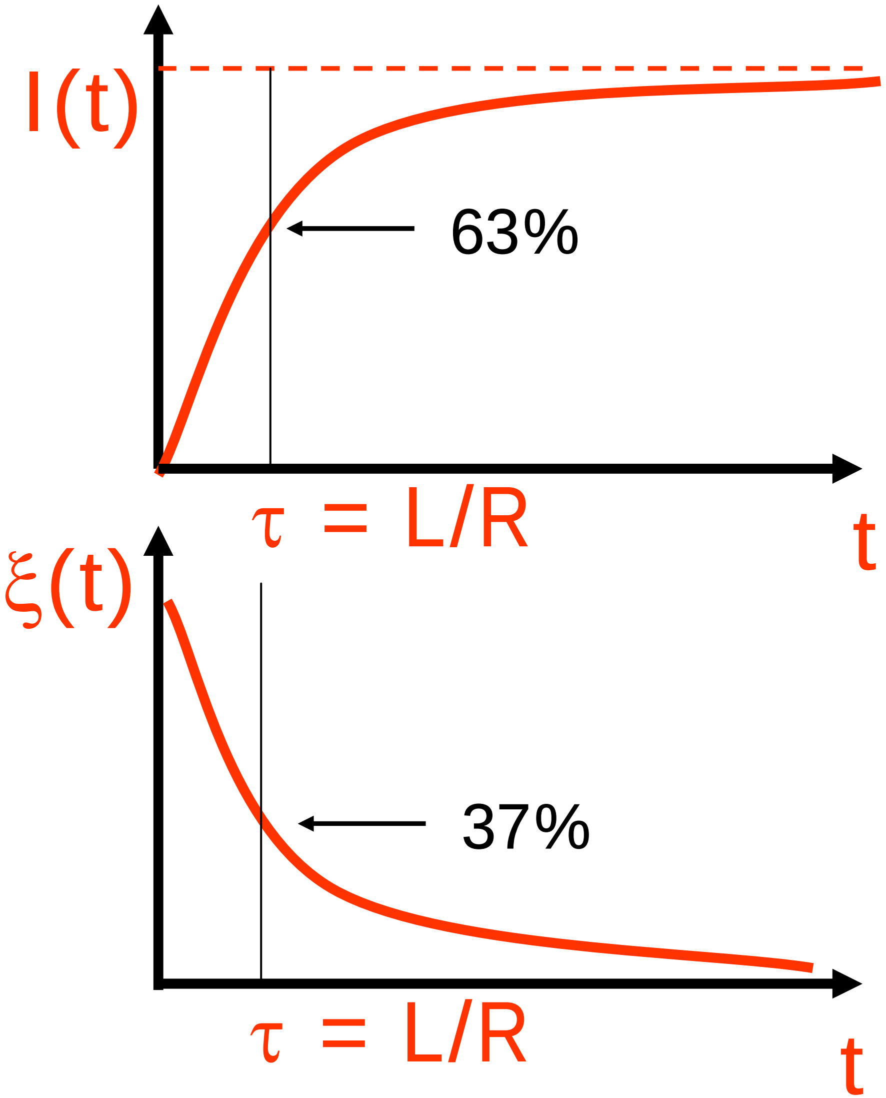




Kirchoffs Rule: $V_0 + \xi_{ind} = R\ I$ -> $V_0 = L\ dI/dt + R\ I$

Q: What is $I(t)$?

RL Circuits



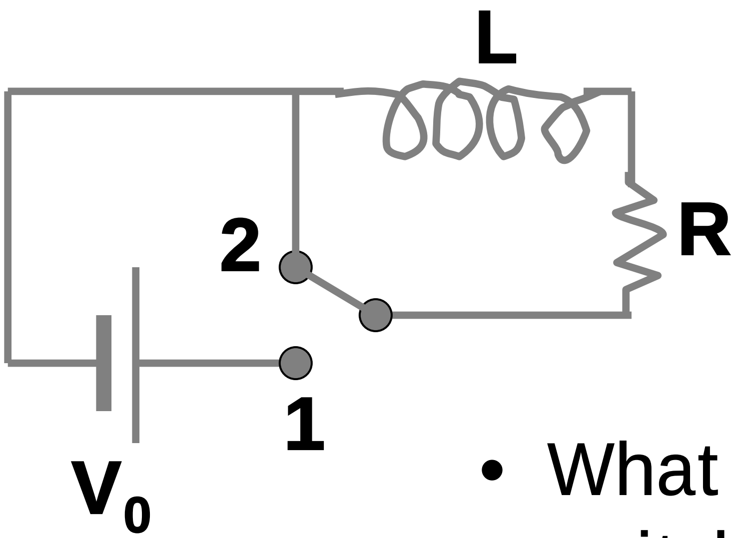
$$I(t)=V_0/R\ [1-\exp(-t/\tau)]$$

$$\xi(t)=V_0\ \exp(-t/\tau)$$

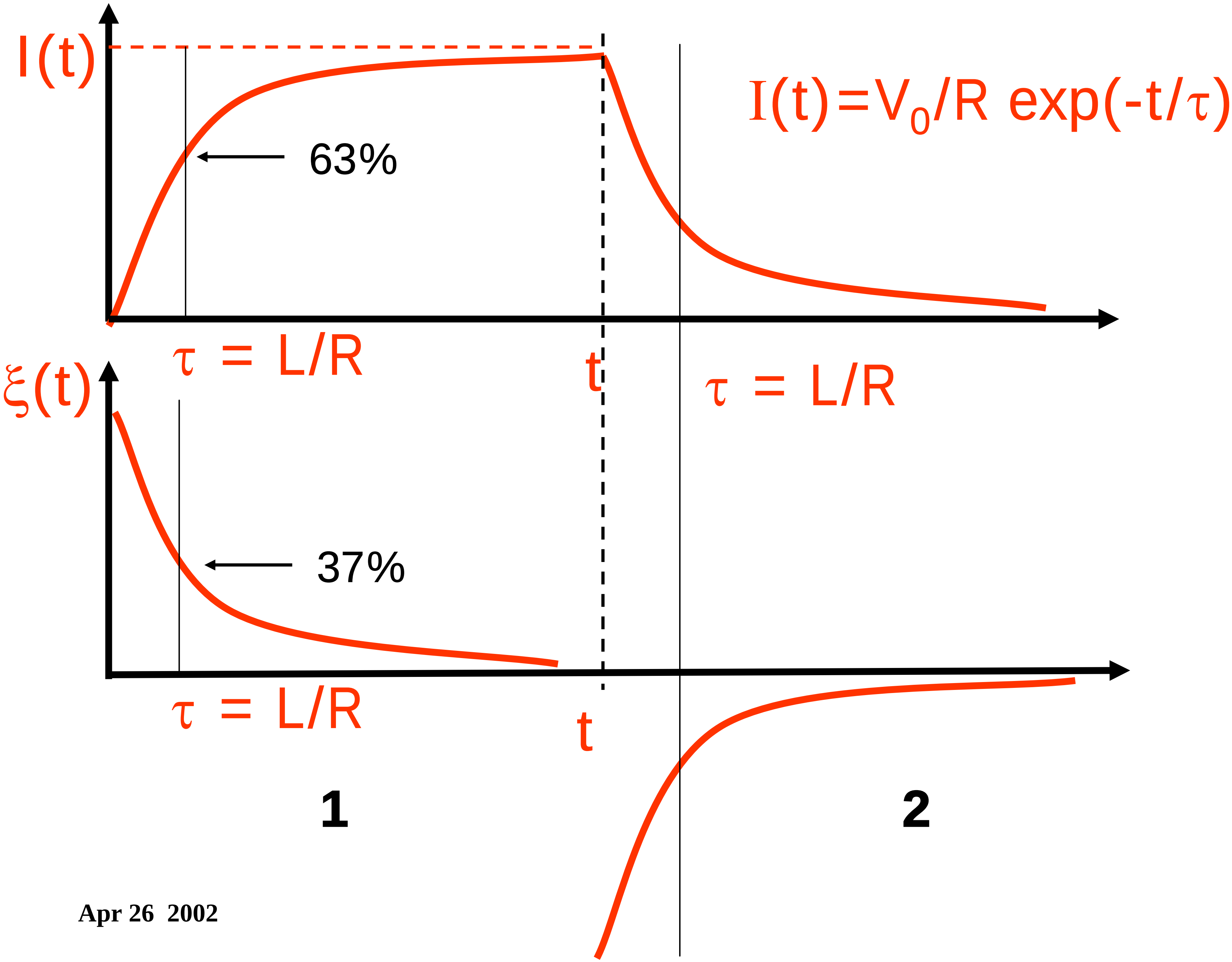
RL Circuits

- Inductance leads to ‘delay’ in reaction of current to change of voltage V_0
- All practical circuits have some L and R
  - change in I never instantaneous

‘Back EMF’



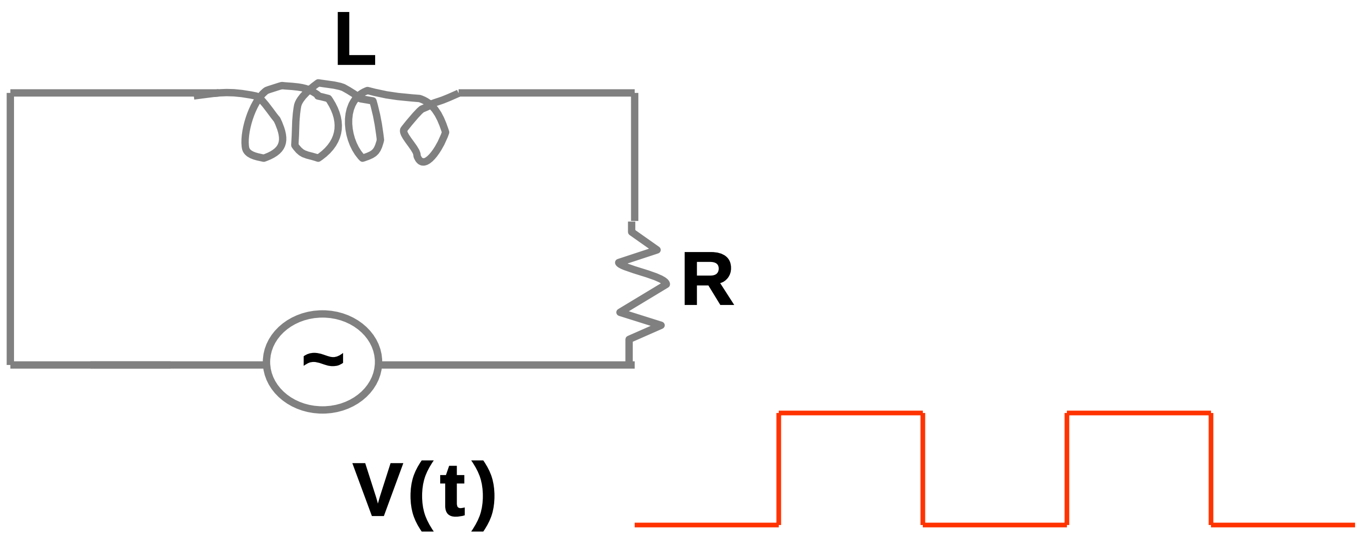
- What happens if we move switch to position 2?


I(t)
63%
τ = L/R
t
τ = L/R
I(t)=V0/R exp(-t/τ)
ξ(t)
37%
τ = L/R
t
1
2

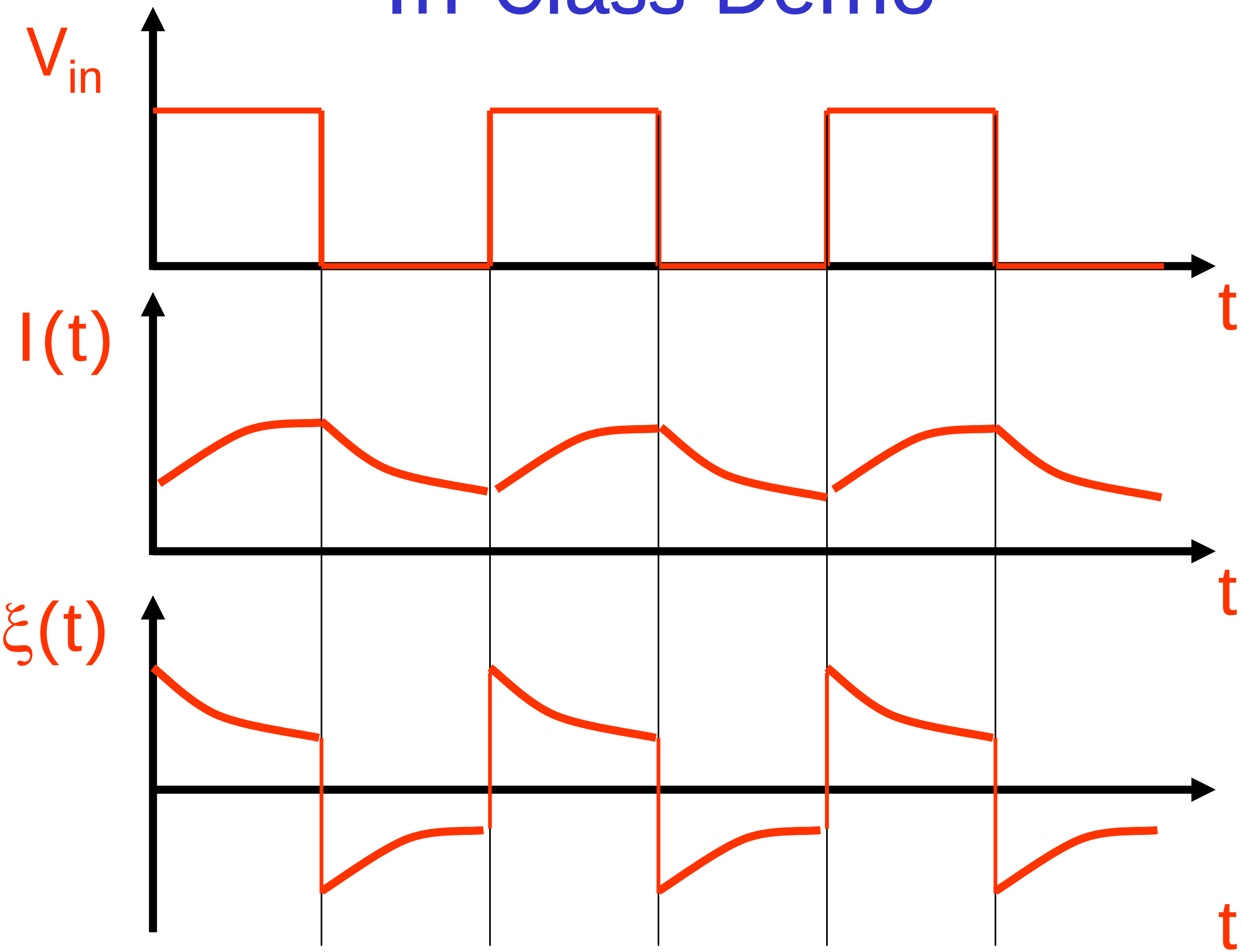
RL Circuits

- L counteracts change in current both ways
  - Resists increase in I when connecting voltage source
  - Resists decrease in I when disconnecting voltage source
  - 'Back EMF'
- That's what causes spark when switching off e.g. appliance, light

In-Class Demo: Square Wave V_0



In-Class Demo



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RL as low-pass filter

- Again, like RC circuits, RL circuits act as low-pass filters
- Sharp edges/high frequencies are removed
  - > In-Class Demo…
- RC circuit: Energy gets stored in C when Voltage switched on, released when Voltage switched off
- Energy storage in RL circuits?

Energy Storage in Inductor

- Energy in Inductor
  - Start with Power $P = \xi I = L\, dI/dt\, I = dU/dt$

  -> $dU = L\, dI\, I$

  -> $U = \frac{1}{2} L I^2$

- Where is the Energy stored?
  - Example: Solenoid

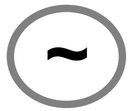
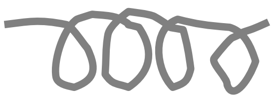
  $U/Volume = \frac{1}{2} B^2/\mu_0$

RLC circuits

- Combine everything we know…

- Resonance Phenomena in RLC circuits
  - Resonance Phenomena known from mechanics (and engineering)
  - Great practical importance
  - video…

Summary of Circuit Components

	V	$V(t)$
	R	$V_R = IR$
	L	$V_L = L\, dI/dt$
	C	$V_C = 1/C \int I\,dt$

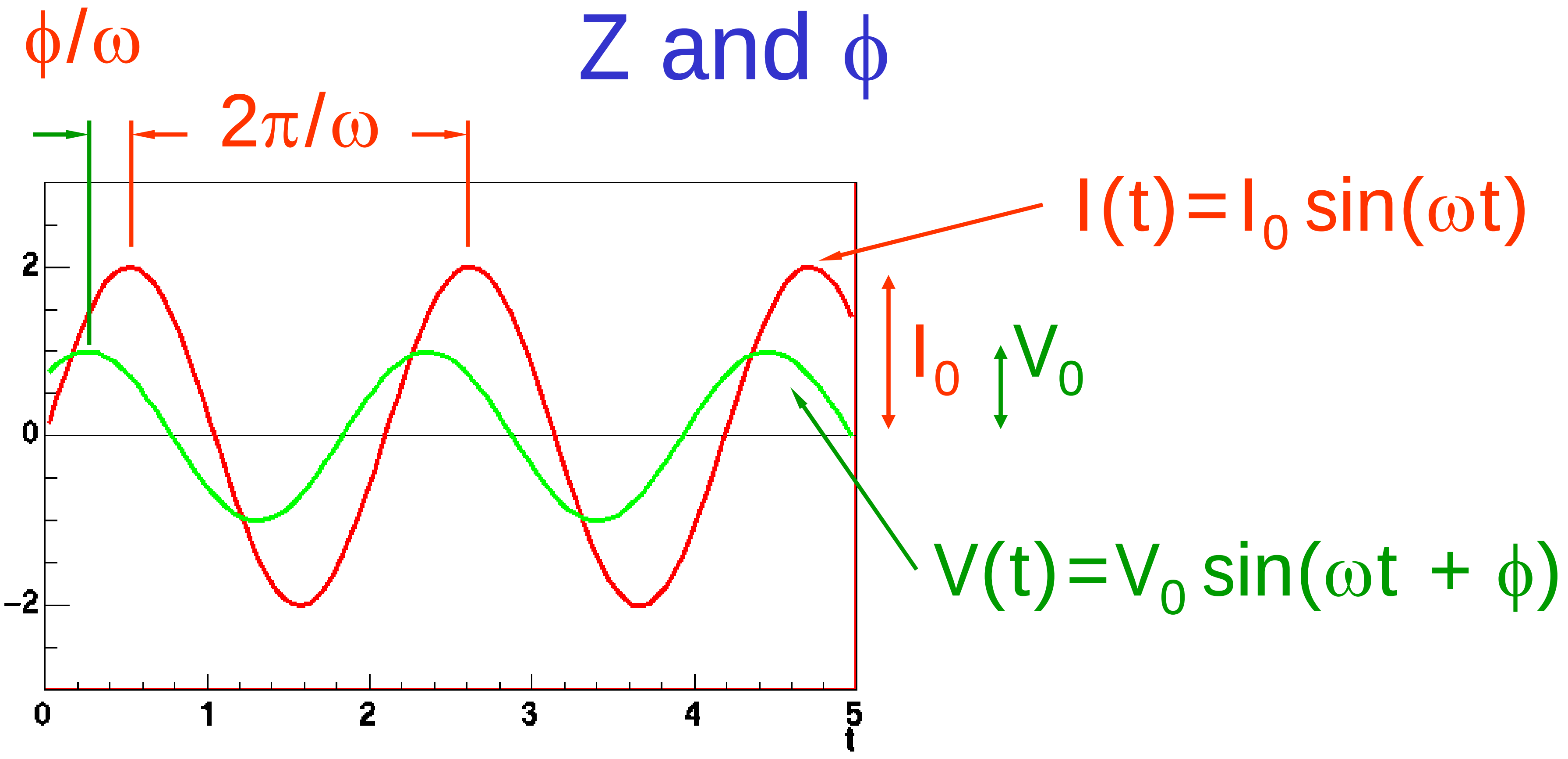
R,L,C in AC circuit

- AC circuit
  - $I(t) = I_0 \sin(\omega t)$
  - $V(t) = V_0 \sin(\omega t + \phi)$

  } same ω!

- Relationship between V and I can be characterized by two quantities
  - Impedance $Z = V_0/I_0$
  - Phase-shift ϕ

Z and ϕ

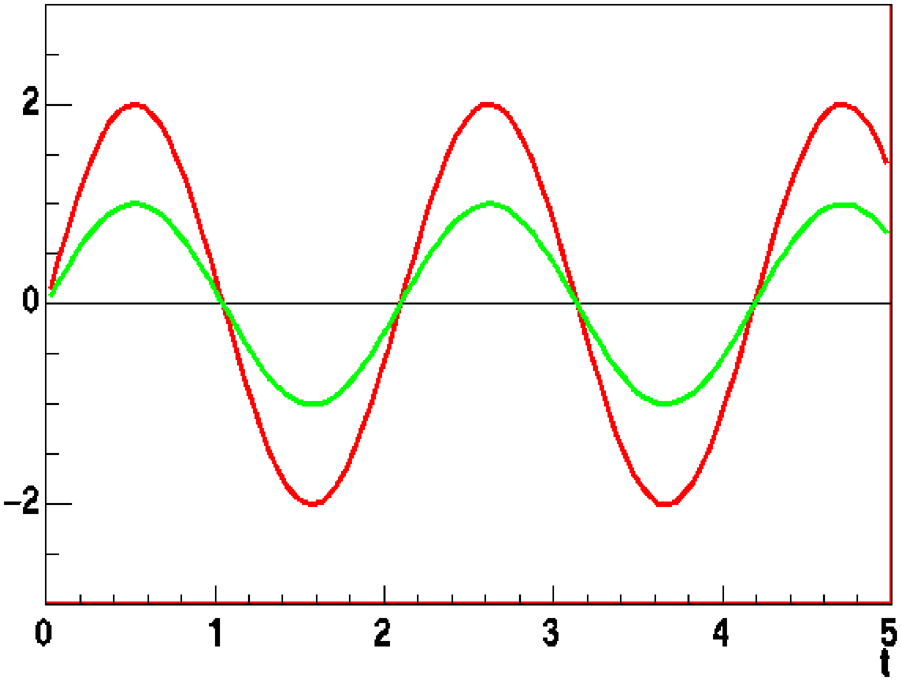


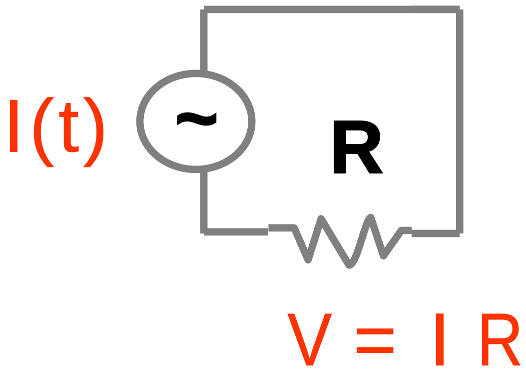
Impedance $Z = V_0/I_0$

Z and ϕ

- First look at impedance and phase-shift for circuits containing only R,C or L
- Then RLC circuit…

Z and ϕ: Capacitance C

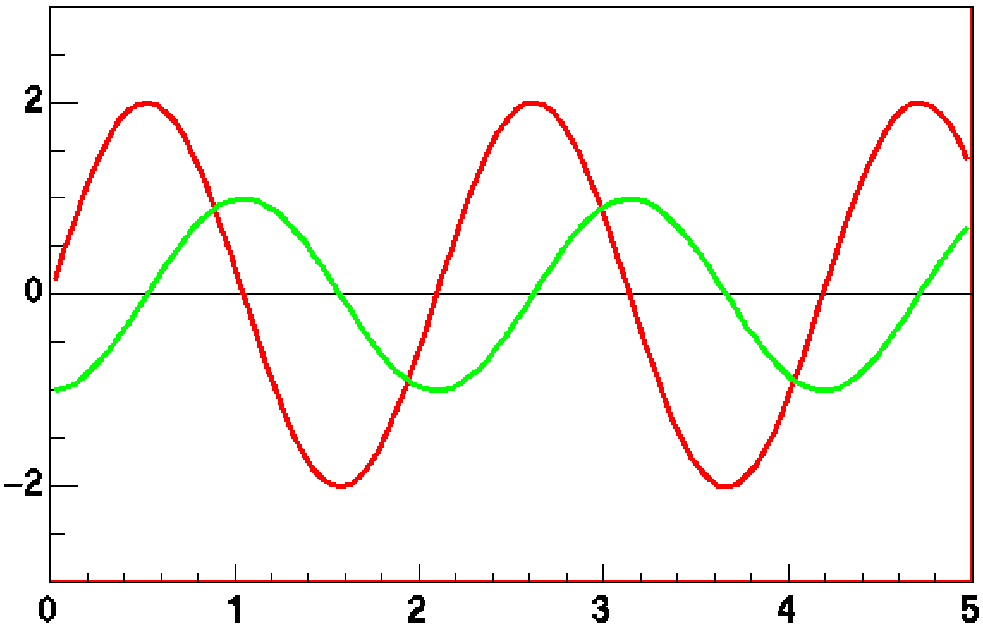


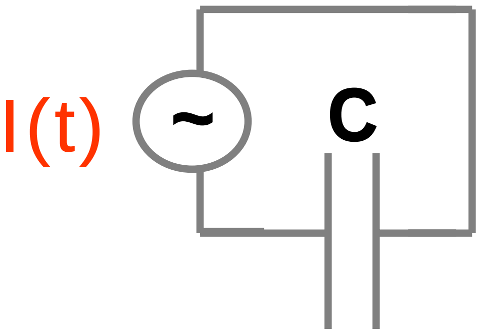


Impedance Z = V / I = R

Phase-shift ϕ = 0

Z and ϕ: Capacitance C



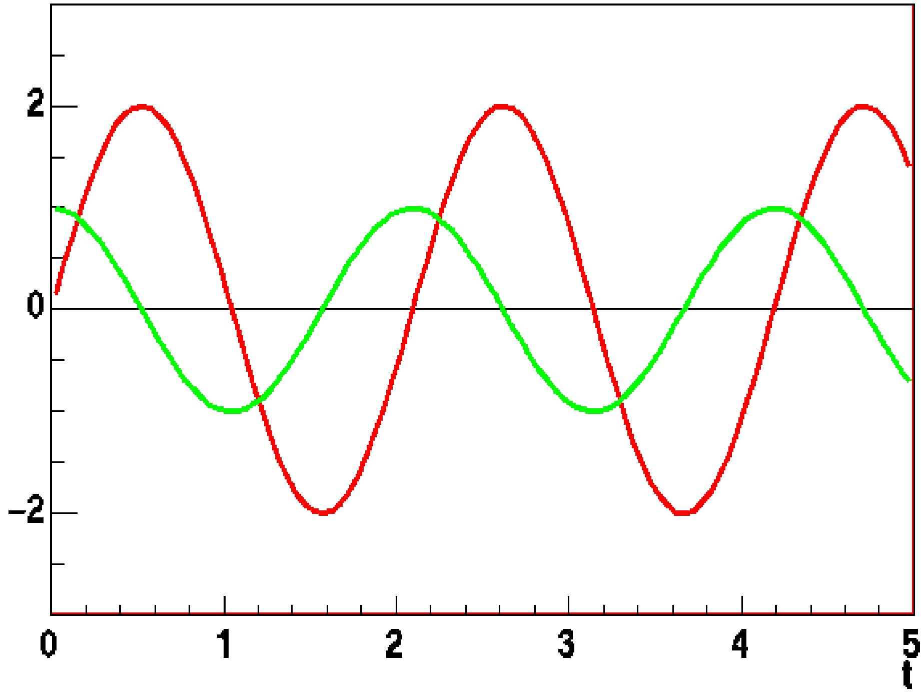


$V = Q/C = 1/C \int I dt$

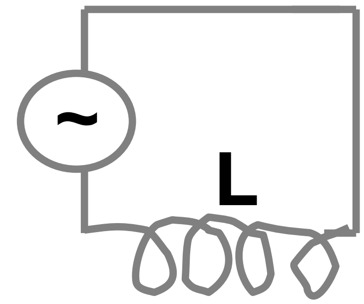
Impedance $Z = 1/(\omega C)$

Phase-shift $\phi = -\pi/2$ *V lags I by 90°*

Z and ϕ: Inductance L



I(t)

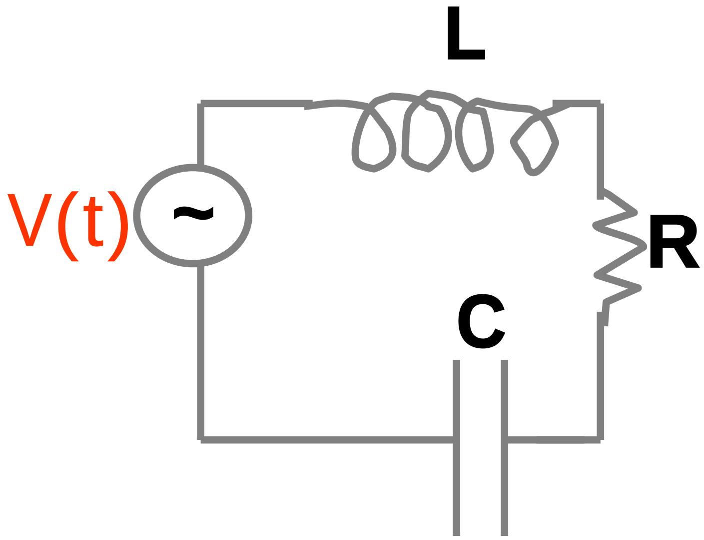


V = L dI/dt

Impedance Z = ω L

Phase-shift ϕ = $\pi/2$ *I lags V by 90°*

RLC circuit

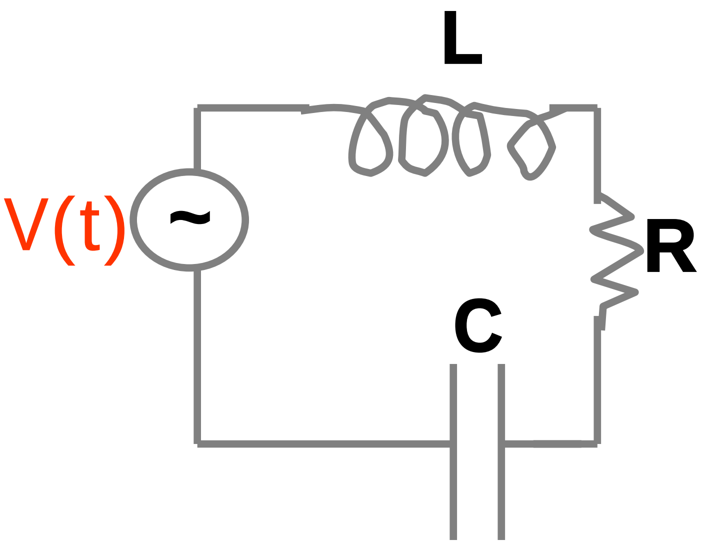


$$V - L\,dI/dt - IR - Q/C = 0$$

$$L\,d^2Q/dt^2 = -1/C\,Q - R\,dQ/dt + V$$

2nd order differential equation

RLC circuit

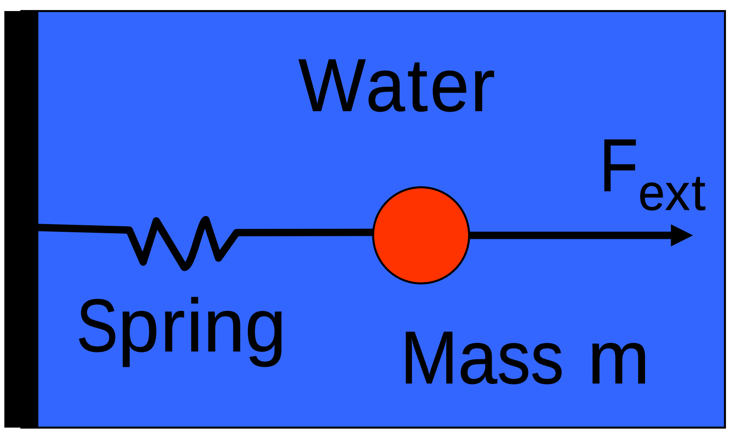


$V - L\, dI/dt - IR - Q/C = 0$

$L\, d^2Q/dt^2 = -1/C\, Q - R\, dQ/dt + V$

'Inertia' 'Spring' 'Drag'

$m\, d^2x/dt^2 = -k\, x - f\, dx/dt + F_{ext}$



RLC circuit

- Solve $L\, d^2Q/dt^2 = -1/C\, Q - R\, dQ/dt + V$
  - for AC circuit: $V = V_0 \sin(\omega t)$, $I = I_0 \sin(\omega t - \phi)$
- If $I = I_0 \sin(\omega t - \phi)$ then
  - $Q(t) = - I_0 / \omega \cos(\omega t - \phi)$
  - $dQ/dt = I_0 \sin(\omega t - \phi)$
  - $d^2Q/dt^2 = I_0\, \omega \cos(\omega t - \phi)$

RLC circuit

$V_0 \sin(\omega t) = I_0\{[\omega L - 1/(\omega C)] \cos(\omega t - \phi) + R \sin(\omega t - \phi)\}$

Solution (requires two tricks):

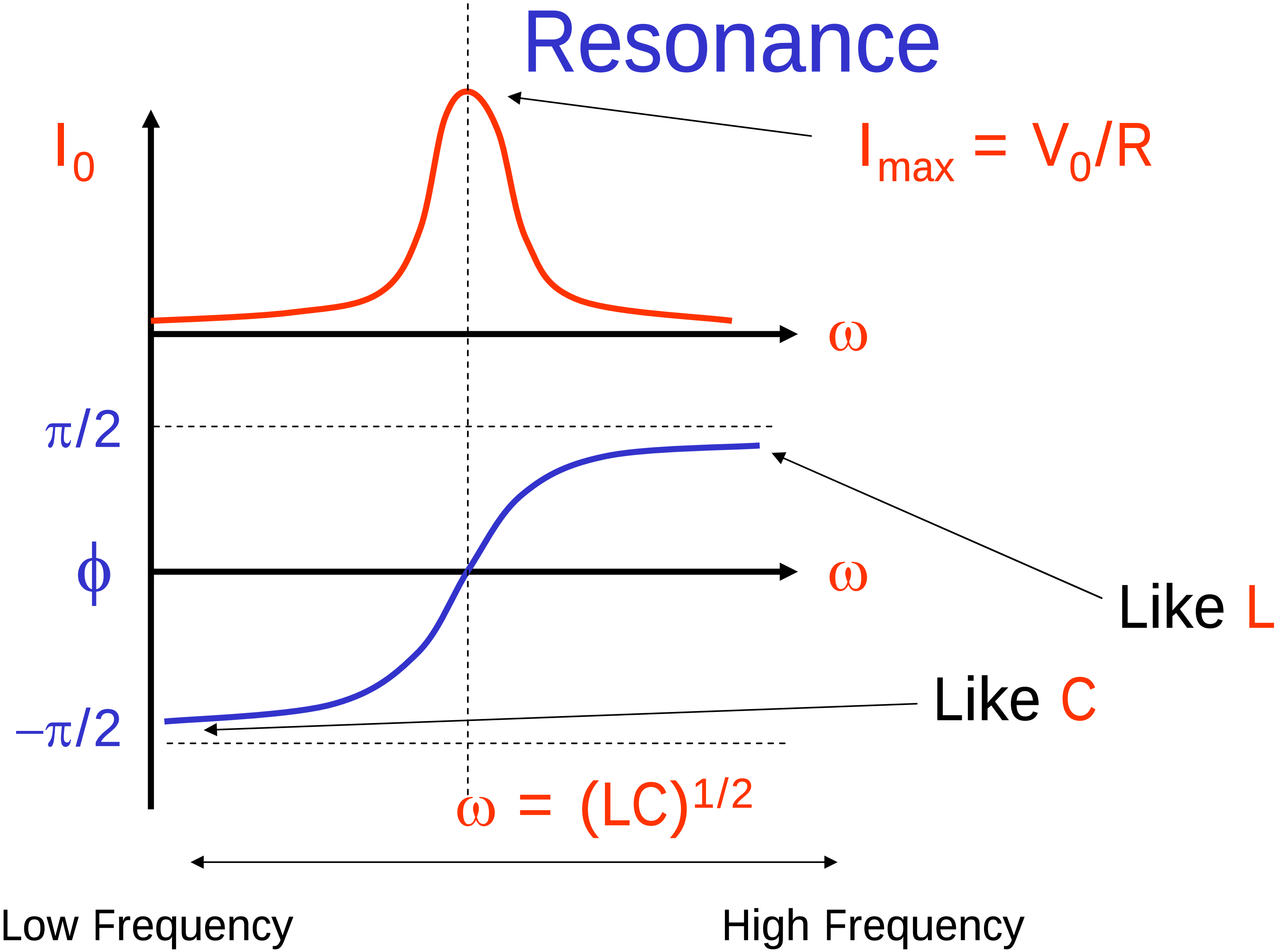
$I_0 = V_0/([\omega L - 1/(\omega C)]^2 + R^2)^{1/2} = V_0/Z$

$\tan(\phi) = [\omega L - 1/(\omega C)]/R$

-> For $\omega L = 1/(\omega C)$, Z is minimal and $\phi = 0$

i.e. $\omega = 1/(LC)^{1/2}$ <u>Resonance Frequency</u>

Resonance



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In-Class Demo (on scope)