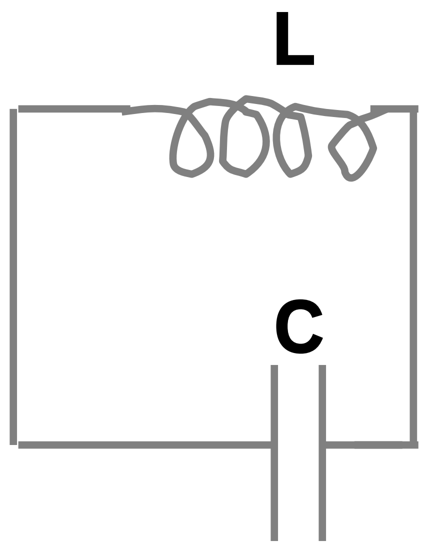
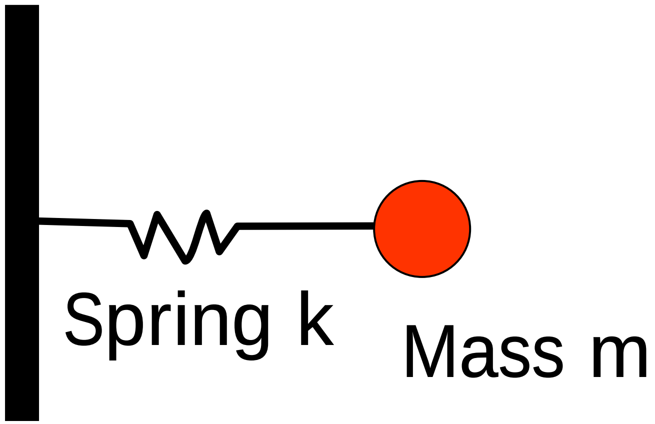
Electricity and Magnetism

- Reminder
  - LC circuits / Oscillations
  - Displacement current
  - Maxwell's equations
- Today
  - More on Maxwell's equations
  - Electromagnetic waves

LC Circuit





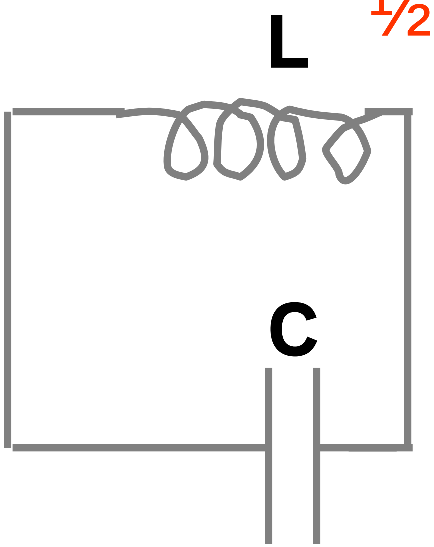
$d^2Q/dt^2 + 1/(LC)\, Q = 0$

$\omega_0^2 = 1/(LC)$

$d^2x/dt^2 + k/m\, x = 0$

$\omega_0^2 = k/m$

LC Circuit



$\frac{1}{2}\ L\ I^2$

$\frac{1}{2}\ Q^2/C$

Energy in E-Field

↕ Oscillation

Energy in B-Field

- Total energy U(t) is conserved:

$Q(t) \sim \cos(\omega t)$

$dQ/dt \sim \sin(\omega t)$

$U_L \sim (dQ/dt)^2 \sim \sin^2$

$U_C \sim Q(t)^2 \sim \cos^2$

$\cos^2(\omega t) + \sin^2(\omega t) = 1$

Electromagnetic Oscillations

- In an LC circuit, we see oscillations:

Energy in E-Field

$\updownarrow$

Energy in B-Field

- Q: Can we get oscillations without circuit, i.e. when we have just the fields?
- A: Yes!
  - **Electromagnetic Waves**

Maxwell's Equations (almost)

$$\oint_{A_{closed}} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Charges are the source of Electric Flux through close surface

$$\xi = \oint_{L_{closed}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Changing magnetic field creates an electric field

$$\oint_{A_{closed}} \vec{B} \cdot d\vec{A} = 0$$

There are no magnetic monopoles

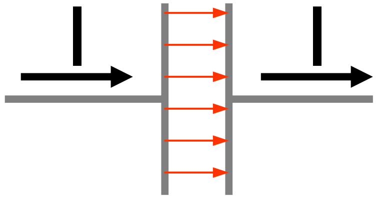
$$\oint_{L_{closed}} \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Moving charges create magnetic field

- Connection between electric and magnetic phenomena
- But not symmetric
- -> James Clerk Maxwell (~1860)

Displacement Current

- Ampere's Law broken – How can we fix it?



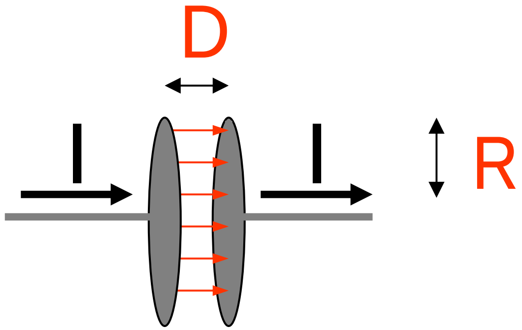
$Q = C V$

Displacement Current $I_D = \varepsilon_0 \, d\Phi_E/dt$

Changing field inside C also produces B-Field!

Displacement Current

- Example calculation: B(r) for r > R



Q = C V

-> $B(r) = R^2/(2rc^2)\ dV/dt$

Maxwell's Equations

$$\oint_{A_{closed}} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint_{L_{closed}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint_{A_{closed}} \vec{B} \cdot d\vec{A} = 0$$

$$\oint_{L_{closed}} \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$1/c^2$

- M.E.'s ***predict*** electromagnetic waves, moving with speed of light
- Major triumph of science

Electromagnetic Waves

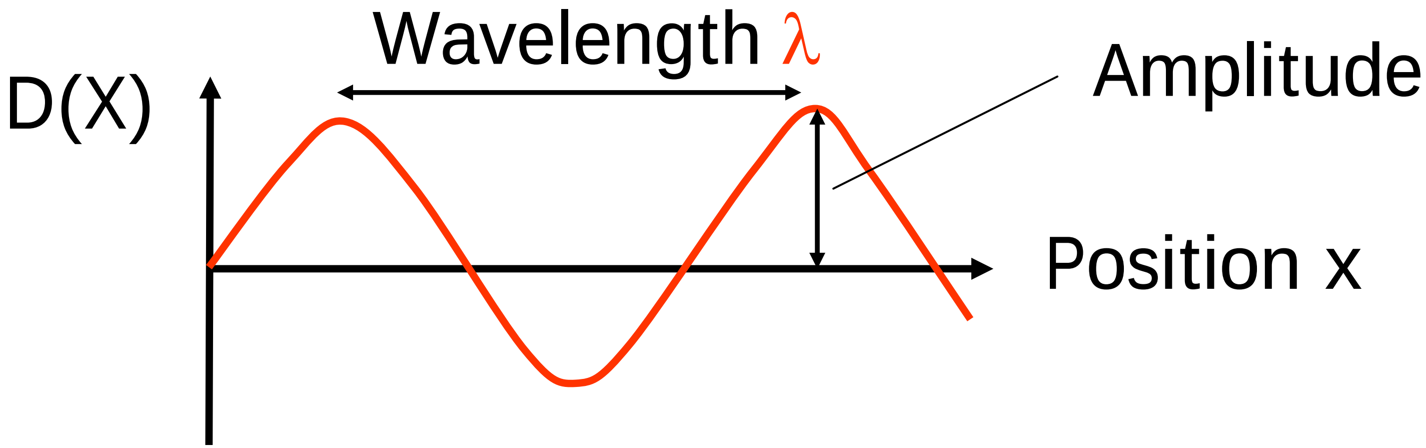
- Until end of semester:
  - What are electromagnetic waves?
  - How does their existence follow from Maxwells equations?
  - What are the properties of E.M. waves?
- Prediction was far from obvious
  - No hint that E.M. waves exist
  - Involves quite a bit of math

Reminder on waves

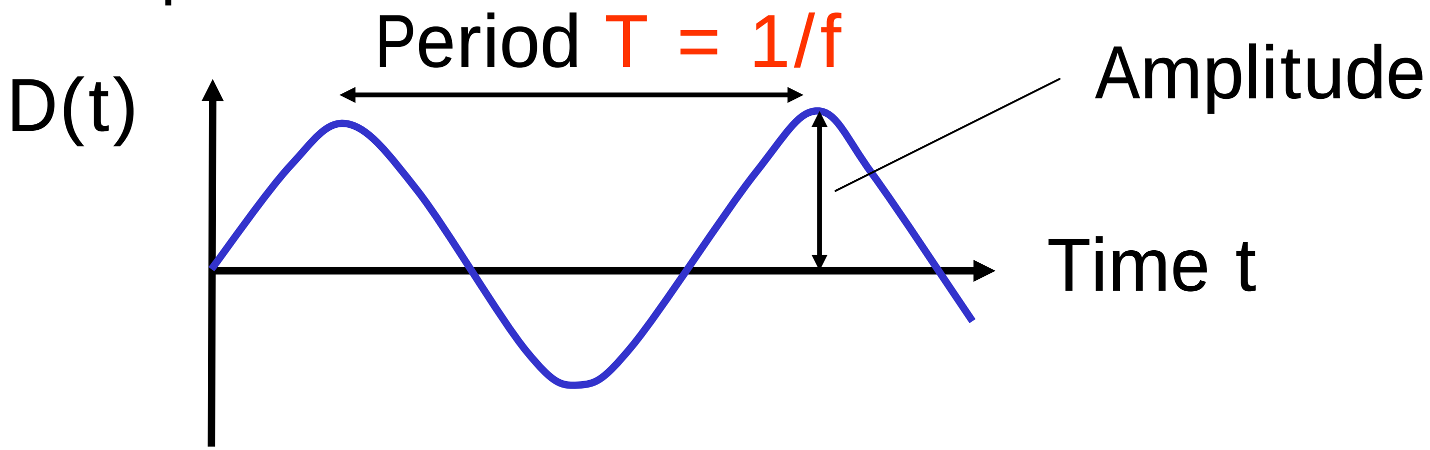
- Examples of waves
  - Mechanical waves
  - Pressure waves
  - E.M. waves
- In-Class Demo…

Reminder on waves

At a moment in time:



At a point in space:



Reminder on waves

- Types of waves
  - Transverse
  - Longitudinal
    - compression/decompression

Reminder on waves

- For a travelling wave (sound, water)

  Q: What is actually moving?

- **-> Energy!**
- Speed of propagation: $v = \lambda f$
- Wave equation:

$$\frac{\partial^2 D(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D(x,t)}{\partial t^2}$$

Couples variation in time and space

Electromagnetic Waves

- Is light an electromagnetic wave?
  - Check speed and see if we can predict that

Back to Maxwell's equation

- Wave equation is differential equation
- M.E. (so far) describe integrals of fields

$$\oint_{A_{closed}} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint_{L_{closed}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint_{A_{closed}} \vec{B} \cdot d\vec{A} = 0$$

$$\oint_{L_{closed}} \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\,\epsilon_0 \frac{d\Phi_E}{dt}$$

Transform into differential eqn's

Differential Form of M.E.

- Need two theorems: Gauss and Stokes
  - Gauss

Flux/Unit Volume

$$\int_{V(A)} \vec{\nabla} \cdot \vec{F} dV = \oint_A \vec{F} \cdot d\vec{A} = \Phi_F$$

Divergence

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Differential Form of M.E.

- Need two theorems: Gauss and Stokes
  - Stokes

Loop Integral/Unit Area of Loop

$$\oint_L \vec{F} d\vec{l} = \int_{A(L)} \vec{\nabla} \times \vec{F} d\vec{A}$$

Curl

$$\vec{\nabla} \times \vec{F} = \vec{i}(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}) + \vec{j}(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}) + \vec{k}(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y})$$

Differential Form of M.E.

$$\oint_{A_{closed}} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint_{L_{closed}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint_{A_{closed}} \vec{B} \cdot d\vec{A} = 0$$

$$\oint_{L_{closed}} \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \ \epsilon_0 \frac{d\Phi_E}{dt}$$

Gauss, Stokes

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \ \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

Differential Form of M.E.

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

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$$\vec{\nabla} \times \vec{B} = \mu_0 \, \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

In Vacuum

$\longrightarrow$

no charge

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$